Optimization of GEANT4 settings for Proton Pencil Beam Scanning simulations using GATE

L. Grevillot^{1,2,3,6}, T. Frisson^{1,2,3}, N. Zahra^{1,3,4}, D. Bertrand⁶, F. Stichelbaut⁶, N. Freud^{1,5} and D. Sarrut^{1,2,3}

E-mail: loic.grevillot@creatis.insa-lyon.fr

¹ Université de Lyon, F-69622 Lyon, France.

² Creatis, CNRS UMR 5220, F-69622, Villeurbanne, France.

³ Léon Bérard Cancer Centre, F-69373 Lyon cedex 08, France.

⁴ IPNL, CNRS UMR 5822, F-69622, Villeurbanne, France.

⁵ CNDRI, INSA-Lyon, F-69621, Villeurbanne Cedex, France

⁶ IBA, B-1348, Louvain-la-Neuve, Belgium.

1	Abstract.
2	Purpose: This study investigated the different GEANT4 settings for proton-
3	therapy applications in view of Treatment Planning System (TPS) comparisons. The
4	GEANT4.9.2 release was used through the GATE platform. We focused on the
5	Pencil Beam Scanning (PBS) delivery technique, which allows for Intensity Modulated
6	Proton-Therapy (IMPT) applications.
7	Methods: The most relevant options and parameters (range cut, step size, database
8	binning) that influence the dose deposition were investigated, in order to settle a
9	robust, accurate and efficient simulation environment. In this perspective, simulations
10	of depth-dose profiles and transverse profiles at different depths and energies between
11	$100~{\rm MeV}$ and $230~{\rm MeV}$ have been assessed against reference measurements in water
12	and PMMA. These measurements were performed in Essen in Germany, using the
13	IBA dedicated Pencil Beam Scanning system, with integral Bragg-peak chambers and
14	radiochromic films. GEANT4 simulations were also compared to the PHITS.2.14 and
15	MCNPX.2.5.0 Monte Carlo (MC) codes.
16	Results: Depth-dose simulations reached 0.3 mm range accuracy, with a dose
17	agreement around 1% over a set of 5 different energies. The transverse profiles
18	simulated using the different MC codes showed discrepancies, with up to 15% beam
19	widening difference between GEANT4 and MCNPX in water. A 8% difference between
20	the GEANT4 multiple scattering (MS) and single scattering (SS) algorithms was
21	observed. The simulations showed the inability of reproducing accurately the measured
22	transverse dose spreading with depth in PMMA, corroborating the fact that GEANT4
23	underestimates the lateral dose spreading.
24	Conclusion: A reference physics-list and an optimized parameters-list have been
25	proposed. Satisfactory agreement against depth-dose profiles measurements was
26	obtained. The simulation of transverse profiles using different MC codes showed
27	inconsistencies. This point is crucial for PBS delivery simulations and suggests that the
28	GEANT4 MS algorithm should be revised. GATE was found to be a very convenient
29	interface to perform this GEANT4-based study.

30 1. Introduction

The main asset of ions over photons is their inverse depth-dose profiles, allowing higher 31 doses to tumors, while better sparing healthy tissues. The most attractive and advanced 32 technique in hadron therapy consists in irradiating patients with small pencil beams of a 33 few millimeters in diameter. The pencil beams are scanned transversally in the patient 34 using two scanning magnets. Several iso-energy layers are then used to cover the whole 35 tumor volume. An active beam delivery system has been used since 1997 at the Paul 36 Scherrer Institute (PSI) in Switzerland for proton therapy and at the Gesellschaft für 37 Schwerionenforschung (GSI) in Germany for carbon-ion therapy [1]. 38

When computing dose distributions with passive beam techniques, one critical point is the Bragg peak range uncertainty. For active beam delivery, the lateral spreading of each single pencil beam also needs to be correctly accounted for. In this setting, MC simulations have become increasingly important for evaluating treatment plans and dose distributions in patients. The dose accuracy reached with analytical algorithms implemented in TPS is limited, for instance near heterogeneities and MC simulation ⁴⁵ can be considered as a powerful Quality Assurance (QA) tool. It has been extensively ⁴⁶ used at the Massachusetts General Hospital in Boston for TPS comparisons, mostly ⁴⁷ for passive irradiation techniques, using the GEANT4 toolkit [2]. From a research ⁴⁸ standpoint, MC simulation allows also for a better understanding of the dose deposition ⁴⁹ mechanisms in the patient and opens many research areas.

In this study, we used GEANT4 version 9.2 through the GATE application[3]. 50 This study was conducted in order to analyze the physics implemented in GEANT4 51 and to select the appropriate settings for patient dose calculation, with an ultimate 52 objective of treatment planning benchmarking. This work dedicated to proton-therapy 53 applications builds upon a previous study oriented toward carbon ion simulations using 54 older GEANT4 releases [4]. Firstly, a short review of the relevant physics models and 55 parameters available in GEANT4 is presented in section 2. Secondly, the influence of 56 different settings on dose calculation is investigated in section 3. A reference physics-list 57 and an optimized parameters-list are then proposed. Comparisons with two other MC 58 codes: MCNPX2.5.0 [5] and PHITS2.14 [6] are presented in section 4. Experimental 59 measurements in water and PMMA are described and compared to GEANT4 simulations 60 in sections 5 and 6. 61

