



Digital Image Processing

Image Analysis: Part 3

Summary

- I. Introduction
 - II. Digital image fundamentals
 - III. Discrete 2D processing
 - IV. Image improvement
-
- V. Image analysis (6 heures)**



Image analysis

■ Books

- ★ ***Gonzales* : Digital Image Processing, Prentice Hall (3th Ed.)**

Summary

V. Image Analysis

- ✓ *Part 1*
 - Mathematical Morphology
 - Contour detection and analysis

- ✓ *Part 2*
 - Region-based segmentation

- ✓ *Part 3*
 - Shape analysis
 - Shape recognition

Summary

V. Image Analysis

- ✓ *Part 1*
 - Mathematical Morphology
 - Contour detection and analysis

- ✓ *Part 2*
 - Region-based segmentation

- ✓ **Part 3**
 - **Shape analysis**
 - Shape recognition

Shape analysis

■ Context

- Definition
- Properties

■ Classical methods

- Including boxes
- Fourier descriptors

Shape analysis

■ Context - Definition

Step that consists on identifying an object using only its shape

- ▶ **This step is usually performed after a segmentation procedure**

Shape analysis

■ Contexte - Définition

- ▶ **Shape analysis procedures are based on the use of shape descriptors**
- ▶ **There exists several shape descriptors:**
 - **Including boxes**
 - **Moment descriptors**
 - **Guzman polygons**
 - **Freeman chain coding**
 - **Fourier descriptors**

Shape analysis

■ Contexte - Propriétés

- ▶ A good shape descriptor should have the following properties:
 - good fidelity to the initial shape
 - good discrimination between different shapes
 - good behavior with shape recognition operations :
 - invariance with translation
 - invariance with rotation
 - invariance with scaling

Shape analysis

■ Contexte - Applications

- ▶ **Shape recognition step can be useful in different fields :**
 - **sorting procedures in industry**
 - **Characters recognition for book scanning**
 - **Guidance and/or supervising tasks**

Shape analysis

■ Context

- Definition
- Properties

■ Classical methods

- Including boxes
- Fourier descriptors
- Others

Shape analysis

■ Including boxes

Designing of a box that includes the object of interest and whose axis are aligned with the image axis

- ▶ **This descriptor is only defined by its dimensions**

$$\left\{ \begin{array}{l} (x_{\max} - x_{\min}) \\ (y_{\max} - y_{\min}) \end{array} \right.$$

Shape analysis

- Including boxes - Example
 - ▶ Detection of filarial structures



Binary image obtained after segmentation

Shape analysis

■ Including boxes - Example

- ▶ *1st step: objects labeling*



Involved algorithm

- *Morphological tool (labeling)*

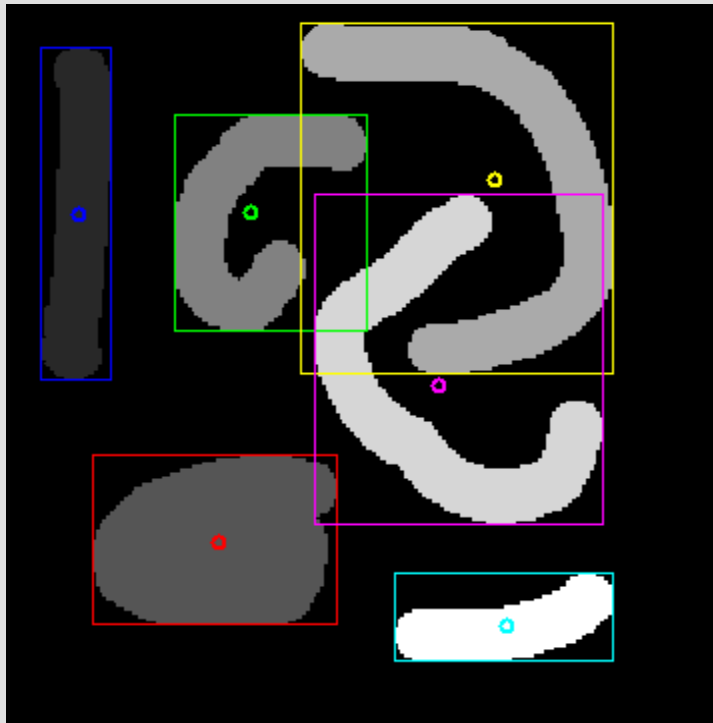
```
% Load image
img = imread('pieces.png');
img = double(img(:,:,1));

% create labeling
result = bwlabel(img,8);
```

Shape analysis

■ Including boxes - Example

- ▶ *2nd step: characterization of each object*



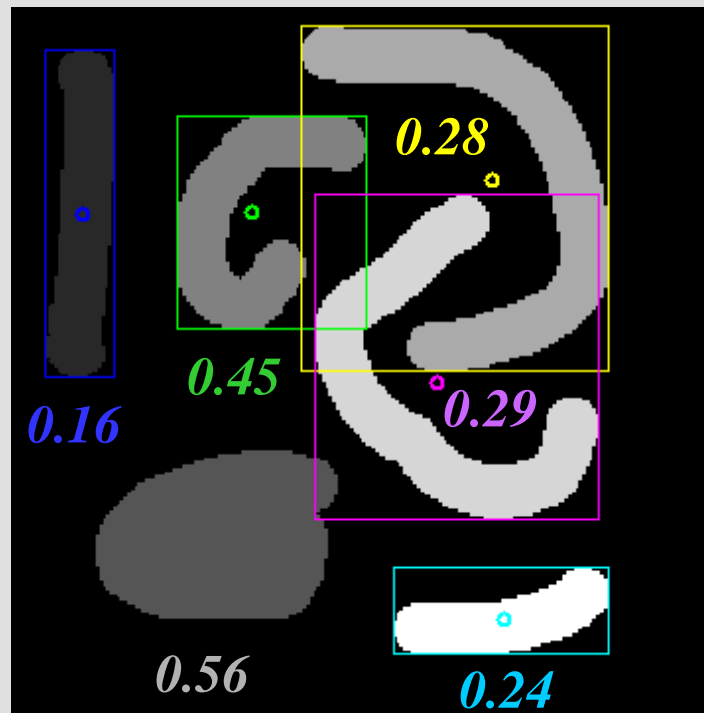
Involved algorithm

- *Characterization using including boxes*

Shape analysis

■ Including boxes - Example

- ▶ *3rd step: computation of discriminate parameters*



Parameters used

- *Ratio between object area / including square area*
- *One keeps objects whose corresponding parameter has a value lower than 0.5*

Shape analysis

■ Including boxes - Properties

- ▶ These descriptors are used when the shape of interest is particularly irregular
- ▶ Invariance with translation only

Shape analysis

■ Context

- Definition

- Properties

■ Classical methods

- Including boxes

- Fourier descriptors**

Shape analysis

■ Fourier descriptor

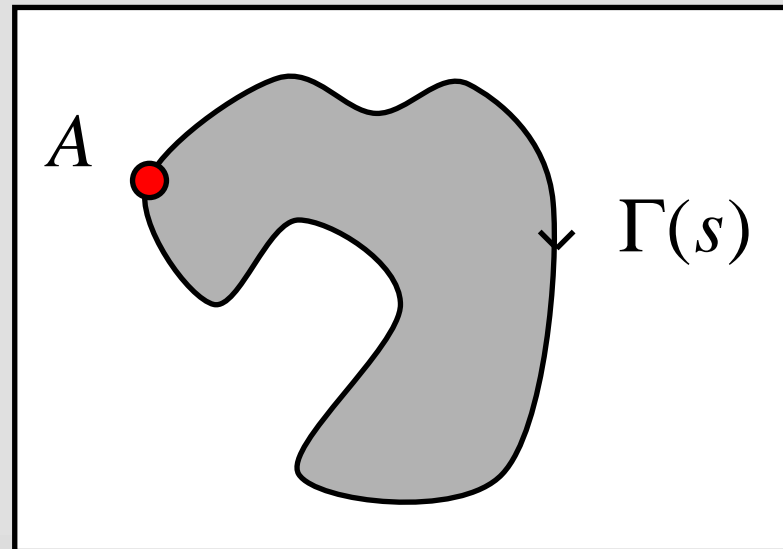
Tool which allows to describe the external shape of an object, that is to its contour

- ▶ **There exists two kinds of Fourier descriptors**
 - **tangential descriptors**
 - **complex descriptors**

Shape analysis

■ Tangential descriptor - Principle

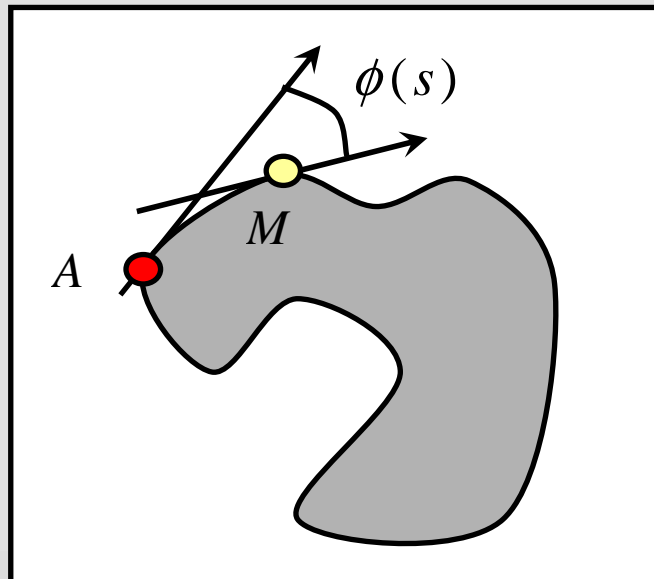
- ▶ One considers the shape of the contour to describe as a continuous curve defined by its curvilinear parameter s from an reference A



