Deconvolution for limited-view streak artifacts removal: improvements upon an existing approach

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***Abstract*–Cardiac C-Arm computed tomography leads to a view-starved reconstruction problem because of electrocardiogram gating. Reconstruction with the Feldkamp, Davis and Kress method (FDK) generates large streak artifacts in the reconstructed volumes, hampering the medical interpretation. In order to remove these artifacts, deconvolution techniques have been proposed. In this paper, we start from a recent inverse filtering method developed for streak artifact removal. Two ways to improve upon this method are described. It is then proposed to replace inverse filtering with an iterative deconvolution scheme. Finally, we show that the iterative deconvolution method can itself be replaced by iterative filtered back projection (IFBP). The IFBP approach is flexible and could be used in a broad range of applications, while the improved inverse filtering approaches are computationally less demanding and better suited for time-critical applications.**

I. INTRODUCTION

In the context of acute and chronic coronary artery disease, it would be of great clinical interest to obtain a 4D representation of the myocardium and of the cardiac motion directly from a C-arm system in the interventional lab. The main challenge arises from the synchronization with the patient's electrocardiogram (ECG) that is necessary to avoid the blurring induced by the cardiac motion. Cardiac synchronization consists in selecting the projections where the heart is in a given motion state, and discarding the others. This approach, called "retrospective gating", drastically reduces the number of available projections and creates large gaps in their angular distribution. This leads to an ill-posed reconstruction problem where traditional techniques, such as FDK [1] or ART [2], give disappointing results: the reconstructed images are affected by streak artifacts, which hamper the medical interpretation (see Fig.1).

In order to remove these streak artifacts, several techniques based on deconvolution have been proposed in the 1980's [3], and recently reintroduced by Badea et al. in a paper on 4D micro-CT [4]. The current paper is focused on improving upon this method, and is organized as follows: in section II, we demonstrate that a Radon transform followed by a gated FBP is equivalent to a convolution, and highlight the main challenges a streak removal method has to take on. Section III provides a short description of the method presented in [4], aimed at explaining how it deals with the specific problems highlighted in section II. Section IV contains the proposed improvements. Section V describes an iterative deconvolution method that could replace the one used in [4]. Section VI compares the results of all the methods we describe, and finally section VII contains a discussion of the results and limitations of the deconvolution approach.

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II. NOTATIONS AND PROBLEM STATEMENT

A. Notations

Let f be a 2D or 3D image and p its projections defined by an operator R (the Radon transform in 2D, and the X-ray transform in 3D). Let p_θ be the projection along the angle θ. Tomography consists in trying to reconstruct f such that $Rf = p$. Gating consists in selecting a limited number of projections and discarding the others, and can be modeled by a sampling operator H. Thus the gated reconstruction problem is equivalent to finding f such that $Hp = HRf$. Let Q be the filtered back projection operator, and g the gated reconstruction image, $g = QHp$.

Throughout the paper, although it is a slightly abusive notation, we will also use H for the $[-\pi;\pi] \to \{0;1\}$ gating function, and for its extension to an $\mathbb{R}^2 \to \{0;1\}$ function (the extension is simply $H(R\cos\theta, R\sin\theta) = H(\theta)$).

B. Problem statement

In this section, we show that in a continuous setting, the gated FBP of the Radon transform of an image is the convolution of this image with the Point Spread Function (PSF) of the "Radon transform followed by gated FBP" process.

The Fourier Slice Theorem[5] states that the 1D Fourier transform $\mathcal{F}_{1D}$ of a line of the Radon transform $\mathcal{R}$ of a function f is identical to a radial line in the 2D Fourier transform of the function $F = \mathcal{F}_{2D} f$. For a given projection angle θ, we have:

$$\forall R \in \mathbb{R}, \quad F(R\cos\theta, R\sin\theta) = \mathcal{F}_{1D}[p_\theta](R)$$

The gated sinogram is obtained by multiplying the sinogram of f by a gating function H, defined by:

$$H: \theta \to \begin{cases} 1 \text{ if } \theta \in \Theta \\ 0 \text{ otherwise} \end{cases}$$

where Θ is the set of projection angles for which we keep the projection data. The 2D Fourier transform of g can be expressed as:

$$G(R\cos\theta, R\sin\theta) = \mathcal{F}_{1D}[p_\theta H(\theta)](R)$$

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Using the Fourier Slice Theorem, and since $H(\theta)$ is constant for a given θ, we get:

$$G(R\cos\theta, R\sin\theta) = F(R\cos\theta, R\sin\theta)H(\theta)$$

Going back to Cartesian coordinates, we obtain:

$$G(u,v) = F(u,v)H(u,v)$$

or equivalently

$$g = f * h$$

where $*$ is the convolution product and $h = \mathcal{F}_{2D}^{-1}H$. Thus the gated reconstruction problem can be seen as a deconvolution problem starting from the image reconstructed by gated FBP.

