# UNIFYING VARIATIONAL APPROACH AND REGION GROWING SEGMENTATION

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## ABSTRACT

Region growing is one of the most popular image segmentation methods. The algorithm for region growing is easily understandable but criticized for its lack of theoretical background. In order to overcome this weakness, we propose to describe region growing in a new framework using a variational approach that we called *Variational Region Growing* (VRG). Variational approach is commonly used in image segmentation methods such as active contours or level sets, but is rather original in the context of region growing. It relies on an evolution equation derived from an energy minimization, that drives the evolving region towards the targeted solution. Here, the energy minimization and the VRG robustness to the initial seeds location are performed on gray-level and color images.

***Index Terms***— Segmentation, region growing, variational approach, region-based energy

## 1. INTRODUCTION

Since its introduction by Zucker [1], the region growing method has become a popular algorithm for 2D and 3D segmentation. In this approach, a homogeneous region is presumed to correspond to a semantic object. Starting from a seed, manually or automatically located, the iterative process of region growing extracts a region of interest by merging all pixels satisfying an aggregation criterion and located in the neighborhood of the region. At each step, candidate pixels neighboring the evolving region, or already belonging to it, are tested. The algorithm converges when no more pixels are added to the evolving region during an iteration.

In classical region growing methods, aggregation criterion can be categorized into two groups. In the first group, the criterion governs the growth of a single region. This criterion measures either a similarity between a candidate pixel and another pixel or the homogeneity of the whole resulting segmented region [2]. Such a criterion requires the use of an arbitrary threshold value to fix the minimum value of homogeneity. This method is attractive due to its simplicity, but the choice of the threshold requires further knowledge about the grey-level distribution to avoid trial and error adjustment. In the second group, the criterion governs a competitive growth between several regions. This kind of region growing called *seeded region growing* was introduced by Adams and Bischof [3] in 1994. At each iteration, the most similar pixel compared to mean intensity of a region is looked up in the set of all candidate pixels and merged. This method is free from tuning parameters [4]. More elaborated merging criteria based on statistics were also proposed [5, 6].

The originality of this work is to apply the region growing process to solve an energy minimization arising from segmentation problems. Through a discrete variational approach, we formalize the iterative algorithm of region growing, and we set out a theoretical framework for the definition of the aggregation criterion. Our *Variational Region Growing* (VRG) describes a generic framework which relies on a region-based energy minimization. The major relevance of this framework is its ability to deal with whatever kind of aggregation criterion, provided that can be converted in a minimization energy problem.

## 2. DEFINITIONS

Let us start by introducing some definitions and notations further used in our framework.

### 2.1 Region representation

In our formalism, the evolving region is represented by a characteristic function $\phi_{\mathbf{x}}$ defined by:

$$\phi_{\mathbf{x}} = \phi(\mathbf{x}) = \begin{cases} 1 & if\,\mathbf{x} \in \Omega_{in} \\ 0 & if\,\mathbf{x} \in \Omega_{out} \end{cases} \quad (1)$$

where $\mathbf{x}$ is a $\mathbb{R}^d$ element of the image domain $\Omega$, $\Omega_{in}$ the segmented region in $\Omega$ and $\Omega_{out}$ the background, defined as the absolute complement of $\Omega_{in}$: $\Omega_{out} = \Omega \backslash \Omega_{in}$. The initial region ($t = 0$) is described by the characteristic function $\phi^0$:

$$\{\mathbf{x} \in \Omega \mid \phi(\mathbf{x}, t = 0) = 1\} \quad (2)$$

### 2.2 Neighborhood definition

In a metric space $\Omega$, the neighborhood of a given point $\mathbf{u}$ is defined by the set of all points close to $\mathbf{u}$ according to the euclidian distance. We define the $\varepsilon$-neighborhood of a point $\mathbf{u}$ as the set:

$$N_\varepsilon(\mathbf{u}) = \{\mathbf{v} \in \Omega \mid \|\mathbf{v} - \mathbf{u}\| \le \varepsilon\} \quad (3)$$

In the 2D-image domain, the well-known $N_4$ and $N_8$ neighborhoods are obtained with respectively $\varepsilon = 1$ and $\varepsilon = \sqrt{2}$.