Shape analysis

■ Tangential descriptor - Principle

- ▶ Characterization of the curve from the angle $\phi(s)$ defined by the tangential vector at each point and the tangential vector at the reference point A



Shape analysis

■ Tangential descriptor - Expression

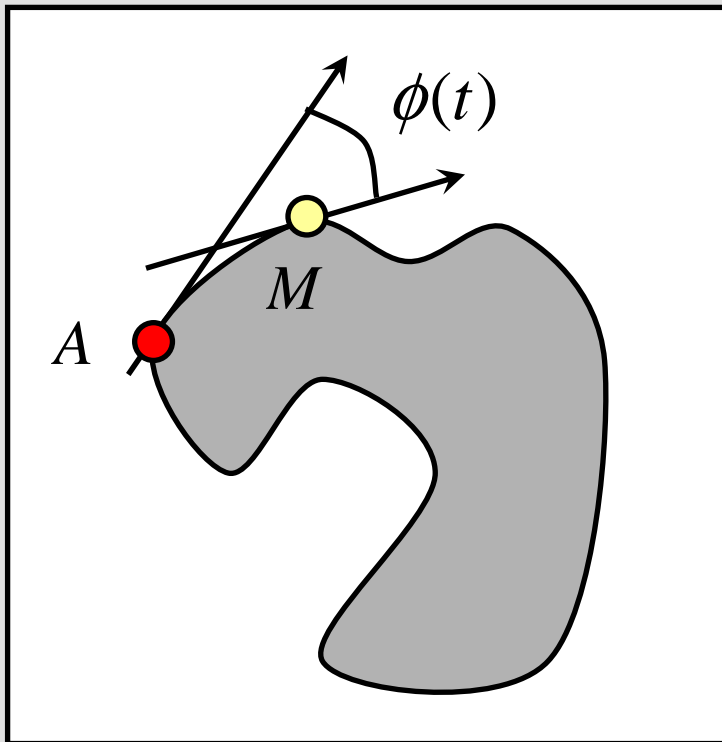
- ▶ Definition of a variable t whose values are defined between 0 and 2π

$$t = \frac{2\pi \cdot s}{L}$$

Where L : length of the curve

Shape analysis

■ Tangential descriptor - Expression



$$\phi(t) = \phi \left[\frac{2\pi \cdot s}{L} \right] - \frac{2\pi \cdot s}{L}$$

- The additional term takes into account the periodic property of the tangent function (which occurs after one loop of the curve)

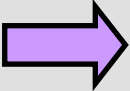
Shape analysis

■ Tangential descriptor - Properties

- ▶ This descriptor is invariant with :
 - translation
 - scaling (as soon as t is normalized)
 - rotation (as soon as one uses the angle difference between 2 tangents)
 - change of reference A

Shape analysis

■ Tangential descriptor - Properties

- ▶ In order to compare two shape, one has to compare there descriptor values in increasing order
- ▶ Unfortunately, the geometric explanation is not easy
- ▶ Idea  Define an other representation which allows an easiest geometric explanation

Shape analysis

■ Complex descriptor - Principle

- ▶ Discrete representation of the shape from a set of points $\{M_i\}$ that describes the contour of the object
- ▶ Characterization of the shape in a complex plan : each point M_i is associated with a complex number $z_i = x_i + j y_i$

Shape analysis

■ Complex descriptor - Principle

- ▶ In this context, the Fourier descriptors correspond to the coefficients of the discrete Fourier transform Z of z

$$Z_k = \sum_{i=0}^{N-1} z_i \exp(-j2\pi ik)$$

Where $k \in [-N/2 + 1, N/2]$

Shape analysis

■ Complex descriptor - Properties

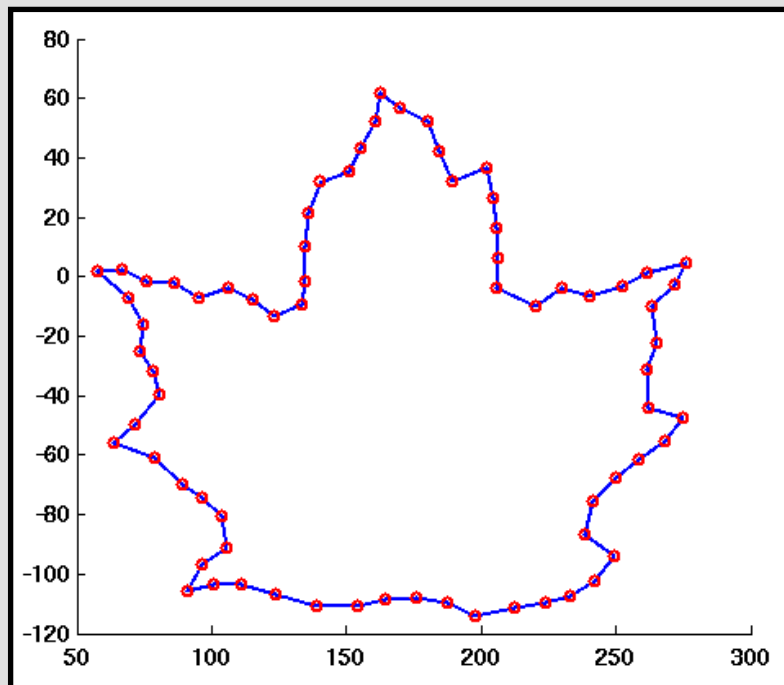
▶ Influence of coefficient Z_0

- Z_0 corresponds to the center of gravity of the shape
- If we force that $Z_0 = 0$, the shape description is invariant with translation

Shape analysis

■ Complex descriptor - Properties

▶ Influence of coefficient Z_0



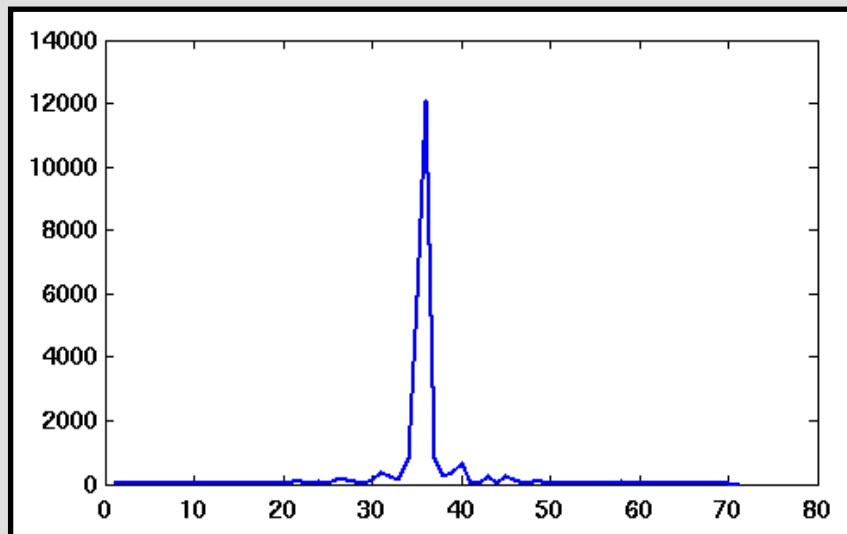
Initial contour

```
% Creation of a list of complex  
% numbers  
ListCmp = complex(ListPoint(:,1), ...  
                  ListPoint(:,2));  
  
% Display of the list  
figure; plot(ListCmp,'or');
```

Shape analysis

■ Complex descriptor - Properties

▶ Influence of coefficient Z_0



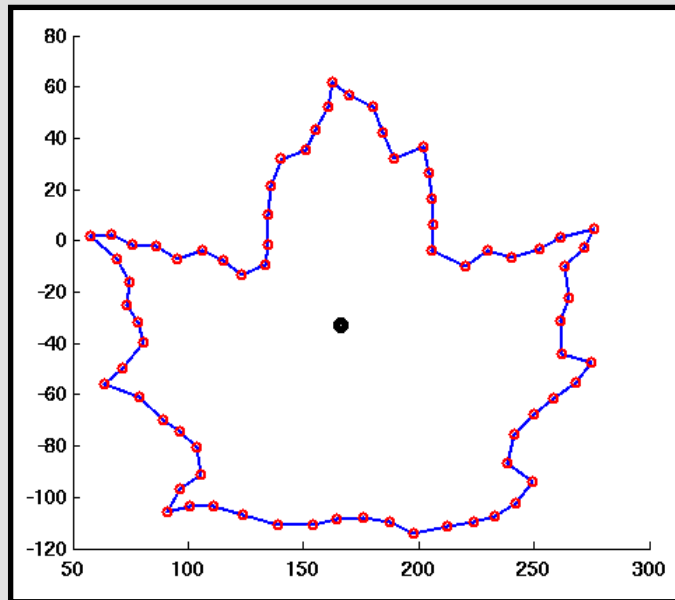
Spectrum of the initial contour

```
% Computation of the corresponding  
% spectrum  
FD = fft(ListCmp);  
  
% Display of the spectrum  
figure; plot(abs(fftshift(FD)));
```

Shape analysis

■ Complex descriptor - Properties

▶ Influence of coefficient Z_0



```
% Computation of the new spectrum
```

```
FDNew = FD;  
FDNew(2:end) = 0;
```

```
% Display of the corresponding  
% contours
```