Let us highlight three important points: first, if $\theta \notin \Theta$, the information contained in $F(R\cos\theta, R\sin\theta)$ is lost during the gated reconstruction because $H(R\cos\theta, R\sin\theta) = 0$. Second, in a real case, F, G, H, f, ... are of finite extent, and the border effects have to be taken into account. Third, the Fourier Slice Theorem holds only in parallel beam geometry: in fan beam or cone beam geometry, forward projection followed by gated filtered back projection is a linear but not shift-invariant process, and thus cannot be modeled by a convolution (one could say it is a convolution with a variable PSF).

III. A RECENTLY PROPOSED APPROACH

From this interpretation of the gated reconstruction as a convolution, the reconstruction of the image can be seen as a deconvolution problem. The first approaches proposed [3] performed the deconvolution through inverse filtering.

In [4], Badea et al. suggested an important modification: to avoid divisions by zero, they perform the voxel-by-voxel division in the Fourier domain only when $|H|$ is above a certain threshold. In the other voxels, the results of the division are considered irrelevant and replaced by the corresponding Fourier coefficients from the ungated reconstruction. The volumes are reconstructed in a field of view twice as large as the object, and multiplied by a 3D cosine window. This helps mitigate the border effects.

IV. IMPROVEMENTS ON INVERSE FILTERING

Throughout this paper, we will call “central area” of a convolution image the area that is computed without using the padding values.

In order to perform inverse filtering correctly, it is important to be able to model precisely the forward projection followed by gated filtered back projection in the discrete and finite-extent case. For inverse filtering to yield the best possible results, we must be in the very specific conditions presented in Fig 1. This means the volume to deconvolve must be the circular convolution of an “ideal volume” with a PSF that has the right size and zero-padding.

We propose two methods to make the gated reconstruction close to a circular convolution, which we call “merging” and “masking”. In both cases, we perform the inverse filtering using the Fourier coefficients from the ungated reconstruction when $|H|$ is below a certain threshold, as proposed by Badea et al. and explained in section III.

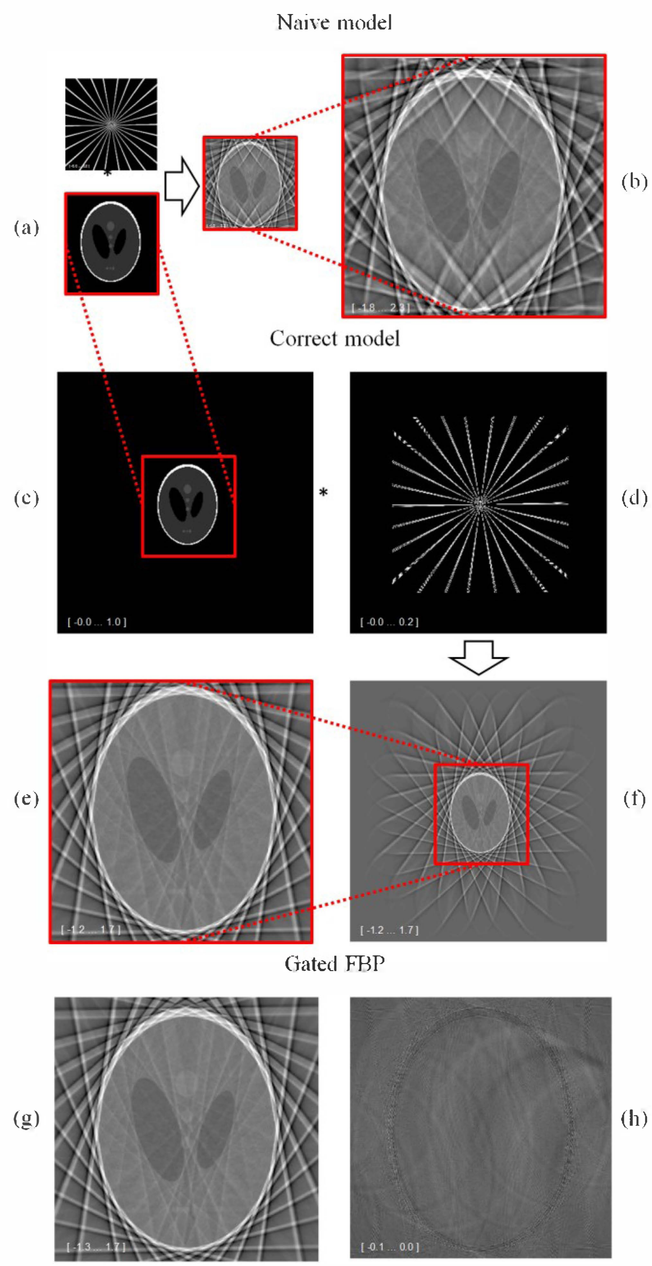

Fig 1. A gated reconstruction is the central area of a zero-padded convolution. The object y (a) is zero-padded to three times its size (c). The PSF is reconstructed in a volume twice as large as the object, and zero padded too (d). The convolution, computed by multiplication of the DFTs, is a zero-padded convolution (f), and its central area (e) matches the gated reconstruction volume (g) almost exactly. The difference between both images, amplified 20 times, is shown in (h), and can be explained by interpolation errors. In contrast, the naïve approach that consists in convolving the object with a PSF of its size yields a circular convolution that is not a zero-padded convolution (b), and contains additional streaks caused by the replicas of the phantom used as padding voxels.