Let us note $\partial_\varepsilon^+$ the outer boundary of $\Omega_{in} \subset \Omega$ defined as follows:

$$\partial_\varepsilon^+ = \{\mathbf{v} \in \Omega_{out} \mid \exists \mathbf{u} \in \Omega_{in},\ \mathbf{v} \in N_\varepsilon(\mathbf{u})\} \quad (4)$$

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In the same way, let us note $\partial_\varepsilon^-$ the inner boundary of $\Omega_{in}$ defined as follows:

$$\partial_\varepsilon^- = \{\mathbf{v} \in \Omega_{in} \mid \exists \mathbf{u} \in \Omega_{out},\ \mathbf{v} \in N_\varepsilon(\mathbf{u})\} \tag{5}$$

We also note $\partial_\varepsilon^\pm$ the union of the inner and outer boundaries.

## 3. VARIATIONAL REGION GROWING APPROACH

The aim of this paper is to describe the region growing segmentation as an energy minimization process. We particularly focus on variational models. Variational models, and functional analysis in general, belong to mathematical theories closely related to physics such as propagation equations or conservation laws. Variational models are linked to optimization problems and are frequently used for solving image segmentation such as active contours and level sets [7].

### 3.1 Variational approach in segmentation

In a variational approach, image segmentation can be formulated by the following expression:

$$\phi^* = \arg\min_{\phi}\ J(\phi) \tag{6}$$

where $\phi^*$ is an optimal solution of an energy minimization process and leads to the best segmentation according to the considered energy $J(.)$. A way to compute the optimal solution is to define the variation of $\phi$ called $\Delta_t\phi$ by introducing an artificial time-variable $t$ and to estimate iteratively the energy variation $\Delta J(.)$ for a small variation of $\phi$ noted $\tilde{\phi}$:

$$\Delta_t\phi + F\left(\phi, \Delta J\left(\tilde{\phi}\right)\right) = 0 \tag{7}$$

where $F$ is a functional that governs the region evolution.

In our approach, this functional depends on a special function $c(\phi)$ (explained below) and the energy variation $\Delta J(\tilde{\phi})$. Starting from $\phi^0$, $\phi$ is updated at each time by solving iteratively equation (8), over the whole the domain $\Omega$, until a steady state solution is reached.

$$\Delta_t\phi - c(\phi).H\left(-\Delta J\left(\tilde{\phi}\right)\right) = 0 \tag{8}$$

$H$ is the one-dimensional Heaviside function defined as follows:

$$H(z) = \begin{cases} 0 & \text{if } z < 0 \\ 1 & \text{if } z \geq 0 \end{cases} \tag{9}$$

The Heaviside function activates the function $c(\phi)$ when the estimated energy variation $\Delta J(\tilde{\phi})$ is negative, and inhibits it otherwise. The function $c(\phi)$ is defined in order to induce the switch of $\phi$ values, so it is equal to $(1-2\phi)$. For clearness sake, $c(\phi)$ is represented in figure 1 in the case of a mono-dimensional space $\Omega$. More details are given in the next subsection.

In our discrete framework, the continuous-time variable $t$ is replaced by a discrete-time variable $n$. Therefore, $\phi^n$ represents the current values of $\phi$. At the next iteration, $\phi^{n+1}$ the update of $\phi^n$ consists in determining the new values of $\phi$ for each grid points $\mathbf{x}$. Let us note $\phi^n_\mathbf{x}$ the value of $\phi^n$ at point $\mathbf{x}$. Equation (8) is iteratively solved by numerical methods making $\phi$ evolve jointly to the segmented region. The evolution of the region function $\phi$ is given by the following equation:

$$\phi^{n+1} = \phi^n + c(\phi^n).H\left(-\Delta J\left(\tilde{\phi}^n\right)\right), \tag{10}$$