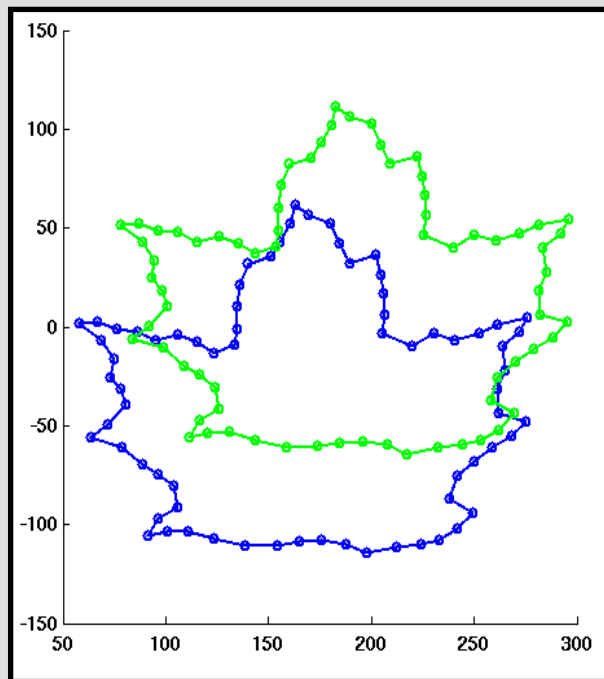
```
figure; plot(ifft(FD),'or');  
hold on; plot(ifft(FDNew),'ok');
```

*Initial contour + reconstructed contour
(black) which corresponds to the
center of gravity of the contour*

Shape analysis

■ Complex descriptor - Properties

▶ Influence of coefficient Z_0



Initial contour (blue) + translated contour (green)

```
% Computation of the translated contour
ListTransCmp(:,1) = ListCmp(:,1) + 20;
ListTransCmp(:,2) = ListCmp(:,2) + 50;

% Display of the corresponding contours
figure; plot(ListCmp,'ob');
hold on; plot(ListTransCmp,'og');

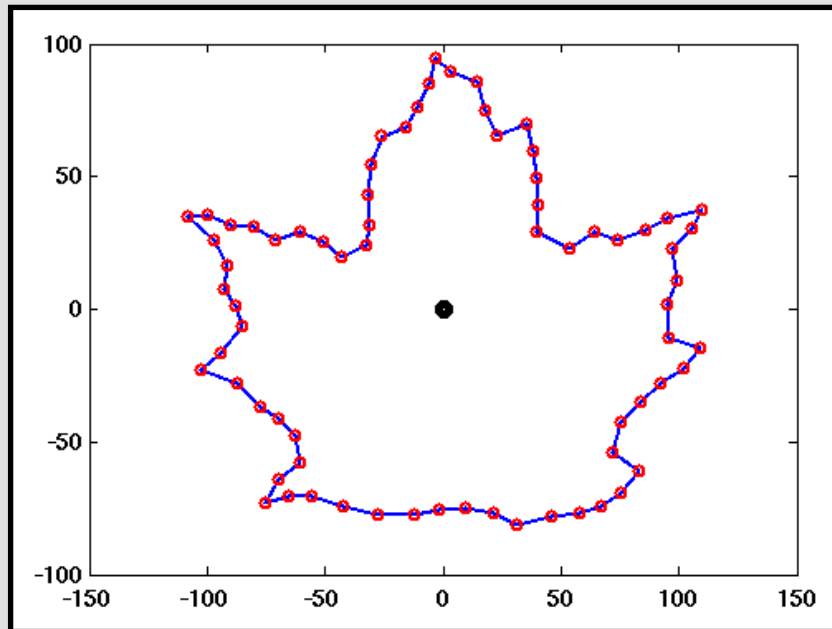
% Computation of the corresponding
% spectrum
FD = fft(ListCmp);
FDTrans = fft(ListTransCmp);

% invariance with translation
FD(1) = 0;
FDTrans(1) = 0;
```

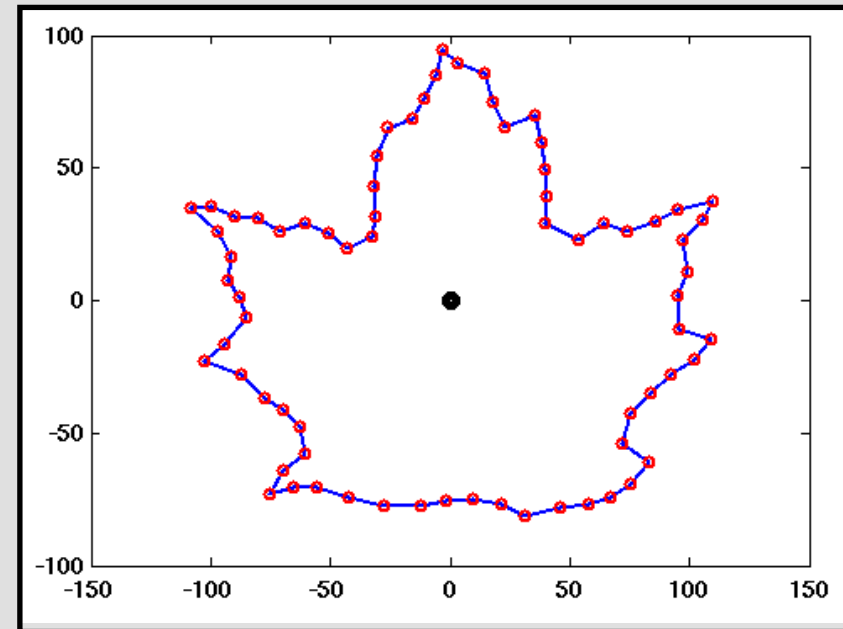
Shape analysis

■ Complex descriptor - Properties

▶ Influence of coefficient Z_0



*Reconstruction of the initial contour
after applying invariance with
translation*



*Reconstruction of the translated
contour after applying invariance
with translation*

Shape analysis

■ Complex descriptor - Properties

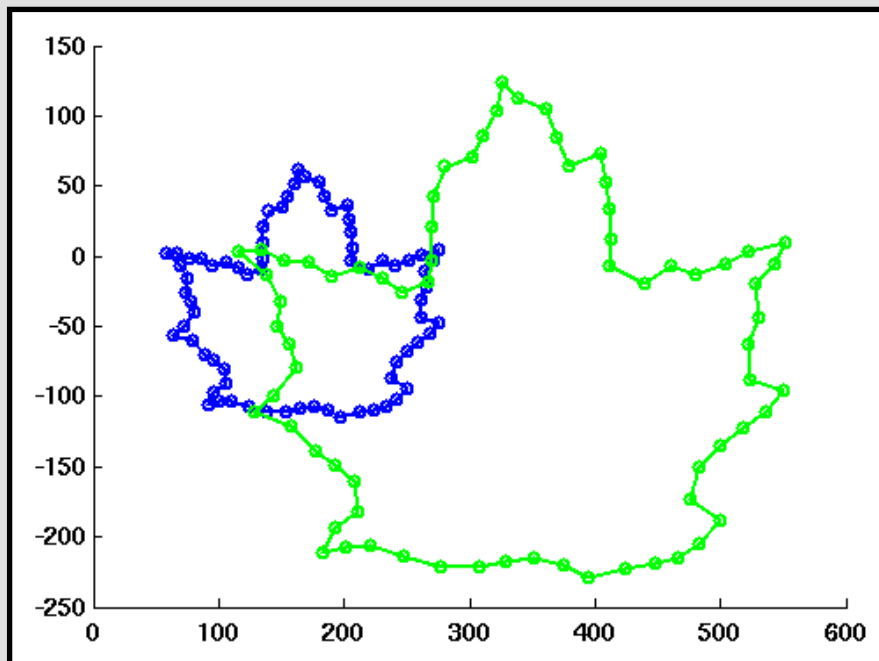
▶ Influence of coefficient Z_1

- In the case where all the coefficients Z_k are equal to 0 except for $k=1$, the reconstructed shape corresponds to a circle with radius equal to Z_1
- Z_1 acts as a scaling factor
- The normalization with $|Z_1|$ make the reconstructed shape invariant with homothecy

Shape analysis

■ Complex descriptor - Properties

▶ Influence of coefficient Z_1



Initial contour (blue) + contour with a scaling factor (green)

```
% Computation of the contour with a
% scaling factor
ListCECmp(:,1) = ListCmp(:,1) * 2;
ListCECmp(:,2) = ListCmp(:,2) * 2;

% Display of the corresponding contours
figure; plot(ListCmp,'ob');
hold on; plot(ListCECmp,'og');

% Computation of the corresponding
% spectrum
FD = fft(ListCmp);
FDCE = fft(ListCECmp);

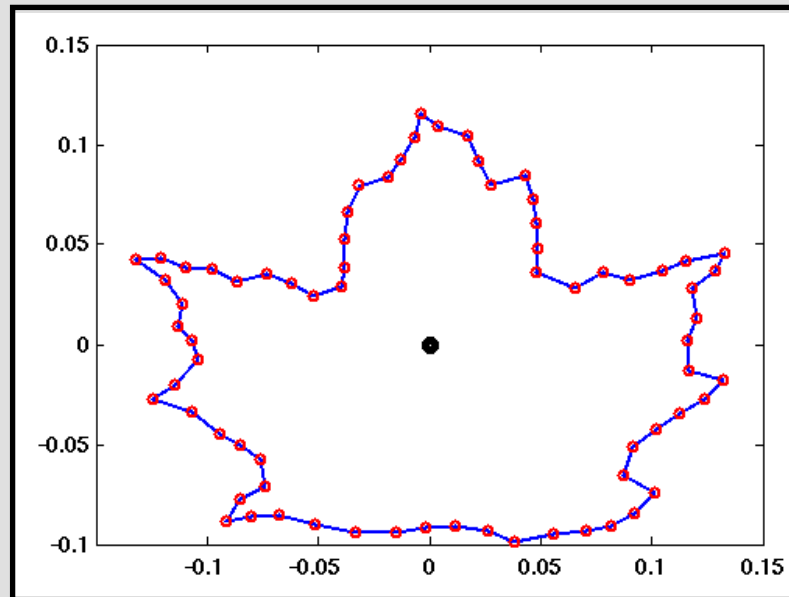
% invariance with translation
FD(1) = 0; FDCE(1) = 0;

% invariance with scaling
FD = FD / abs(FD(2));
FDCE = FDCE / abs(FDCE(2));
```