62 2. Simulation settings

63 2.1. Pencil beam model

In this study, a simple pencil beam model was used in order to reproduce the nozzle 64 output beam based on reference measurements. The energetic spectrum was assumed 65 Gaussian and adjusted over depth-dose measurements, as presented in section 6.1. The 66 2D probability density function (PDF) of protons was considered normal (Gaussian 67 distibution) and adjusted over transverse profile measurements performed at the 68 isocenter (section 6.2). By convention, Cartesian coordinates were used, with z the 69 beam direction, x and y the lateral position. In this paper, the expression "spot size" 70 will refer to the PDF parameters σ_x and σ_y : the standard deviation in the x and y 71 directions at the isocenter. The role of the beam divergence on the lateral dose spreading 72 in water was estimated negligible compared to the effect of multiple Coulomb scattering. 73 This assumption has been checked by simulating a realistic beam divergence of $\sigma = 3$ 74 mrad. Thus, the intrinsic beam divergence was neglected. 75

76 2.2. Physics-list selection

For medical physics applications the electromagnetic (EM) standard package with the option 3 (Opt3) parameters-list is recommended by the GEANT4 Electromagnetic Standard working group [7]. Opt3 refers to options/processes which are described in the next sections and proposes reference parameters to reach a high level of accuracy. Our physics-list is mainly based on a reference paper dedicated to proton-therapy applications [8], using the standard package for EM interactions and a recently

implemented process (G4UHadronElasticProcess) combined with the G4HadronElastic 83 model for elastic hadronic (HAD) interactions. Details about available models have 84 been discussed elsewhere [9, 10, 8, 11, 12]. The only difference in our physics-list 85 compared to the one proposed in [8] is the choice of the inelastic HAD model. By 86 comparing the Bertni, binary cascade, precompound and QMD models against depth-87 dose measurements in water, the precompound model was found to best match the 88 No difference between the different models was observed for the measurements. 89 transverse dose profile simulation. Therefore, the precompound model has been selected 90 for the rest of the study. The satisfactory agreement obtained with the precompound 91 model has been pointed out in two recent studies investigating respectively a model of a 92 proton magnetic beam scanning delivery nozzle [13] and the prompt-gamma production 93 during proton irradiation [14]. 94

⁹⁵ 2.3. Multiple scattering (MS) and single scattering (SS)

Charged particles transported through matter are disrupted by the surrounding 96 electromagnetic field, which is produced by the nucleus and orbiting electrons 97 encountered. The simulation of each single interaction (SS algorithm) increases 98 drastically the number of steps and simulation time, but is considered as the reference, 99 since it is based on cross-section measurements. It is shown in section 4 that SS 100 multiplies the simulation time by a factor of 10^3 . To overcome this issue, condensed 101 algorithms (MS theories) have been developed, so that the global effect of multiple 102 collisions is computed at the end of each step. This global effect encloses the net 103 displacement, energy loss and change of direction [12]. MS algorithms are considered 104 exact if they reproduce the SS behavior. Most of the MC codes implemented the 105 MS theories of Molière, Goudsmit-Saunderson or Lewis [12]. Besides the angular 106 distribution after a step, the advantage of the Lewis theory over the others is the 107 computation of the moment of the spatial distribution as well [12]. The computation of 108 the spatial displacement is not part of those theories and each MC code has to develop 109 its own algorithm. MS theories are subjects of interest and recent investigations on 110 the scattering power, considered as a key quantity for beam transport in matter, may 111 improve the accuracy of the MS algorithms implemented in MC codes [15, 16]. It 112 was shown that the MS algorithm implemented in GEANT4 release 9.1 depends on 113 the step size [16]. Improvements of the scattering power calculation may avoid this 114 dependence in the future and hence, improve the MS accuracy of MC codes. The 115 management of geometrical boundaries is also a complex task, because particles are 116 not allowed to cross a boundary without performing a step. In GEANT4, several 117 stepping algorithms [12], wich are included in the MS model can be selected: "simple". 118 "safety" and "distanceToBoundary", depending on the accuracy required. The MS 119 model implemented in GEANT4 is based on the Lewis theory and is detailed in [12]. 120

121 2.4. Relationship between simulation parameters

The two main parameters in a GEANT4 simulation are the *step*, which corresponds to the distance to the next interaction and the *range cut*, which corresponds to the production threshold for secondary particles (gammas, electrons and positrons).

The energy loss of ions in matter is governed by the ionization process. Below the range cut threshold, the energy loss occurs continuously along the ion track (at each step), while above the threshold, it is caused by the explicit production of secondary particles (discrete component) [12, 11]. All the particles generated are then tracked until no energy is left. The continuous energy loss of charged particles is calculated by the restricted stopping power equation, defined in GEANT4 as the Bethe-Bloch formula integrated between 0 and the range cut value.

Before starting a simulation, GEANT4 initializes tables to describe EM processes: 132 lambda (mean free path), $\frac{dE}{dx}$ (restricted stopping power), range and inverse range 133 tables [11]. These tables are pre-calculated according to the simulation parameters 134 defined by the user, in order to save time during the simulation. By default, 84 bins 135 are stored between 100 eV and 100 TeV, corresponding to a resolution of 7 bins/decade 136 for each material, but the *binning* parameter can be adjusted by the user. In fact, the 137 lambda table should be called cross-section (σ) table, because it stores the cross-sections 138 and indirectly indicates the mean free path (λ) values via equation (1). 139

$$\lambda(Z, E) = \frac{1}{n_{at} \times \sigma(Z, E)} \tag{1}$$

where n_{at} is the number of atoms per unit volume. Range and inverse range tables show 141 the correspondence between range cut and energy. The step length is sampled at each 142 step using the lambda table for EM processes and directly in the database for the HAD 143 processes. Hence, a sufficient number of bins in the different tables is mandatory to 144 accurately describe EM interactions. In the GEANT4 Opt3 parameters-list, 220 bins 145 between 100 eV and 10 TeV, i.e 20 bins/decade, are advised. It is important to note 146 that the range cut threshold influences the values stored in both the lambda and $\frac{dE}{dx}$ 147 tables. Consequently, the step lengths sampled, continuous energy loss along the steps 148 and δ_{e^-} production of charged particles depend on the range cut threshold. 149

¹⁵⁰ 2.5. Dealing with the continuous energy loss

The continuous energy loss imposes a limit on the step length, because of the energy dependence of the cross sections. It is assumed in many MC codes that the crosssection is constant during a step, consequently, the continuous energy loss is computed via equation (2).