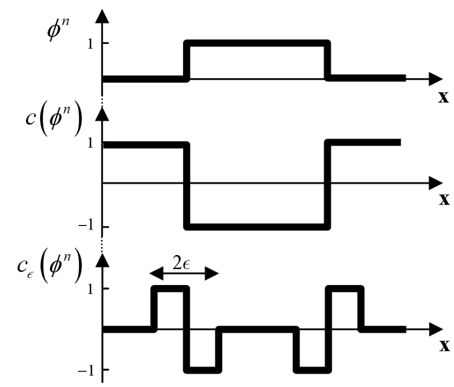



**Fig. 1**. The characteristic function $\phi^n$ (also called region function), the function $c(\phi^n)$ and the cut-off function $c_\varepsilon(\phi^n)$ are plotted with a solid line in the case of a mono-dimensional domain $\Omega$.

At a specific point $\mathbf{x}$ of $\Omega$, it can be also written:

$$\phi^{n+1}_\mathbf{x} = \phi^n_\mathbf{x} + c(\phi^n_\mathbf{x}).H\left(-\Delta J\left(\tilde{\phi_\mathbf{x}}^n\right)\right), \tag{11}$$

where the sign of $\Delta J(\tilde{\phi_\mathbf{x}}^n)$ indicates whether the value $\phi_\mathbf{x}$ has to be switched or not.

### 3.2 Adaptation for region growing method

In this section, we focus on the VRG algorithm implementation. Previously, we defined $\Delta J(\tilde{\phi}^n)$ for each point $\mathbf{x}$ of $\Omega$. In the particular case of region growing, the set of candidate points for the state switch is restricted to a neighborhood around the boundary of the evolving region. The size of neighborhood where the region can evolve, is parameterized by $\varepsilon$ value (see equation (3)). That leads us up to propose a cut-off function $c_\varepsilon(\phi^n)$ defining a spatial bandwidth for the candidates points by:

$$c_\varepsilon(\phi^n_\mathbf{x}) = \begin{cases} 1 - 2\phi^n_\mathbf{x} & \text{if } \mathbf{x} \in \partial_\varepsilon^\pm \\ 0 & otherwise \end{cases} \tag{12}$$

where $c_\varepsilon(\phi^n_\mathbf{x})$ depends on $\varepsilon$-value as shown in figure 1. The aim of $c_\varepsilon$ is similar to the narrow band used in level sets. Note that in most of region growing methods, only the outer boundary $\partial_\varepsilon^+$ is considered for the evolution of the region.

### 3.3 Energy variation

VRG recovers an object of interest by means of a discrete function $\phi$ that switches according to the minimization of an energy $J$. In the literature, many region-based energies were introduced into the variational framework. Jehan-Besson [8] gives a general definition of a region-based energy computed from a "region-independent" descriptor $k_\mathbf{x}$ as:

$$J(\Omega_{in}) = \int_{\Omega_{in}} k_\mathbf{x} d\mathbf{x} \tag{13}$$

In our framework, we express the previous energy by:

$$J(\phi^n) = \sum_{\mathbf{x}\in\Omega} k_\mathbf{x} \cdot \phi^n_\mathbf{x} \tag{14}$$

Starting from this energy, we evaluate $J\left(\tilde{\phi}^n\right)$ the energy that would result from the state switch of a candidate pixel $\mathbf{v}$. We define the assessed state switch $\tilde{\phi}^n_\mathbf{v}$ of a pixel by:

$$\tilde{\phi}_{\mathbf{v}}^{n} = 1 - \phi_{\mathbf{v}}^{n}, \tag{15}$$

thus,

$$\tilde{\phi}_{\mathbf{x}}^{n} = \phi_{\mathbf{x}}^{n} \;\; if \quad \mathbf{x} \neq \mathbf{v}. \tag{16}$$

From equation (14), the energy of $\tilde{\phi}^n$ can be expressed as:

$$J\left(\tilde{\phi}^{n}\right) = \sum_{\mathbf{x}\in\Omega} k_{\mathbf{x}} \cdot \tilde{\phi}_{\mathbf{x}}^{n} \tag{17}$$