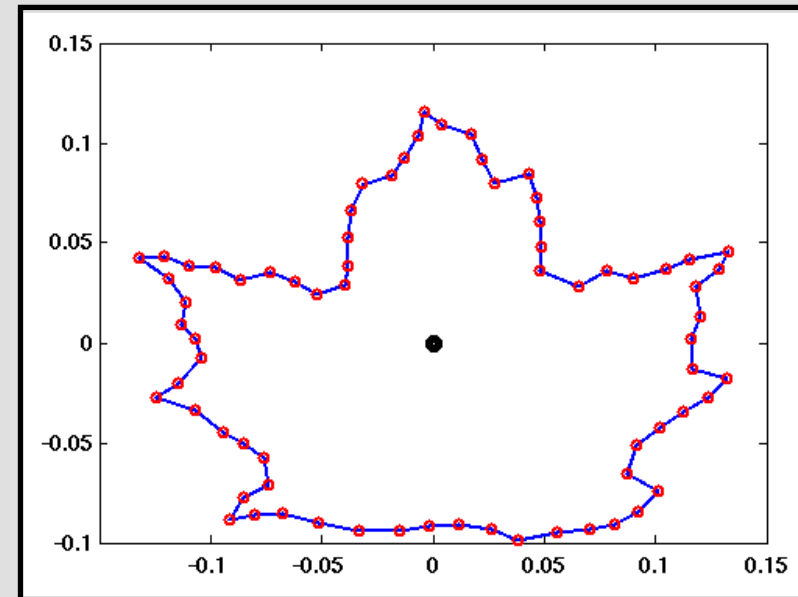
Shape analysis

■ Complex descriptor - Properties

▶ Influence of coefficient Z_1



Reconstruction of the initial contour after applying invariance with both translation and scaling



Reconstruction of the modified contour after applying invariance with both translation and scaling

Shape analysis

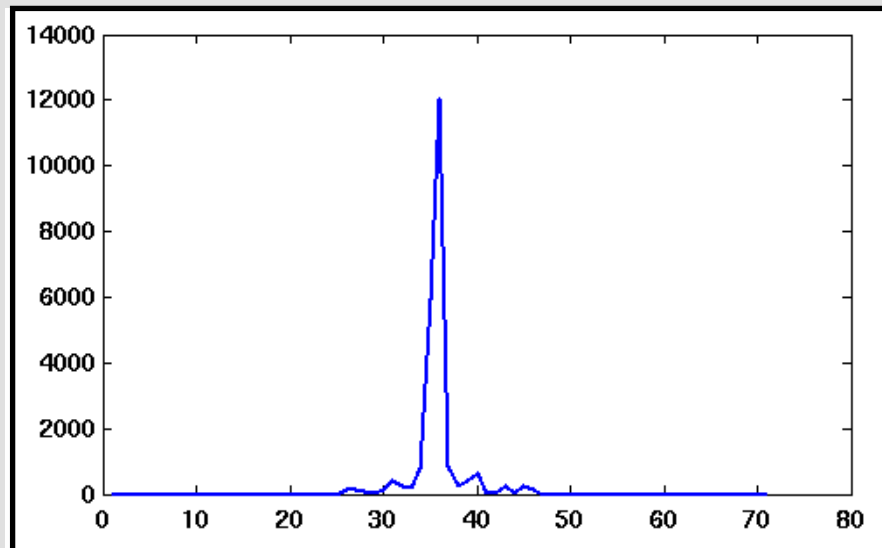
■ Complex descriptor - Properties

- ▶ Influence of the others coefficients Z_k
 - The truncation of Fourier descriptors provides the reconstruction of smoothed version of shapes which remain closed

Shape analysis

■ Complex descriptor - Properties

► Influence of the others coefficients Z_k



Spectrum of the simplified contour

```
% Computation of the corresponding
% contour
FDS = fftshift(fft(ListCmp));

% Simplification of the
corresponding spectrum
FDS(1:25) = 0;
FDS(47:71) = 0;

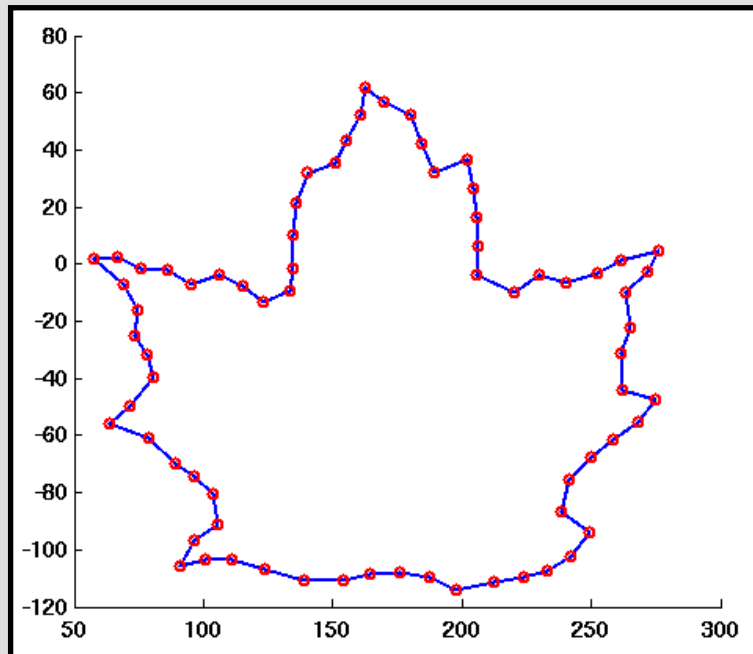
% Reconstructed contour
Contour = ifft(ifftshift(FDS));

% Display of the reconstructed
% contour
figure; plot(Contour, 'or');
```

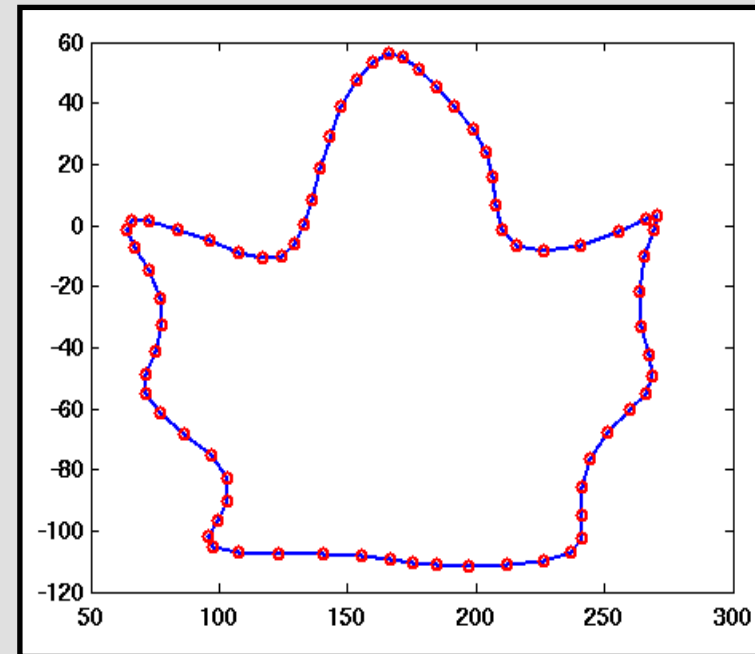
Shape analysis

■ Complex descriptor - Properties

▶ Influence of the others coefficients Z_k



Reconstruction of the contour before simplification



Reconstruction of the contour after simplification

Summary

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- ✓ *Part 1*
 - Mathematical Morphology
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- ✓ **Part 3**
 - Shape analysis
 - **Shape recognition**



Shape recognition

- **Context**

- Definition**

- **Classical methods**

- Line approximation**

- Hough transform**



Shape recognition

■ Context - Definition

From a set of points (pixels) obtained after some image processing, one aims at finding a prior shape which described the best the extracted points