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$$Continuous Energy Loss = Step \times \frac{dE}{dx}$$
(2)

with $\frac{dE}{dx}$ the restricted stopping power of the charged particle at the beginning of the step. In high-gradient cross-section regions, the approximation of constant cross-sections along 169

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the step may lead to inaccurate dose deposit. This point is very significant in hadron-158 therapy in the Bragg-peak region. The step length can be limited by two user-defined 159 limits: maximum allowed step [17] and stepping function [11]. The maximum allowed 160 step is managed like a process in competition with the other processes and limits the 161 maximum step length according to a user-given value. The stepping function described 162 in equation (3) is a dynamic step limitation which decreases the particle step limit 163 (ΔS_{lim}) in parallel with the particle decreasing range (Figure 1). The stepping function 164 is defined by 2 parameters: "dRoverRange" and "finalRange". The "dRoverRange" 165 (α_R) parameter defines the maximum step size allowed as a $\frac{step}{range}$ ratio. As the particle 166 travels, the maximum step allowed decreases until the particle range (R) becomes lower 167 than the "finalRange" (ρ_R) parameter. 168



$$\Delta S_{lim} = \alpha_R \cdot R + \rho_R \cdot (1 - \alpha_R) (2 - \frac{\rho_R}{R}) \tag{3}$$

Figure 1. This figure shows the maximum step length allowed for a 230 MeV proton beam in water, with the stepping function and default parameter ($\alpha_R = 0.2$ and $\rho_R = 1$ mm) in green (StepFunction2); with the stepping function and $\alpha_R = 0.01$ and $\rho_R = 10$ μ m in blue (StepFunction1); with a 1 mm maximum allowed step in red (StepLimiter). The left scale corresponds to the step limit and the right scale corresponds to the normalized dose of the proton beam in dark (Dose).

Instead of limiting the step, one can also integrate the mean cross-section and the mean energy loss along the step, so that equation (2) becomes equation (4):

$$Continuous Energy Loss = \int^{step} \frac{dE}{dx} dx$$
(4)

This solution enables to sample the exact cross-section and mean energy loss via a MC technique [11]. This function is used when the $\frac{E_{loss}}{E}$ ratio is larger than the user-defined *linear loss limit* [17], with E_{loss} and E the particle continuous energy loss and particle kinetic energy. A low threshold can lead to a significant calculation time increase. Differences between GEANT4.9.2 default options and Opt3 are summarized in Table 1.

		Default values	
	e^- / e^+	Proton	GenericIon
range cut	$1 \mathrm{mm}$	-	-
stepping function - finalRange	$1 \mathrm{mm}$	$1 \mathrm{mm}$	$0.1 \mathrm{mm}$
stepping function - dRoverRange	0.2	0.2	0.1
binning (bins/decade)	7	7	7
linear loss limit	0.01	0.01	0.15
stepping algorithm	safety	minimal	minimal
		GEANT4 Opt3	
	e^- / e^+	Proton	GenericIon
stepping function - finalRange	0.1 mm	$0.05 \mathrm{~mm}$	0.02 mm
binning (bins/decade)	20	20	20
stepping algorithm	distance To Boundary	-	-

 Table 1. Summary of the GEANT4.9.2 default and Opt3 parameters.

178 3. Influence of GEANT4 settings on dose computing

The first objective of our study was to understand the influence of the different parameters and functions on proton dose simulation, with a focus on the proton range, the simulation time and the dose fluctuations. For all simulations, the geometry was a single volume of water. Proton ranges were defined as the position of 80% of the maximum dose in the distal fall-off region of the Bragg peak. We evaluated the simulation times by comparing the proton source rate (in protons·s⁻¹) for different configurations.

¹⁸⁶ 3.1. Influence of the range cut and maximum allowed step values

¹⁸⁷ A 230 MeV mono-energetic proton beam was simulated in a $60 \times 60 \times 60 \text{ cm}^3$ water tank ¹⁸⁸ and the depth-dose profiles were integrated along the z axis with a 1 mm step. For a 230 ¹⁸⁹ MeV proton beam, the NIST [18] CSDA range is 329.4 mm, while the GEANT4 ranges ¹⁹⁰ vary from 329.4 mm for a 1 μ m range cut to 334.9 mm for a 0.1 mm range cut without ¹⁹¹ fixed step limitation. Results are presented in Figure 2. Simulations were performed for ¹⁹² different range cut values between 1 μ m and 1 mm, using different maximum allowed ¹⁹³ step values, while other parameters were set to default values.