Using equations (15) and (16), we can also write:

$$J\left(\tilde{\phi}^{n}\right) = k_{\mathbf{v}} \cdot \tilde{\phi}_{\mathbf{v}}^{n} + \sum_{\mathbf{x}\neq\mathbf{v},\mathbf{x}\in\Omega} k_{\mathbf{x}} \cdot \phi_{\mathbf{x}}^{n} \tag{18}$$

$$J\left(\tilde{\phi}^{n}\right) = k_{\mathbf{v}} \cdot (1-\phi_{\mathbf{v}}^{n}) - k_{\mathbf{v}} \cdot \phi_{\mathbf{v}}^{n} + \underbrace{k_{\mathbf{v}} \cdot \phi_{\mathbf{v}}^{n} + \sum_{\mathbf{x}\neq\mathbf{v}} k_{\mathbf{x}} \cdot \phi_{\mathbf{x}}^{n}}_{J(\phi^{n})} \tag{19}$$

The energy variation $\Delta J(\tilde{\phi}^n)$ resulting from promoting a single point at position $\mathbf{v}$ is obtained by identification and is defined by:

$$\Delta J\left(\tilde{\phi}_{\mathbf{v}}^{n}\right) = (1-2\phi_{\mathbf{v}}^{n}) \cdot k_{\mathbf{v}} \tag{20}$$

Note that this variation of energy is defined whatever the region-independent descriptor used.

## 4. APPLICATION

Here, we present our algorithm to solve energy minimization through a region growing process. And then we apply it to image segmentation using the probability distribution separation energy introduced by Paragios [7].

### 4.1 Variational region growing algorithm

The outline of our algorithm is presented in few steps:

**Step 1.** Initialization. Construct the initial partition $\phi^0$ (see eq. (2)). This partition corresponds to the initial seeds.

**Step 2.** Compute the set $\partial_{\varepsilon}^{\pm}$ of candidate points for the switch state (inner and outer boundaries of $\Omega_{in}$).

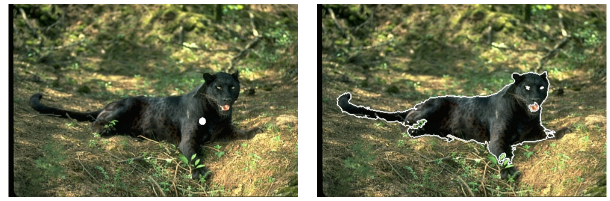
**Step 3.** Evolution. Switch $\phi$-value of each point $\mathbf{v}$ in $\partial_{\varepsilon}^{\pm}$, if $\Delta J(\tilde{\phi}_{\mathbf{v}}^{n})$ the energy variation is negative.

**Step 4.** Repeat the step 2, until the energy variation remains unchanged (i.e. no more points are switched).

### 4.2 Probability distribution separation energy

To illustrate our variational method, we use it to Paragios image segmentation model [7]. We consider a more complex energy that looks simple means and compares the full probability distributions of the foreground and background. We show that its incorporation into our framework is simple.

Consider $p_{in}(I_{\mathbf{x}})$ and $p_{out}(I_{\mathbf{x}})$ be the two estimated probability density functions at pixel $\mathbf{x}$ computed from the global inner $\Omega_{in}$ and the outer region $\Omega_{out}$ of a partitioned image $I_{\mathbf{x}}$ using Parzen window. Parzen window method is a non-parametric kernel density estimation. This estimation is asymptotically unbiased, uniformly consistent in probability, and consistent in the mean-square sense.



(a) (b)

**Fig. 2**. Segmentations of the LEOPARD color image. (a) Shows the initialization (white circle region); (b) shows contour of segmentation using probability distribution separation in our variational approach.