Shape recognition

- Context

- Definition

- **Classical methods**

- Line approximation**

- Hough transform**

Shape recognition

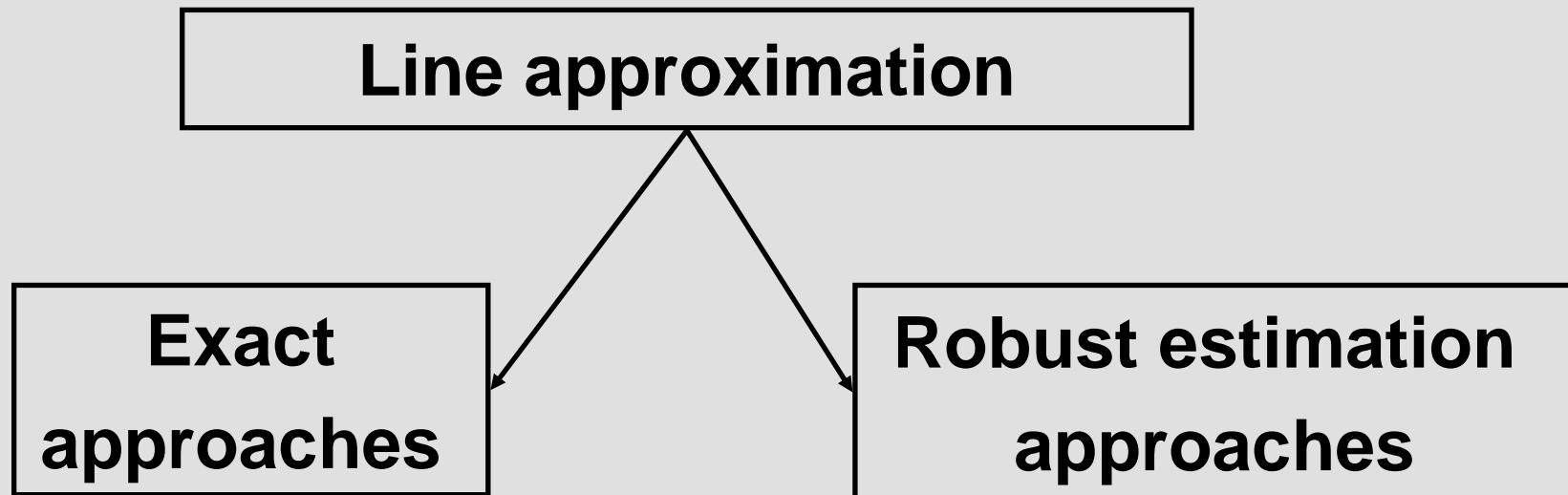
■ Line approximation

Least square methods derived from the minimization of a distance measure

- ▶ **There exists several methods which deals with this particular problem**

Shape recognition

- **Line approximation - groups**



Shape recognition

■ Exact approaches

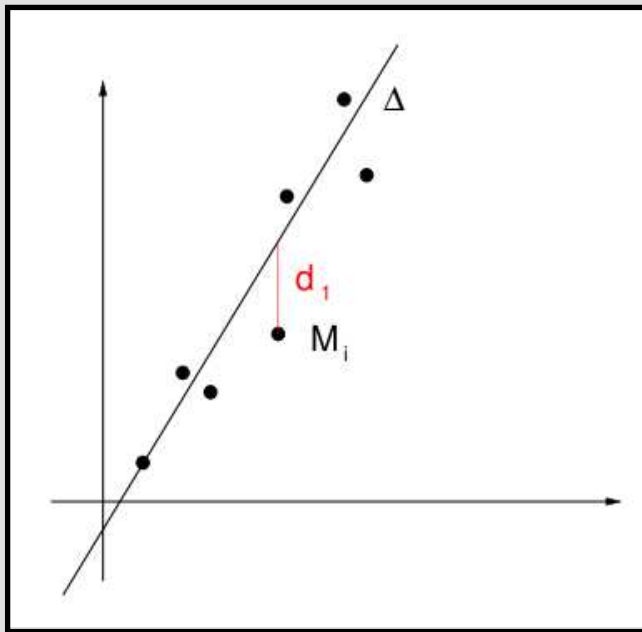
One considers that every extracted points acts with the same influence on the line description

- ▶ **There exists several methods**
 - **Linear regression approaches**
 - **Principal axes of inertia approaches**
 - ...

Shape recognition

■ Linear regression approximation

- ▶ Find the line $\Delta: y = a_0 + a_1x$ which minimized the following distance



$$d_1^2 = \sum_{i=1}^N [y_i - (a_0 + a_1x_i)]^2$$

Shape recognition

■ Linear regression approximation

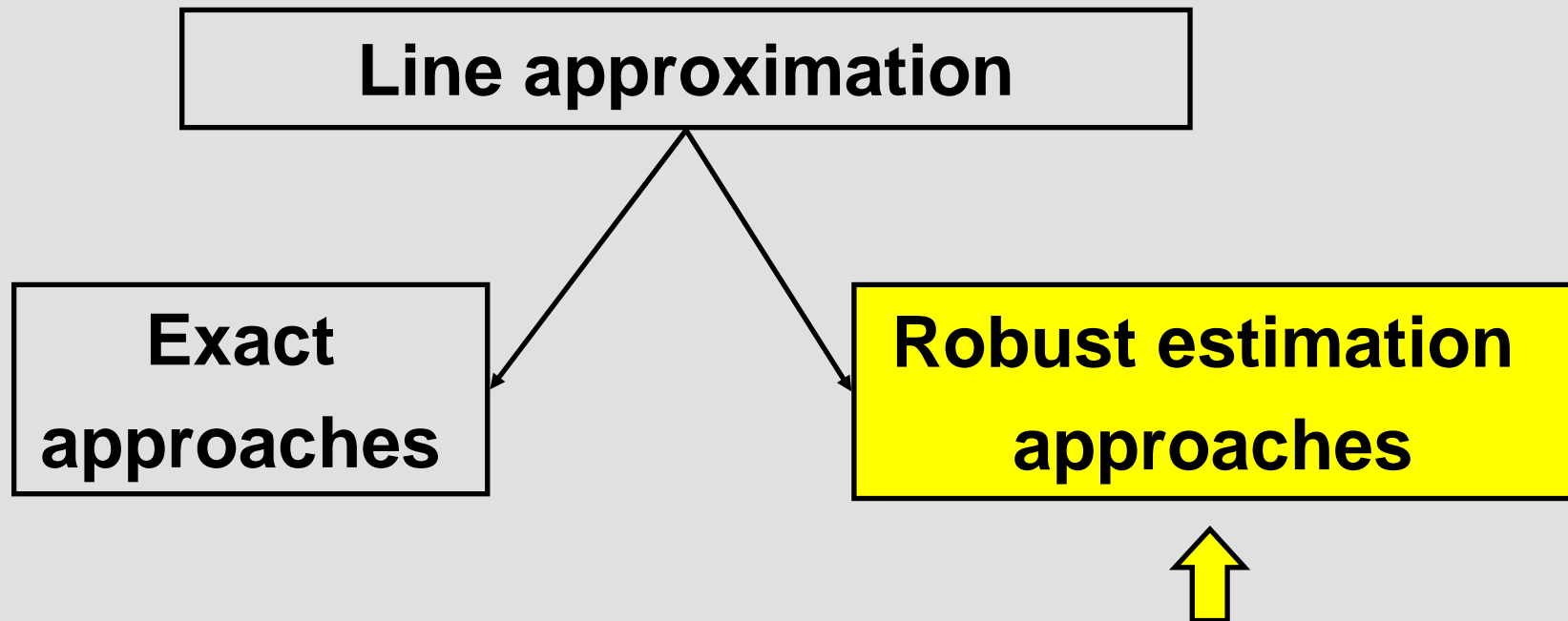
- ▶ The solution is simply obtained from the set of points (x_i, y_i) using the following equation

$$A = (X^t X)^{-1} X^t Y$$

where $X = \begin{bmatrix} 1 & x_1 \\ \dots & \dots \\ 1 & x_N \end{bmatrix}$ $Y = [y_1, y_2, \dots, y_N]^t$ $A = [a_0, a_1]^t$

Shape recognition

- Line approximation - groups



Shape recognition

■ Robust estimation approaches

Find the best line that represents the set of points under the hypothesis that each points position is corrupted by noise

- ▶ There exists several methods:
 - median estimation
 - mixture of lines estimation
 - ...

Shape recognition

■ Median estimation approaches

▶ Principle (1/2)

- One choose two points M_i et M_j from the set
- These points are associated with the corresponding line defined by coefficients a_{ij} and b_{ij}
- From combinations applied on the set of points, one obtain two sets of coefficients $a = \{a_{ij}\}$ et $b = \{b_{ij}\}$