The proton range converges towards the NIST range when the range cut value decreases. This is observed without step limitation, but introducing such a constraint brings more consistency in the convergence. Indeed, since the step size is related to the range cut value, the range convergence observed for decreasing range cut values is in fact indirectly due to step size limitation. The relationship between the two parameters has been checked by varying the maximum allowed step for different range cut values. Not surprisingly, the increased accuracy at very low range cut yields a significant



Figure 2. This figure illustrates the influence of the range cut value on the range of 230 MeV protons in water (a) and simulation time (b) for different maximum allowed step values. Ranges are compared to the NIST reference value and converge to the right range for sufficiently low range cut and step. Low range cut and step values decrease the proton rate drastically.

simulation time increase, as also presented in [4] for carbon ions. The simulation time increase was also related to the step size limitation associated with decreasing range cut value. Consequently, both the proton range and the computation time are strongly related to the step size, while the similar effects observed with low range cut values are mainly due to the step shortening effect. Part of the time increase is also due to the electron tracking process, which increases with low range cut values.

The influence of the range cut and hence indirectly of the step limitation on dose 207 computing artifacts is presented in Figure 3. No fixed step limitation was used. When 208 the range cut is sufficiently low, fluctuations become negligible. The worst case occurs 209 with a range cut value of 0.1 mm (highest fluctuations and range shift). Ideally, the 210 range cut value should neither affect the proton range, nor the dose fluctuations. In 211 theory, the electron range cut should only define the accuracy of the electronic dose 212 distribution in the medium. The fluctuations observed are due to wrong data sampling, 213 as presented in section 3.2. 214

²¹⁵ 3.2. Influence of the pre-calculated table binning

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In a second stage, the influence of the binning parameter on the dose deposition for several simulations initialized between 7 bins/decade and 50 bins/decade was examined. We set the binning energy range between 100 eV and 1 GeV to decrease the total number of bins. The range cut was set to 0.1 mm, without limiting the step, which was the worst case observed previously (section 3.1). Dose calculation errors were evaluated using equation (5):

$$\epsilon = \frac{1}{N} \sum_{i=1}^{N} \left(\frac{|d_i - dref_i|}{dref_i} \right) \tag{5}$$



Figure 3. Influence of the range cut threshold and hence of the step size on dose computing of 230 MeV protons in water. When a sufficiently low range cut is used, the proton range becomes stable and the dose fluctuations negligible. Influence of high range cuts is presented in figure (a) and low range cut in figure (b).

where ϵ is the mean point-to-point error calculated, *i* corresponds to a given curve point, *N* is the number of points in a curve, d_i is the dose computed and $dref_i$ is the dose computed for the reference simulation. Errors were calculated between 0 and the Bragg peak range (ϵ_{80}) to discard Bragg peak tail errors.

The influence of EM table binning on dose computing is presented in Figure 4 (a) and (b).

Fluctuations decreased as the number of bins per decade increased. Based on this 229 result, the 50 bins/decade simulation was considered as the reference. 3×10^5 protons 230 were simulated, leading to a statistical uncertainty around 0.5% from the water tank 231 entrance up to the Bragg-peak distal fall-off. Above 15 bins/decade, the fluctuations 232 became irrelevant, meaning that the 20 bins/decade resolution recommended by the 233 GEANT4 Electromagnetic Standard working group is sufficient. The number of bins 234 increases slightly the initialization time, but this is negligible even with a large number 235 of materials (the initialization time was respectively 3 and 3.5 minutes with 7 and 20 236 bins/decade, for 1000 materials). 237

Then, the influence of the previously studied parameters (range cut and step size) was assessed by comparing dose deposits in the reference simulation described above and in the same simulation with a 1 μ m range cut (Figure 4 (c)). No significant difference was observed. Finally, we checked the influence of the Opt3 (Figure 4 (d)). Results are summarized in Table 2.

If few bins are set, the tables do not accurately describe EM processes. Hence, the interpolated cross-sections are wrong, leading to incorrect step and continuous energy loss sampling. When limiting the step, the dose deposit along the ion track is more frequent, limitting the propagation of inaccurate data sampling to the maximum step size allowed. When a sufficient number of bins is set, the proton range and dose



Figure 4. Figure (a) and (b) show the influence of the binning parameters on dose computing with a range cut of 0.1 mm and no step limitation. When a sufficient number of bins is set, the proton range becomes stable and the dose fluctuations negligible. The influence of a 1 μ m range cut (c) and GEANT4 Opt3 (d) on a simulation using 50 bins/decade is also presented.

Bins/decade	7	10	15	20	20 (Opt3)	50 (range cut 1μ m)	50 (ref)
$\epsilon_{80}~(\%)$	4.8	1.7	0.7	0.7	0.5	0.5	-
Range (mm)	331.9	326.6	325.1	324.8	325.2	324.7	325.0

Table 2. Influence of the number of bins used to initialize the pre-calculated EM tables on dose computation and proton range. Above 15 bins/decade, simulations lie within 0.3 mm in range and 0.7% of ϵ_{80} errors with the reference. The use of a 1 μ m range cut and Opt3 did not affect the simulations.

fluctuations are independent of the range cut and step parameters. Consequently, 248 the range cut parameter can be used at it should, i.e. to define the accuracy of the 249 electronic dose distribution along the ion track. For safety, it makes sense to set the 250 range cut and maximum allowed step equal or lower to the voxel size, around 1 mm for 251 clinical applications. The Opt3 parameters-list did not modify the results, however, the 252 simulation was performed using a simple homogeneous geometry and both the stepping 253 function and stepping algorithm may play a role in heterogeneous and voxelized media 254 like patient CT data. 255

256 3.3. Efficiency-based parameter selection

As regards the previous investigations and in view of the clinical implementation of dose calculation, simulation efficiencies were compared between the four different simulation settings presented below:

(i) 50 bins/decade, range cut and maximum allowed step at 1 mm.