The kernel density estimator $p_{in}(I_{\mathbf{x}})$ of the interior region image intensities ($I_{\mathbf{x}} \in R^d$) at point $\mathbf{x}$ is given by:

$$p_{in}\left(I_{\mathbf{x}}\right)_{\mathbf{H}} = \frac{1}{|\Omega_{in}|} \sum_{\mathbf{i}\in\Omega_{in}} K_{\mathbf{H}}\left(I_{\mathbf{x}} - I_{\mathbf{i}}\right) \tag{21}$$

where $|\Omega_{in}|$ is the cardinality of set $\Omega_{in}$, $\mathbf{H}$ is a $d \times d$ symmetric positive definite matrix called the bandwidth matrix,

$$K_{\mathbf{H}}\left(I_{\mathbf{x}}\right) = |\mathbf{H}|^{-1/2} K\left(\mathbf{H}^{-1/2} I_{\mathbf{x}}\right) \tag{22}$$

and $K$ is a $d$-variate kernel function satisfying:

$$\int K\left(\mathbf{z}\right) d\mathbf{z} = 1 \tag{23}$$

Conditions on the multivariate kernel $K$ and bandwidth matrix $\mathbf{H}$ to guarantee the asymptotic unbiasedness, the mean-square consistency and the uniform consistency of the estimator can be found in [9]. In the following of the paper, we use the Gaussian kernel $K_G$ which satisfies the previous conditions:

$$K_G\left(\mathbf{H}^{-1/2} I_{\mathbf{x}}\right) = (2\pi)^{-d/2}.exp\left(-\frac{1}{2} {I_{\mathbf{x}}}^{T} \mathbf{H}^{-1} I_{\mathbf{x}}\right) \tag{24}$$

Paragios et al. shows that the maximization of the posterior probability is equivalent to the minimization of the corresponding $[-log()]$ function. In our work, this energy is defined by:

$$J\left(\phi^{n}\right) = -\sum_{\mathbf{x}} \log\left(p_{in}\left(I_{\mathbf{x}}\right)\right).\phi_{\mathbf{x}} - \sum_{\mathbf{x}} \log\left(p_{out}\left(I_{\mathbf{x}}\right)\right).\left(1-\phi_{\mathbf{x}}\right) \tag{25}$$

With equation 20, we obtain the energy variation of $J\left(\phi_{\mathbf{x}}\right)$:

$$\Delta J\left(\tilde{\phi}_{\mathbf{v}}^{n}\right) = (1-2\phi_{\mathbf{v}}^{n}) \cdot \left[\log\left(p_{out}\left(I_{\mathbf{v}}\right)\right) - \log\left(p_{in}\left(I_{\mathbf{v}}\right)\right)\right] \tag{26}$$

where $p_{in}\left(I_{\mathbf{v}}\right)$ and $p_{out}\left(I_{\mathbf{v}}\right)$ are updated at each iteration.

## 5. EXPERIMENTS

In order to demonstrate the strengths and limitations of our method, we performed several experiments. First, we compare VRG with a classic region growing approach. We continue with a study of the effects of segmentation initialization. Finally, we examine the convergence properties of the proposed method. For all experiments, the VRG parameters are set to constant values: $\mathbf{H} = 2.25\,\mathbf{I}$, ($\mathbf{I}$ is the identity matrix), and $\varepsilon = 1$.