Shape recognition

■ Median estimation approaches

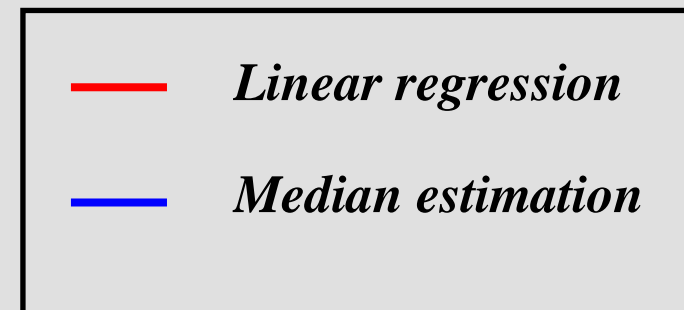
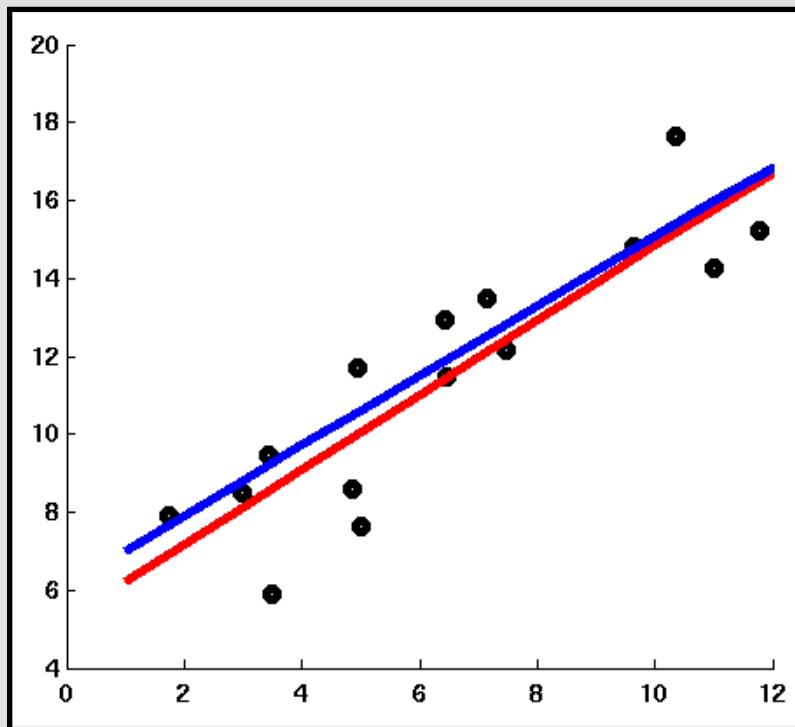
▶ Principle (2/2)

- The sets a et b are sorted separately and one keep the median values a_m et b_m as the parameters of the estimated line
- This estimator can approximate line even with *50%* of incorrect points

Shape recognition

■ Median estimation approaches

▶ Example





Shape recognition

- Context

- Definition

- **Classical methods**

- Line approximation

- Hough transform**

Shape recognition

■ Hough transform

The Hough transform associated with a parametric shape $\{a_i\}$ corresponds to the transformation which defines the way to go from the image space I to the parameter space H

- ▶ This tool can be applied to any parametric shape (ex: line, circle, ellipse, parabola, sinusoidal traces)

Shape recognition

■ Hough transform - Example

▶ Line detection (1/5)

- One associate to any point $M_i = (x_i, y_i)$ in the image space I , all the lines that pass through this point
- This set of lines is defined in the parameter space H through $b = -ax_i + y_i$

Shape recognition

■ Hough transform - Example

▶ Line detection (2/5)

- One point in I is then associated with a line in H

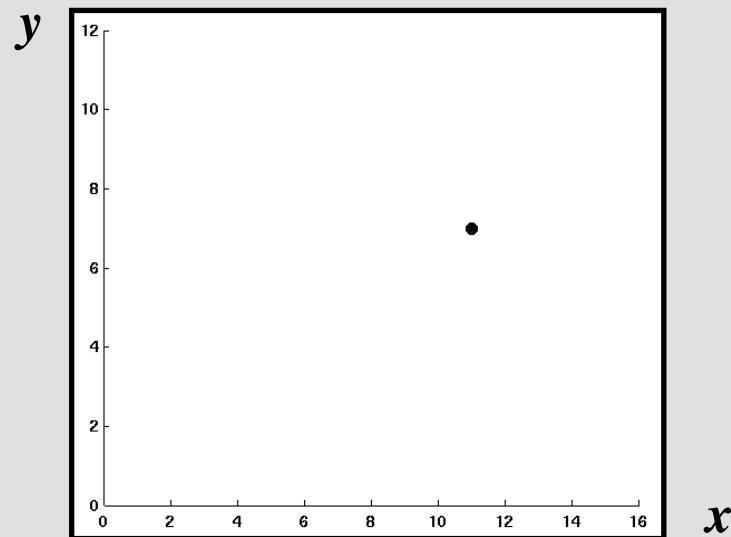
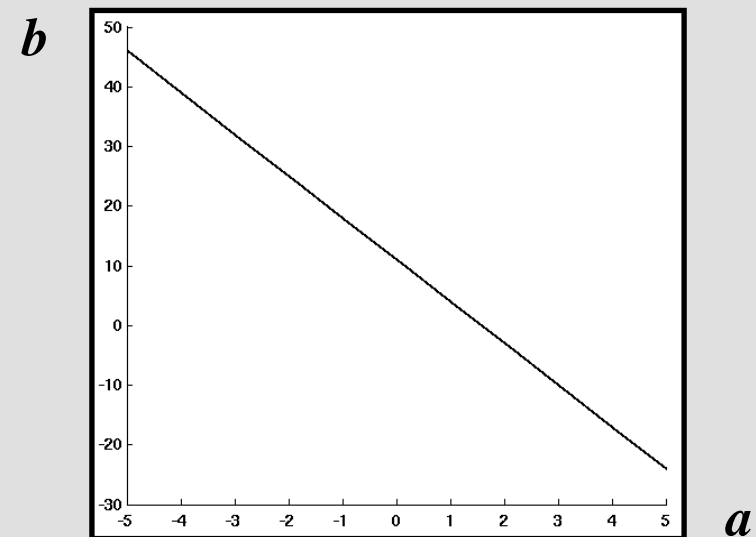


Image space



Parameter space

Shape recognition

■ Hough transform - Example

▶ Line detection (3/5)

- If this procedure is applied to every pixel of the image, one associates I with a set of lines that should cross each others on some points H_k

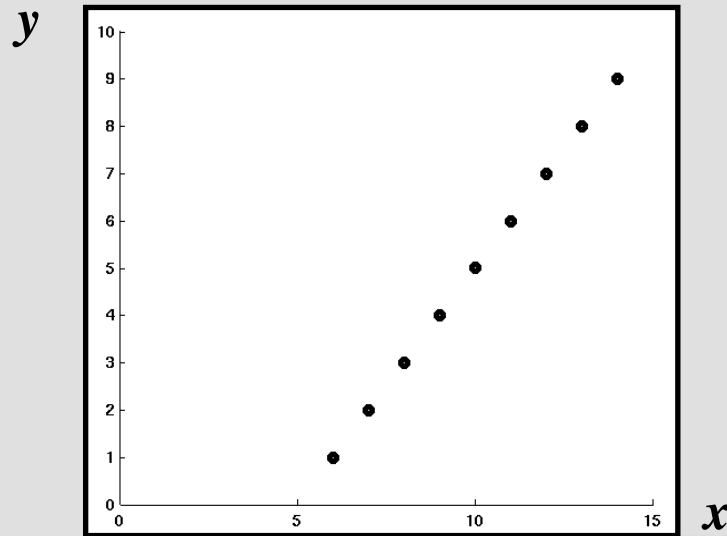
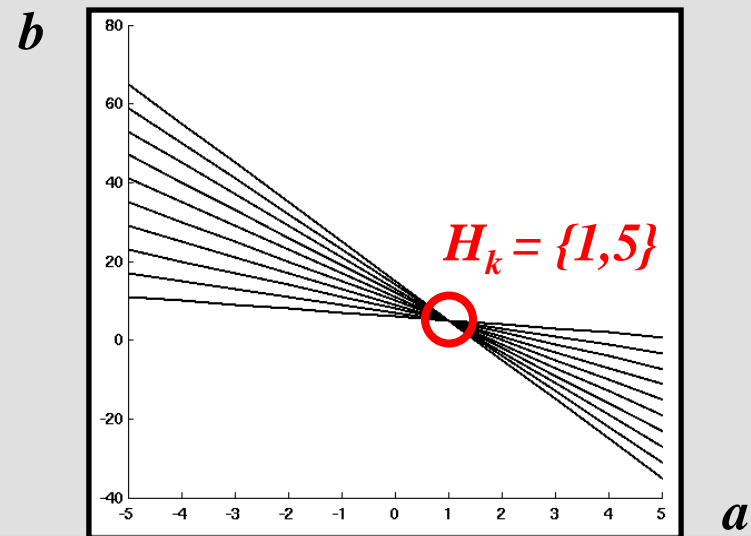


Image space



Parameter space

Shape recognition

■ Hough transform - Example

▶ Line detection (3/5)

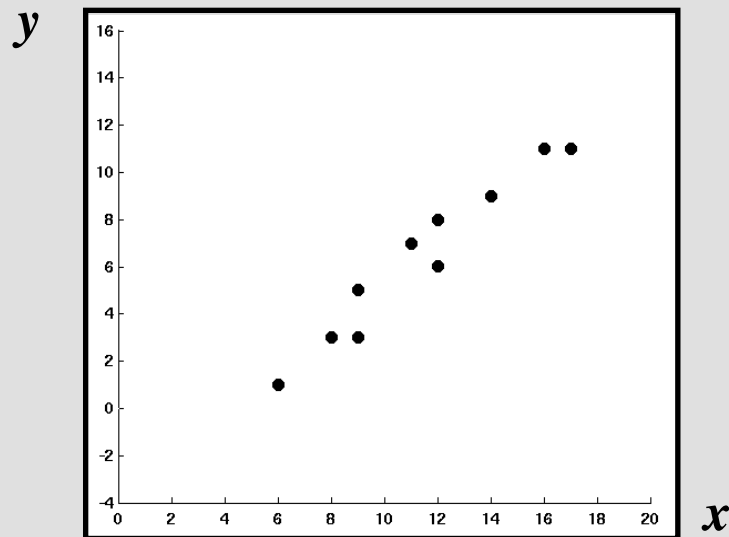
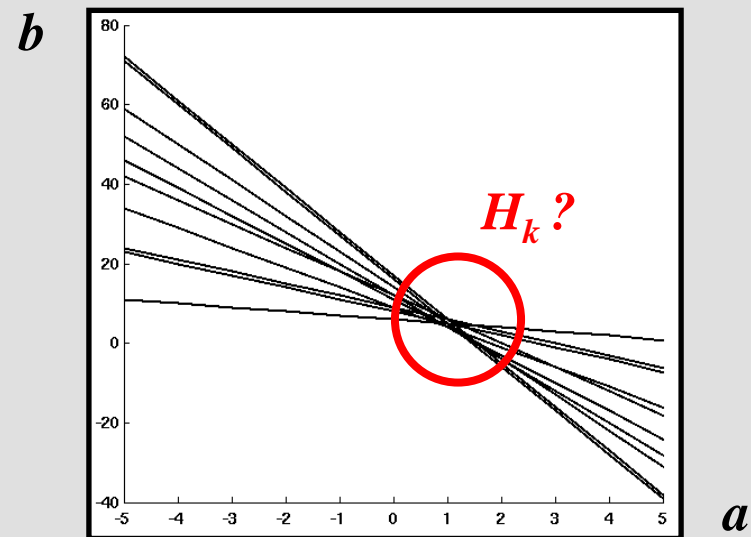