²⁶¹ (ii) 50 bins/decade, range cut and maximum allowed step at 0.1 mm.

²⁶² (iii) 50 bins/decade, range cut at 1 μ m.

²⁶³ (iv) 50 bins/decade, range cut and maximum allowed step at 1 mm, Opt3.

The simulation efficiency (η) was calculated using equation (6), as defined in [19], taking into account the simulation time (T) and statistical uncertainty (σ) . The simulation statistical uncertainty (σ) was defined as the mean dose uncertainty between the entrance and the proton range.

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$$\eta = \frac{1}{\sigma^2 \cdot T} \tag{6}$$

Simulations were performed on a single 1.66 GHz CPU. Results are summarized in Table 3.

Simulation index	i	ii	iii	iv
Statistical uncertainty (%)	1.3	1.2	1.2	1.2
Time (s)	$1.4{ imes}10^2$	$1.2{ imes}10^3$	$5.4{ imes}10^4$	$1.5{ imes}10^2$
Efficiency	4.2×10^1	5.9	1×10^{-1}	4.3×10^1

Table 3. Simulation efficiency for four different settings.

Settings (i) and (iv) had a comparable efficiency, while settings (ii) and (iii) were respectively about 7 and 430 times lower. In settings (iv), Opt3 parameters were added to settings (i) and could only increase the simulation accuracy. Hence, settings (iv) were selected as the reference parameters-list, in order to perform robust and fast simulations.

275 3.4. Ionization potential of water

The proton range depends mainly on the mean ionization potential (I) of the medium. 276 The I value of water is a subject of growing interest and values between 67.2 eV and 85 eV 277 were reported in Table 1 from [20]. A recent study has also evidenced the uncertainty 278 related to the I values of human tissues, stating that this could lead to the use of 279 "sub-centimeter" clinical margins [21]. When the ionization potential of a medium is 280 not known, Bragg's additivity rule is used to compute the I value, by weighting the I 281 values of the different constituents. In GEANT4, the ionization potential is calculated 282 following Bragg's additivity rule by default for all user-defined media and is 70.9 eV for 283 water, however, the user has the possibility of changing this value. We tested different 284 values of I: 70.9 eV, 75 eV and 80 eV, which moved the proton range respectively to 285 324.9 mm, 329.2 and 330.8 mm, while the CSDA range given by NIST is 329.4 mm. We 286 then set the ionization potential of water to 75 eV, which is the value recommended by 287 ICRU reports 37 and 49 [22, 23]. 288

²⁸⁹ 4. GEANT4 comparison with PHITS and MCNPX

Simulation time, depth-dose profiles and transverse profiles at 10 cm, 30 cm and 32 290 cm depth simulated with GEANT4, were compared to PHITS and MCNPX for a 230 291 MeV proton beam, using a circular Gaussian spot of 3 mm sigma. We also assessed the 292 impact of MS on the lateral dose spreading compared to the SS algorithm implemented 293 in GEANT4. Depth-dose profiles were integrated along the z axis with a 1 mm step 294 and transverse profiles were scored in dosels (dose scoring voxels [24]) of $2 \times 2 \times 1 \text{ mm}^3$, 295 respectively in x, y and z (the beam direction). Default parameters were used for 296 PHITS and MCNPX, using a MS model and the ATIMA cross-section database for 297 PHITS. Transverse profiles were normalized to the maximum dose, in order to better 298 visualize the different dose spreading (Figure 5). 299



Figure 5. Comparison of depth-dose and transverse profiles at 30 cm depth using GEANT4, MCNPX and PHITS, for a 230 MeV proton beam in water.

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As regards depth-dose profiles, MCNPX and PHITS are in close agreement. Differences in the plateau and in the Bragg peak regions compared to GEANT4 may be explained by different HAD and EM models. A detailed investigation of these differences is out of the scope of this paper, but it is worthwile to note that the integral

energy deposited by a 230 MeV mon-energetic proton beam between 0 and 40 cm is in
average 215.5 MeV/proton with GEANT4, 204.7 MeV/proton with PHITS and 205.6
MeV/proton with MCNPX. The integral dose deposited by GEANT4 is 5% higher than
PHITS and 4% higher than MCNPX.

As regards transverse profiles, the proton beam spreading with depth in GEANT4 is narrower than in MCNPX and PHITS. Dose spreading was also estimated thanks to an analytical formula based on measurements using equation (4) from [25]. The dose spreading simulated with GEANT4 using the MS model was unable to reproduce the SS behavior (8% difference), which gave results close to the PHITS code. Results are presented in Table 4.

	GEANT4 (MS)	GEANT4 (SS)	MCNPX	PHITS	Szymanowski
$\sigma_{10cm} (\mathrm{mm})$	3.1	3.2	3.1	3.4	3.2
$\sigma_{30cm} (\mathrm{mm})$	6.2	6.8	7.3	6.8	7.1
$\sigma_{32cm} (\mathrm{mm})$	6.9	7.5	8.1	7.5	7.8

Table 4. Comparison of the transverse profile spreading (σ) at 10 cm, 30 cm and 32 cm depth for a 230 MeV proton beam in water using the GEANT4 MS algorithm, GEANT4 SS algorithm, MCNPX, PHITS and an analytical model (Szymanowski). The uncertainty on the σ values was estimated to 0.15 mm using ROOT. GEANT4 beam spreading is significantly lower than in MCNPX, PHITS and Szymanowski's model, even if it gets close to PHITS results using the SS algorithm. MCNPX shows the wider beam spreading.