### 5.1 Comparison with a classic region growing approach

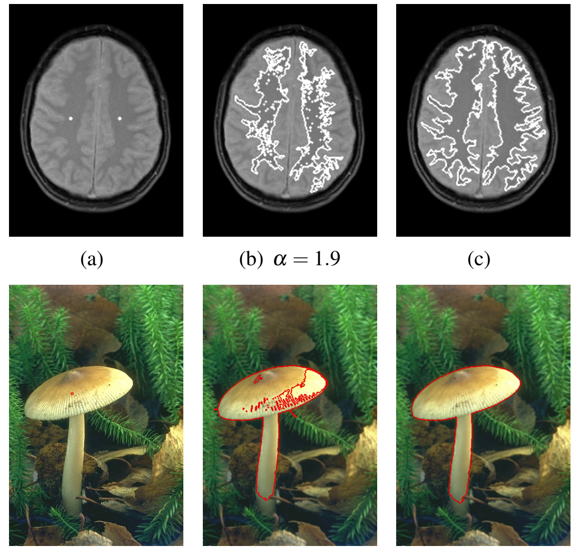
In the previous section, we have described how to integrate region-based energies in our variational region growing. Here, we demonstrate the improvements offered by our algorithm. We propose to compare the variational region growing with a classic region growing approach driven by a homogeneity criterion. Lots of homogeneity criteria were implemented in region growing. These can be classified as local or global criteria depending on the spatial extent where measurements are computed. Difference between grey levels, statistic properties of the evolving region such as mean or variance are well-known features used to assess homogeneity criteria [2], [10]. The purpose of the following experiments is to demonstrate that using energy minimization instead of a simple homogeneity criterion can improve significantly the region growing segmentation .
We compare VRG with a classic region growing algorithm called *Global Mean Region Growing* (GMRG) cited in [10]. In GMRG, the candidate pixels are aggregated to the growing region if their grey levels are included in a range specified by the mean of the grey level distribution of the evolving region and a tolerated variation specified by the standard deviation of this distribution. This approach implicitly assumes that the grey level distribution of the target region can be modeled by a Normal law. The criterion adapted to the inside region $\Omega_{in}$ is expressed by the following equation:

$$\left|I_x - \mu_{\Omega_{in}}\right| \leq \alpha \cdot \sigma_{\Omega_{in}} \qquad (27)$$

where $\alpha$ is a tuning parameter.
In figure 3, we compare the performance of VRG guided by Paragios region-based energy described in section 4.2 with GMRG driven by the previous homogeneity criterion. Notice that the classic approach only extracts the brightest parts of the image while our approach converges to meaningful object boundaries. For both images, $\alpha$ value is manually adjusted in order to get the best results. In the MUSHROOM image [11], the main object and the inhomogeneous background are associated with multimodal distributions of their color components (R,G,B). Lighting variations and complex textures induce both smooth and quick changes onto these components. In the BRAIN image, the VRG is initialized from two small ellipses pointing out a single initial region. Starting from this initial region (chosen to be inside the object), the variational region growing evolves iteratively by testing candidate points specified by $\partial_{\varepsilon}^{+}$ until convergence.

### 5.2 Sensitivity to Initialization

A major advantage of VRG is a lower sensitivity to initialization. This characteristic comes from the use of global region-based functional. Indeed, global region-based energy analyses the whole image, whereas classic methods based on homogeneity criterion only use features of the evolving region and are consequently more sensitive to initialization.
The experiment in figure 4 shows multimodal image in which the leopard was segmented with several different initializations. In this experiment, the analysis of density distribution of regions (inside and outside region) allows the segmentation technique to accurately separate this structure from the rest of the image. With various initialization, our variational approach converges to the same segmentation.



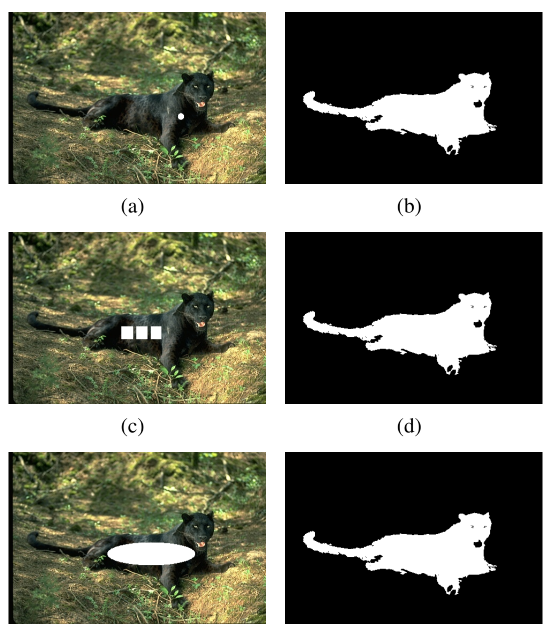
(a) (b) $\alpha = 1.9$ (c)

(d) (e) $\alpha = 2.06$ (f)

**Fig. 3**. Segmentations of a gray-level image (BRAIN) and a color image (MUSHROOM). (a, d) show the same initialization for VRG and GMRG; (b, e) and (c, f) show contours of segmentation using GMRG and VRG, respectively.