Image space



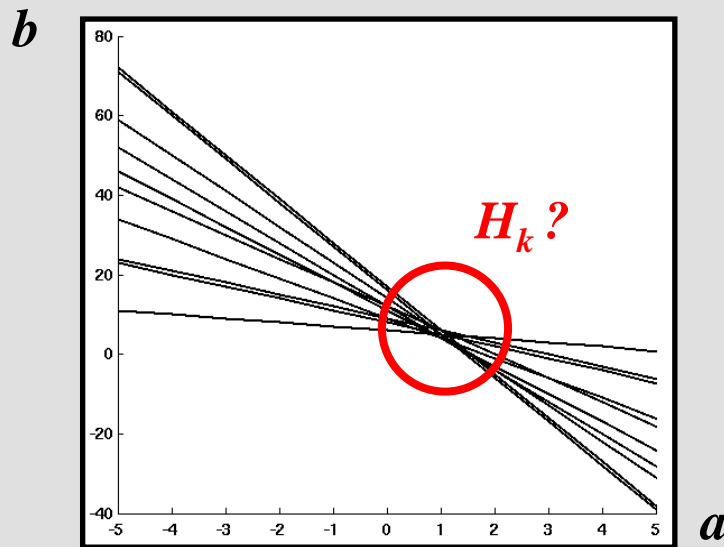
Parameter space

Shape recognition

■ Hough transform - Example

▶ Line detection (4/5)

- The intersection points H_k correspond to the lines of interest in the image space I



Parameter space

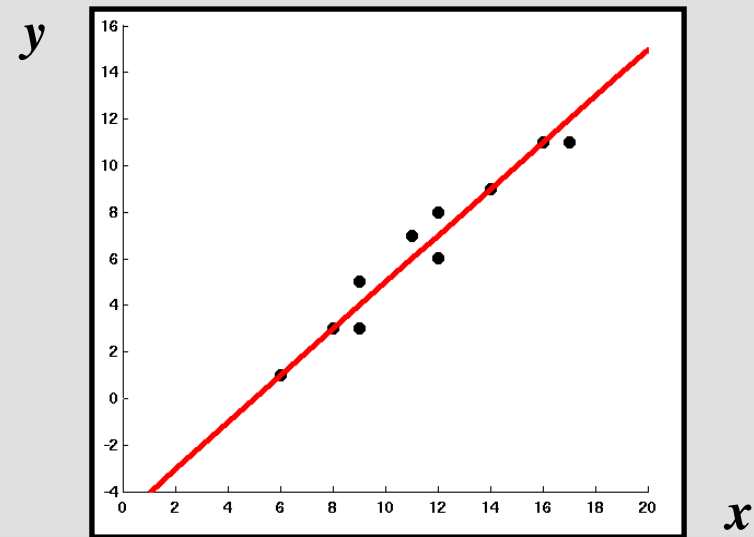


Image space

Shape recognition

■ Hough transform - Example

▶ Line detection (5/5)

- In this context, one just has to detect in the parameter space the intersection points (or a small dense area) in order to locate lines in the image space
- One uses to work with this line representation $\{\rho, \theta\}$

$$\rho = x_i \cos \theta + y_i \sin \theta$$

➡ In this case, the space H is bounded

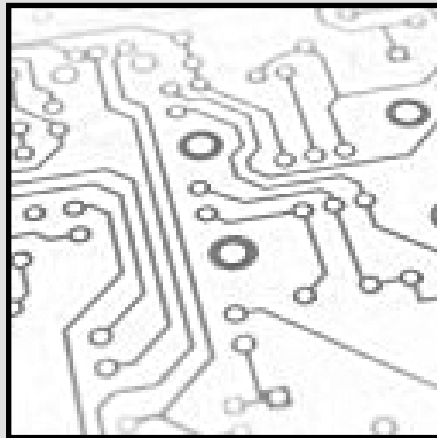
$$\Delta\theta = [-\pi, +\pi] \text{ et } \Delta\rho = [0, \sqrt{2}L] , \text{ where } L \text{ is the image width}$$

Shape recognition

■ Hough transform

▶ Example: study of an integrated circuit

```
% Loading image  
img = imread('circuit.pgm'); img = double(img);  
  
% Extraction of a set of points of interest  
BW = edge(img,'canny');
```



Original image

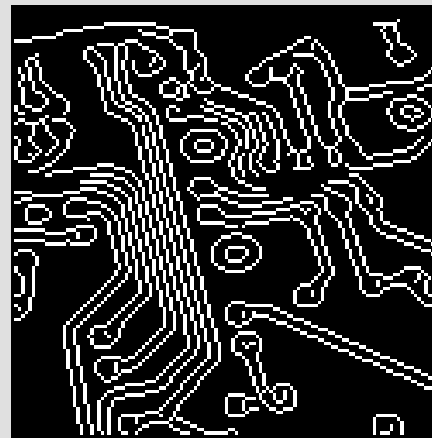
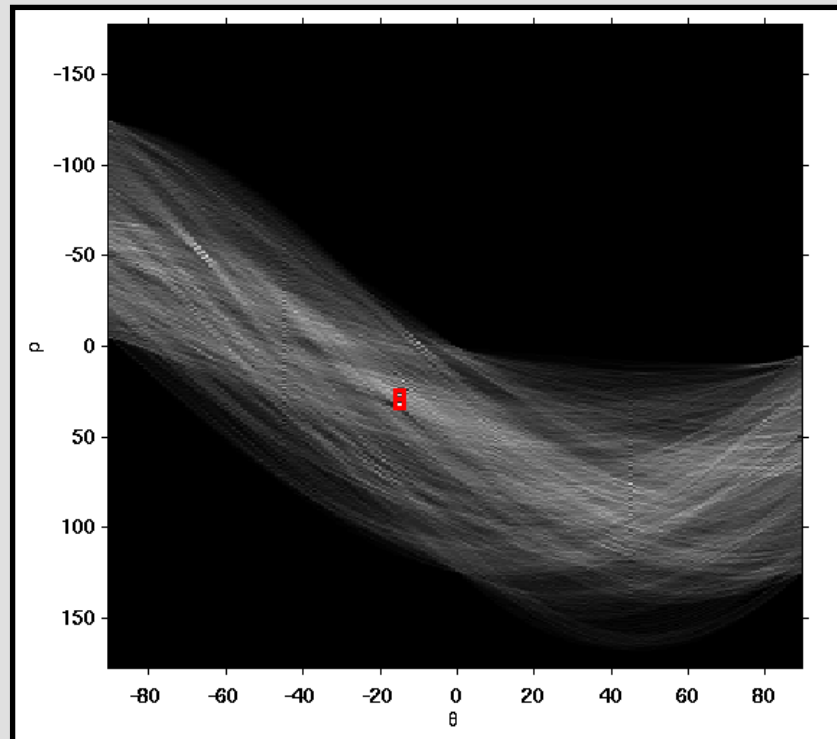


Image of contour

Shape recognition

■ Hough transform

▶ Example: study of an integrated circuit



Parameter space

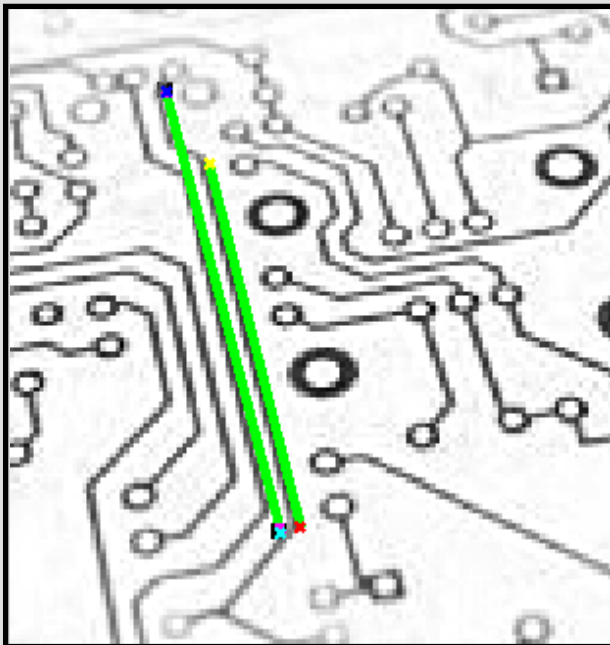
```
% Computation of the Hough transform
[H,T,R] = hough(BW);

% Detection of the highest values in the
Hough space
P = houghpeaks(H,2,'threshold', ...
               ceil(0.8*max(H(:))));
```