The comparisons between the different MC codes and Szymanowski's analytical 314 model showed inconsistencies, with up to 15% difference (2.8 mm in FWHM) in the 315 lateral dose spreading simulated with GEANT4 and with MCNPX, at 32 cm depth in 316 water. Using SS instead of MS multiplies the number of steps and the simulation time 317 by three order of magnitude (330 steps per incident proton were recorded using the 318 MS model and more than 700×10^3 with the SS). The same simulations were performed 319 without the proton MS process and almost no beam spreading was observed, suggesting 320 that proton scattering is mainly due to the MS process, even if HAD collisions may 321 affect the profiles. As the computation of the spatial displacement is not part of the 322 MS theories, each MC code has to develop its own algorithm [12], which may explain 323 part of these discresspancies. Our first conclusion is that the proton MS process used in 324 GEANT4.9.2 should be revised. 325

As regards computation time, MCNPX and PHITS proton rates were respectively estimated to 127 and 29 protons \cdot s⁻¹ on a single 3.06 GHz CPU, using detailed simulation settings. On a comparable machine with a 2.33 GHz CPU, the GATE/GEANT4 proton rate was estimated to 263 protons·s⁻¹ using optimized settings. These simulation times have only an indicative purpose, since the simulations were performed on different machines, using different MC parameters.

332 5. Experimental measurements

Measurements were performed in Essen in Germany, with the new IBA PBS dedicated nozzle mounted on a rotating gantry. This nozzle allows for delivering circular spots of a few millimeters in diameter at the treatment isocenter. The Water Equivalent Thickness (WET) of the different media within the nozzle were estimated using equation (7). The resulting Nozzle water Equivalent Thickness (NET) was estimated to 1.7 mm.

$$WET_m = L \times \frac{\rho_m}{\rho_w} \times \frac{S_m}{S_w}$$
(7)

where the index m stand for medium and w for water. S and ρ are respectively the mass stopping powers (in MeV.cm².g⁻¹) and densities (in g.cm⁻³). WET_m is the medium WET (in cm) and L its thickness (in cm).

The Energy Selection System (ESS) is designed to provide one given beam of range R_{ESS} and energy E_{ESS} at the nozzle entrance. The corresponding range R_{Noz} and energy E_{Noz} at the nozzle output were therefore obtained by subtracting the NET. The R_{ESS} and R_{Noz} given correspond to ranges in water. Range to energy conversion was determined with a fit from the NIST PSTAR database [18].



Figure 6. Illustration of the measurement set-up of depth-dose profiles in water (a) and transverse profiles in PMMA (b). Figure (a) shows the nozzle (1), the proton beam direction (2) and the water phantom (3), with the reference Bragg-peak chamber (a), the sensitive Bragg-peak chamber (c) and the phantom entrance wall (b). Figure (b) presents the transverse profile set-up of the 98.71 MeV proton beam, with 4 radiochromic films (in blue) inserted between the PMMA slabs (3).

347 5.1. Reference pristine Bragg peak in water

Reference pristine Bragg peaks were measured in a $60 \times 60 \times 60$ cm³ water phantom 348 (Blue Phantom[®]), IBA-Dosimetry) for 5 energies, as presented in Table 5. Two large 349 Bragg peak chambers (PTW type 34070) with a 10.5 cm² sensitive area were used, so 350 that the proton beams were always fully integrated within the sensitive volume of the 351 chamber (Figure 6 (a)). The first chamber was placed at the phantom entrance and 352 used as a reference chamber to eliminate beam fluctuations. The second chamber was 353 placed in the phantom and moved along the z axis with a 1 mm increment. Measured 354 depth-dose profiles were shifted by 44.1 mm to account for the total set-up. 355

R _{ESS}	E_{ESS}	R_{Noz}	E_{Noz}	Measured ranges
(g/cm^2)	(MeV)	(g/cm^2)	(MeV)	(cm)
7.72	99.95	7.55	98.71	7.78
13.50	137.72	13.33	136.21	13.59
19.50	169.48	19.33	168.63	19.55
26.50	202.51	26.33	201.75	26.44
32.54	228.35	32.37	227.65	32.50

Table 5. Pristine Bragg peak measured for 5 energies. The settings at the nozzle exit (R_{Noz} and E_{Noz}) were estimated from the nozzle entrance parameters (R_{ESS} and E_{ESS}) and then measured in water (Measured ranges). Problems with nozzle calibration at the time of the measurements explain the discrepancies between set ranges and measured ranges.

³⁵⁶ 5.2. Reference transverse profiles in PMMA

Reference transverse profiles were measured with ISP self-developing EBT Gafchromic® 357 films inserted between uncalibrated PMMA slabs of 1 cm thickness (Figure 6 (b)) and 358 1.19 g.cm^{-3} density. The exact positions of the films between the slabs were recorded. 359 Transverse profiles were measured for 3 different energies, with 4 or 5 films inserted 360 between the slabs as summarized in Table 6. The film optical densities (OD) were 361 recorded using a Vidar scanner. For each film, the mean OD of a non-irradiated film, 362 considered as the background, was subtracted before normalization to the maximum 363 OD. Transverse profiles were measured with a grid resolution of $1 \times 1 \text{ mm}^2$, to mimick 364 the simulated matrix of dosels. At the time of the measurements, only a preliminary 365 version of the PBS system was available and the monitor units were not yet available. 366 Therefore, it was not possible to perform a calibration curve between the film OD and 367 doses. These preliminary measurements were used only qualitatively to illustrate the 368 beam widening increase with depth. 369

E_{Noz}	R_{Noz}	PMMA range	Pos 1	Pos 2	Pos 3	Pos 4	Pos 5
(MeV)	(g/cm^2)	(g/cm^2)	(mm)	(mm)	(mm)	(mm)	(mm)
98.71	7.55	7.74	0	19	49	59	-
153.01	16.33	16.77	0	50	99	128	-
210.56	28.33	29.13	0	88	186	216	226

Table 6. Positions of the transverse profiles measured with EBT radiochromic films inserted in a PMMA phantom. Four films were used at the lowest and medium energies and five at the highest energy.