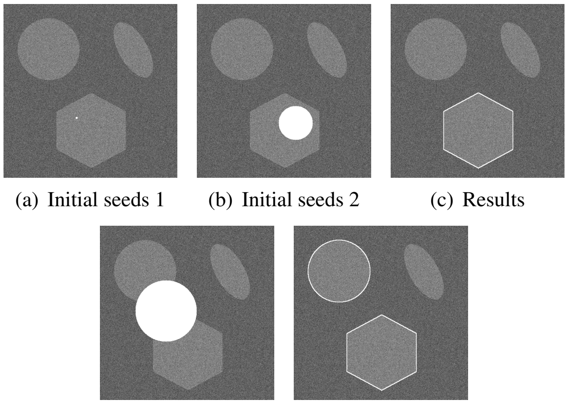
### 5.3 Convergence and properties

Finally, we examine the convergence and properties of our variational region growing. Figure 5 displays the results for different initial conditions. The white contours delineate segmented regions. In figure 5(c), the same result is obtained from two different initialization given by figures 5(a), 5(b).



(a) (b) (c) (d) (e) (f)

**Fig. 4**. Segmentations of the LEOPARD color image. (a, c, e) show different initializations; (b, d, f) show corresponding segmentations using our variational approach.



(a) Initial seeds 1 (b) Initial seeds 2 (c) Results

(d) Initial seeds 3 (e) Results

**Fig. 5**. Three different initial regions (seeds) (a,b,d) and the corresponding segmentation results (c,e).

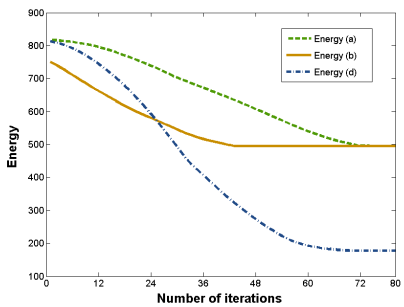



**Fig. 6**. Three different initial conditions and the corresponding energy versus iterations.

Thus, different initial seeds, provided that they were inside the same region of interest, converge to the same result. In the same way, figure 5(e) displays the segmentation result obtained with seeds of figure 5(d). Thanks to the use of $\partial_\varepsilon^-$, some pixels belonging first to the evolving region, can be rejected during the next iterations, enabling the segmentation of non-connected objects. These examples illustrate the ability of VRG to converge towards the desired object, while enabling free changes of topology during the growing.
In figure 6, we plot the energy values versus iterations for each initialization. These evolution curves point out the energy minimization which occurs during the iterative process of the region growing. In the three cases, our variational method detects a local minimum of energy. This intended characteristic comes from the spatial restriction imposed by the algorithm to the evolution of the region at each iteration.

## 6. CONCLUSION

In this work, we propose an original framework based on variational region growing (VRG) for image segmentation. Describing region growing segmentation approaches with the proposed variational framework, we show that region growing algorithms lead to an energy minimization. Our framework is based on the discrete region-based energy variation and allows one to readily take into account energy in the segmentation. An evolution equation of the region is determined from a functional that depends on the variation of the energy and a special cut-off function allowing both the selection of the candidate pixels and the state switch of these pixels during the segmentation process. We choose as criterion the maximization of the posterior probability, and we compare our approach with a classical region growing criteria. VRG gives more accurate results than classical approach on both gray-level and color images. Finally, we assess the influence of the initial seeds on the segmented results and on the minimization of the energy. We show that the resulting segmentation is robust to the position of the initial seeds for a given region as the seed initialization lead the same minimum of energy and then to the same segmented region.

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