Shape recognition

■ Hough transform

▶ Example: study of an integrated circuit



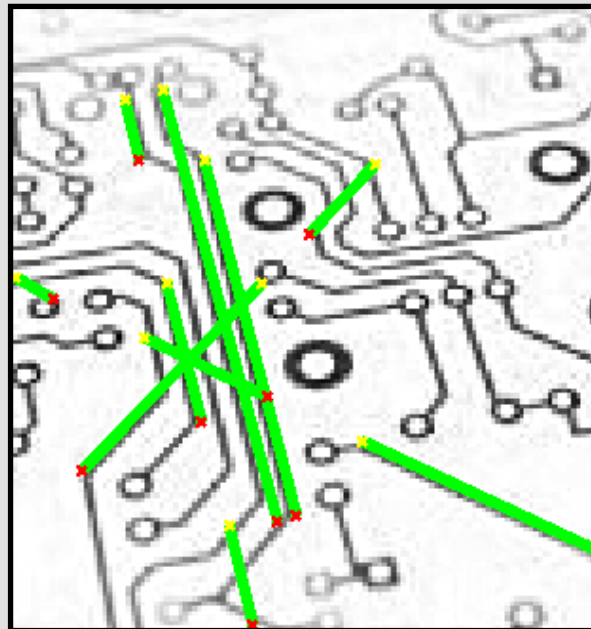
The two curves that are the more expected

```
% Extraction of the corresponding lines  
lines = houghlines(BW,T,R,P, ...  
                  'FillGap',5,'MinLength',8);
```

Shape recognition

■ Hough transform

- ▶ *Example: study of an integrated circuit - difficulties*

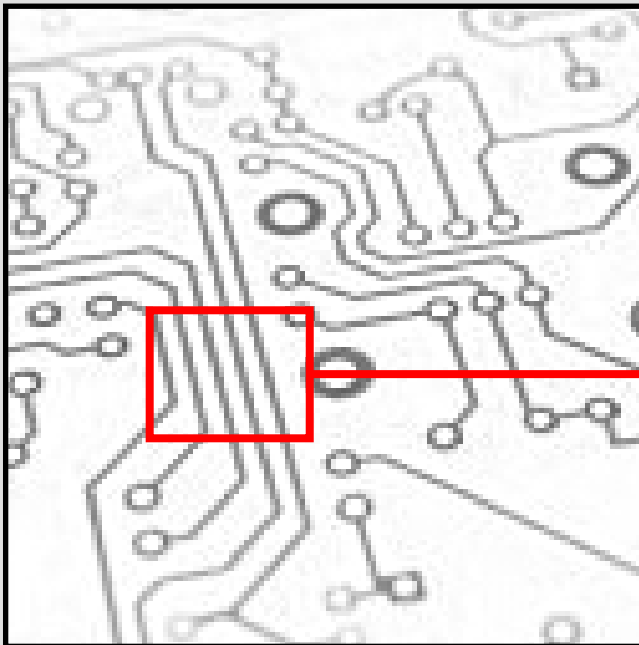


The five curves that are the more expected

Shape recognition

■ Hough transform

- ▶ *Example: study of an integrated circuit - idea*



Original image

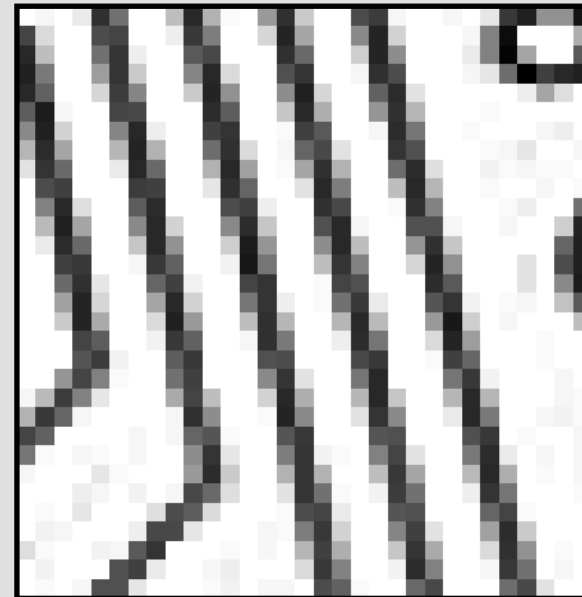


Image with zoom

Shape recognition

■ Hough transform

- ▶ *Example: study of an integrated circuit - idea*

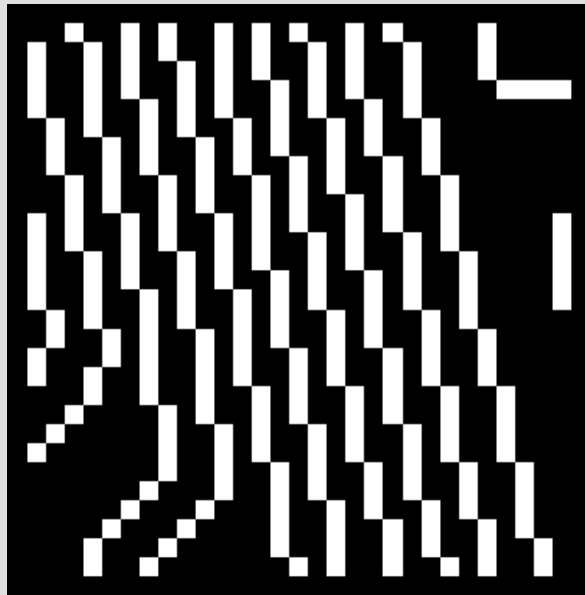
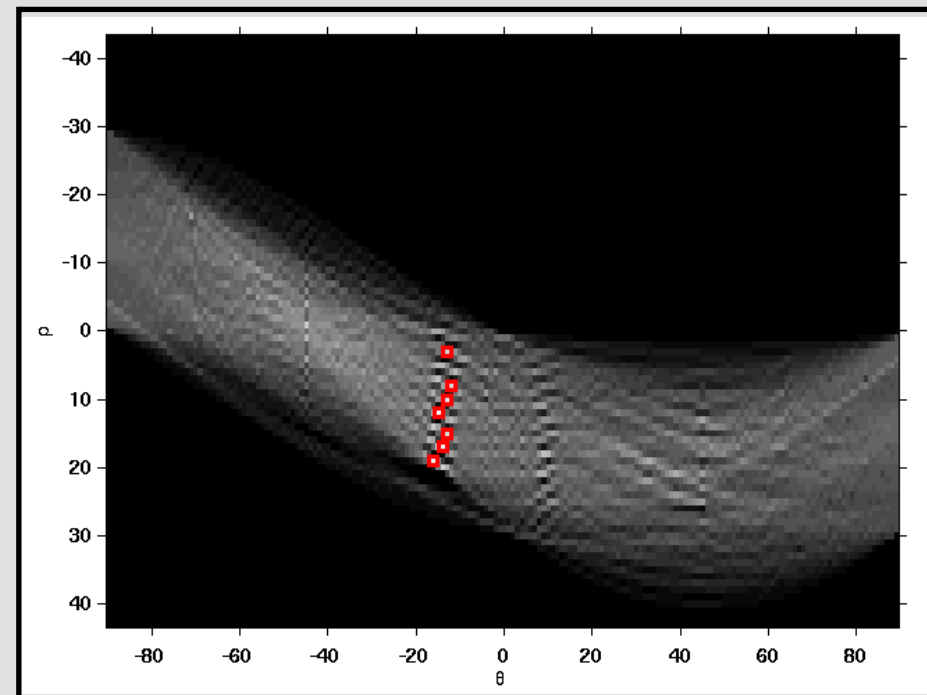


Image space

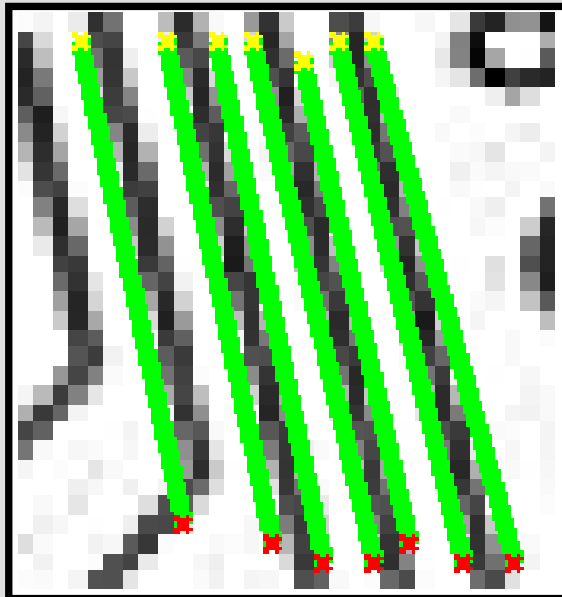


Parameter space

Shape recognition

■ Hough transform

- ▶ *Example: study of an integrated circuit - idea*



The seven curves that are the more expected

Shape recognition

■ Hough transform – General approach

- ▶ The Hough transform works with all parametric shapes
 - parabola (5 parameters)
 - circles (3 parameters)
 - ellipses (5 parameters)
 - sinusoidal traces (5 parameters)



Image analysis

End of third part