370 6. GEANT4 comparison with measurements

371 6.1. Depth-dose in water

The evaluation of depth-dose profile simulations was based on three criteria: the proton 372 range, the peak dose error and the mean point-to-point dose error. Simulated and 373 measured depth-dose profiles were normalized to the integral dose deposited. There 374 was a discrepancy between measured ranges and system ranges, because the nozzle 375 had not been yet properly calibrated at the time of the measurements: the energies 376 were slightly higher than the set values, leading to measured ranges increased by 1.1 to 377 2.6 mm (Table 5). To further assess the dose deposited, we shifted the measurements 378 to compensate for the range difference with the simulations. Then, we adjusted the 379 energy spread of the incident beams in the simulation for the five energies to match the 380 measurements as closely as possible. The tuning stage of the energy spread was done 381 with an energy step of 0.05-0.1 % of the mean energy. The energy spread was adjusted 382 according to two criteria: the peak dose error and the mean point-to-point dose error 383 (ϵ_{80}) calculated using equation (5). Results obtained at the lowest and highest energies 384 are presented in Figure 7. 385

³⁸⁶ 10^5 protons were simulated. Lower peak dose errors were associated with lower ³⁸⁷ mean point-to-point dose errors, as presented for $E_{Noz} = 168.63$ MeV in Figure 8. ³⁸⁸ Simulated ranges lie within 0.3 mm of set ranges. Peak dose errors and mean point-to-³⁸⁹ point dose errors are around 1 %. Results are summarized in Table 7.

The dose statistical uncertainty was around 0.8% in the plateau region, around 0.4%in the Bragg peak region and around 12% in the tail. Consequently, these results were in good agreements with measurements. For absolute dosimetry using plane-parallel chambers, the standard dose uncertainty for clinical proton beams was estimated to 2.3% in [26].

395 6.2. Transverse dose profiles in PMMA

The simulation of the lateral dose spreading with depth of single pencil beams was assessed against measurements at 3 energies (98.71 MeV, 153.01 MeV and 210.56 MeV) using radiochromic films. The beam energy parameters were determined from the



Figure 7. Comparison between measured and simulated depth-dose profiles in water for the highest and lowest energies, respectively 227.65 MeV (b) and 98.71 MeV (a). The left and right axis correspond respectively to normalized doses and point-to-point errors. This figure shows that the simulations overestimate the dose deposit in the plateau entrance and underestimate the dose deposit in the Bragg peak region.



Figure 8. Tuning of the 168.63 MeV proton beam, by adjusting the energy spread of the simulation. The lowest peak error and ϵ_{80} error (referred to as "Mean dose error" in the figure key) correspond to a sigma energy spread of 0.5% of the mean energy.

R _{Noz}	E_{Noz}	σ_E simulation	ϵ_{80}	ϵ_{peak}	R _{Simu}
(g/cm^2)	(MeV)	(%)	(%)	(%)	(cm)
32.37	227.65	0.10	$1,\!1$	$1,\!1$	32.35
26.33	201.75	0.30	0.9	$0,\!4$	26.33
19.33	168.63	0.50	0.8	0.4	19.33
13.33	136.21	0.55	1.2	-0.8	13.31
7.55	98.71	0.60	1.2	0.4	7.52

Table 7. Assessment of depth dose profiles in water, in terms of peak dose error (ϵ_{peak}) , mean point-to-point dose error (ϵ_{80}) and range accuracy. The energy spread $(\sigma_E \text{ in } \%)$ adjusted in the simulations, increased with decreasing energy within a 0.1-0.6%.

17

³⁹⁹ previous depth-dose profile simulations.

The dose response mechanism of radiochromic films is not linear with dose and depends on the particle Linear Energy Transfer (LET) [27, 28]. Radiochromic films show a significant under-response in the Bragg-peak region, because of quenching effects due to high-LET particles [27, 28]. The radiochromic film dose response has been modeled following a logarithmic relation in [27], as shown in equation (8):

405
$$OD^{net}(D_{eff}) = \log(a'.D_{eff} + 1)$$

with a' the film response parameter, $OD^{net}(D_{eff})$ the net optical density after irradiation with an effective dose D_{eff} , which depends on the particle LET and dose deposit D. For low LET, $D_{eff} \simeq D$. As the LET increases, D_{eff} becomes lower than D, illustrating the film under-response.

The particle LET increases as its remaining range decreases with penetration in 410 water. Hence, for depth-dose profile measurements, the film response dependence on 411 LET has to be accounted for. Since our measurements were transverse to the beam 412 direction, the LET lateral variations were neglected in first approximation for relative 413 dosimetry in a given film: we assumed a homogeneous transverse LET spectrum at 414 a given depth. Further simulations showed that the mean proton energy was slightly 415 higher in the centre of the profiles compared to the sides, with increasing depth in 416 PMMA, while being the same at the phantom entrance. This suggests a possible 417 underestimation of the measured FWHM with depth, due to the transversal LET 418 distributions. However, it has been stated in [29], that radiographic films and diodes, 419 which are detectors also sensitive to the energy spectrum of the protons, can be safely 420 used to measure distributions perpendicular to the proton beam direction. 421

We compared the film OD FWHM (FWHM $_{OD}$) increase to the simulated transverse 422 dose profile FWHM (FWHM $_{simu}$) increase with depth. A Gaussian fit on the 423 radiochromic film OD measured at the beam entrance was performed using the ROOT 424 software [30] for the 3 energies. The spot FWHM in the x and y directions were then 425 used as input parameters in the simulations, so that $FWHM_{simu} = FWHM_{OD}$ at the 426 phantom entrance. The measured spot widths (sigma in OD) were between 3 and 427 6 mm depending on the energy. The uncertainty of radiochromic film measurements 428 was estimated to 5% for MD-55-2 films in [31]. The FWHM uncertainty on the fitted 429 measurements was estimated to 0.1 mm using ROOT. 430

Assuming FWHM_{dose} the true dose FWHM, it follows from the logarithmic relationship between OD and dose (equation (8)), that for a fixed FWHM_{dose}, the FWHM_{OD} decreases as the dose decreases(Figure 9 (b)). Hence, the true dose spreading increase with depth should be even higher than the "OD spreading" increase with depth, because the dose at the beam axis decreases with depth (contrary to the integral dose), as illustrated in Figure 9 (a).

Results obtained using GEANT4 for x profiles with $E_{Noz} = 210.56$ MeV at 3 different depths are presented in Figure 10. Similar results were obtained for the 2 other energies. The $\frac{FWHM_{simu}}{FWHM_{OD}}$ ratio at different depths for x and y profiles for the 3

(8)



Figure 9. (a) Simulated dose spreading with depth in PMMA of a 211 MeV proton beam, with a circular spot of 3 mm sigma. While the integral depth dose increases continuously with depth, the depth dose at the beam axis decreases with depth with a factor around 2 between the entrance and the Bragg peak and increases again within the last 2 cm.

(b) Illustration of the FWHM_{OD} increase with increasing dose, compared to a constant FWHM_{dose} of 3 mm, with a maximum dose varying between 1 Gy and 8 Gy, for 2 different film parameters: a = 1 and a = 0.1, using equation (8).



Figure 10. Comparison between simulated transverse dose profiles and measured transverse OD profiles in PMMA, for a 210.56 MeV proton beam at three depths: 0 mm, 186 mm and 226 mm. It shows that the beam spreading with depth is not sufficiently accounted for in the simulation.

energies are presented in Figure 11 and illustrates the lack of dose spreading with depth 440 of the GEANT4 MC code compared to measurements. It is important to notice, that 441 the previous discussions about the film LET and dose response dependences suggested 442 that the qualitative measurements presented, also under-estimate the true lateral dose 443 spreading with depth. This study corroborates the fact that the MS model implemented 444 in GEANT4.9.2 release underestimates the lateral dose spreading with depth, even 445 though further comparisons with quantitative measurements are required to fix the 446 dose spreading accuracy achievable by MC simulation. 447



Figure 11. This figure illustrates the lack of lateral dose spreading with depth compared to measurements in PMMA, using the GEANT4 MS model. Comparisons are shown at 3 energies (210.56 MeV, 153.01 MeV, 98.71 MeV) in 2 lateral directions (x and y).

448 7. Conclusion

This study applied to proton PBS simulations aimed at better understanding the settings 440 of the GEANT4.9.2 MC code. Two major simulation parameters are the maximum 450 step size and the range cut, which should be defined in accordance with the voxel size. 451 Another key parameter is the binning of the EM tables, which needs to be set high 452 enough (> 15 bins/decade) to ensure accurate interactions, independently of the range 453 cut and maximum allowed step values. An optimized parameters-list has been proposed 454 in order to perform robust and efficient simulations, that are competitive in terms of 455 simulation time with other MC codes like MCNPX and PHITS. A reference physics-456 list for proton-therapy has been presented, using the EM standard package combined 457 with the precompound model for inelastic HAD collisions. The ionization potential 458 of water was set to 75 eV, in accordance with ICRU reports 37 and 49. Depth-459 dose profile simulations were in satisfactory agreement with reference measurements 460 performed in water, with 0.3 mm range accuracy. Peak errors were less than 1.1%461 and mean point-to-point errors (ϵ_{80}) were around 1%. Inconsistencies were pointed 462 out for transverse profile simulations using different MC codes, with up to 15% of 463 dose spreading difference between GEANT4 and MCNPX at 32 cm depth in water. 464 Transverse dose profile simulation issues using GEANT4 were attributed to the MS 465 algorithm, which was not able to reproduce the SS dose spreading with depth. Further 466 comparisons against measurements in PMMA corroborated these results, showing that 467 the lateral dose spreading with depth is not sufficiently accounted for in the simulation. 468 The MS algorithm accuracy is currently the limiting factor for PBS simulations, since 469 the dose spreading of each single beam is very important for patient dose calculation. 470 Improvements of the MS algorithm are expected with the new GEANT4.9.3 release, 471

which is being evaluated. Investigations using quantitative measurements are necessary
to fully estimate the lateral dose spreading accuracy achievable by MC simulation.
Further studies investigating the effects of patient heterogeneities, using a MC pencil
beam model of the new IBA PBS dedicated system and TPS comparisons will follow.

476 Acknowledgments

This work was conducted in a collaboration between the IBA company and the Creatis laboratory. The research leading to these results has received funding from the [European Community's] Seventh Framework Programme [FP7/2007-2013] under grant agreement n° 215840-2. We also acknowledge the GEANT4 collaboration, especially Vladimir Ivantchenko for fruitful discussions.

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