A. *Merging*

Merging consists in replacing the missing data (around the central part of the volume) with new data computed by zero-padded convolution between the ungated reconstruction and the PSF. The PSF has to be reconstructed in a sufficiently large volume, so that its convolution with the ungated reconstruction has a central area the size of the gated reconstruction. The volume to be deconvolved is obtained by merging the just computed zero-padded convolution together with the gated reconstruction, as shown in Fig. 2.

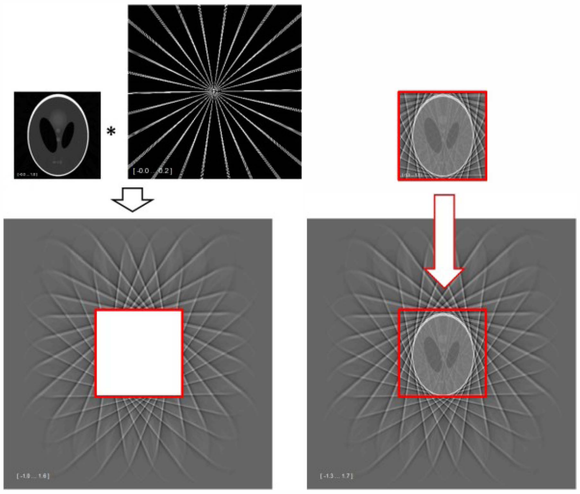

Fig 2. Merging. The ungated reconstruction (top left corner) is convolved with the PSF to obtain a first volume. The central area of this volume is cropped and replaced by the gated reconstruction (top right corner) to obtain a merged volume (bottom right corner).

B. *Masking*

Masking consists in multiplying the gated reconstruction by a mask to simulate the fading to zero on the borders. It is very similar to cosine windowing, except that the mask depends on the object. But a good approximation of the ideal mask can be obtained using the ungated reconstruction.

This approximate mask can be constructed by dividing two convolution volumes: both should be zero-padded convolutions between a PSF and the ungated reconstruction. The first one should be the same size as the gated reconstruction volume, but can have an arbitrarily small central area. The second one should have a central area the same size as the gated reconstruction volume, and be cropped to keep only this central area. The mask is obtained by dividing the former by the latter.

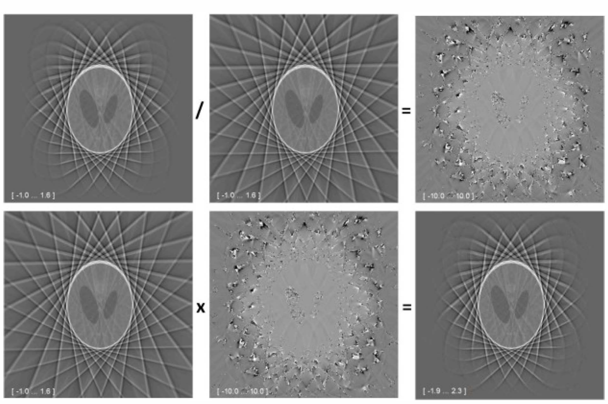

Fig 3. Masking. Volume1 (top left corner) is the convolution of the ungated volume and a small PSF (as large as the ungated). Volume2 (top row, middle) is the convolution of the ungated volume and a large PSF (three times as large as the ungated), cropped to keep only the central area. Multiplying the gated reconstruction with the mask, we obtain a zero-padded-convolution-like volume.

V. Iterative deconvolution approach

The shortcomings of inverse filtering have led people to develop other deconvolution schemes, which are not based on a voxelwise division in the Fourier domain. In this article, we focus on one of them: the Van Cittert approach [6]. We describe how it can be used for streaks removal and how it naturally leads to iterative filtered back-projection. In both methods, we use the ungated reconstruction as starting point.

A. *Van Cittert method for iterative deconvolution*

The Van Cittert method is a very simple iterative deconvolution scheme, defined as follows:

$$f_{k+1} = f_k + \alpha(g - h * f_k)$$

where f_k is the deconvolved image at the k-th iteration, g is the observed image (here the gated reconstruction), and α a relaxation weight (much smaller than one). This scheme is of particular interest for us, as it only requires the ability to compute $h * f_k$, which in our application can be done exactly by computing the forward projection of f_k, followed by a gated reconstruction. Replacing the convolution with a forward projection followed by a gated reconstruction also circumvents the non-stationarity issue, so this method can be applied to fan beam and cone beam data.

B. *Iterative filtered back-projection*

Using the notations of section II, the modified Van Cittert approach can be rewritten as follows:

$$f_{k+1} = f_k + \alpha QH(p - Rf_k)$$

It turns out that this reconstruction scheme has already been studied [7], [8]: it is called "iterative filtered back-projection" (IFBP). In the cone beam case, we replace Q by the FDK reconstruction operator, and we call this method iterative FDK, short IFDK.

VI. Results

Although both Merging and Masking significantly improve the deconvolution results over the method proposed in [4], neither can deal with fan beam or cone beam data. Thus we demonstrate them only on parallel beam data.

A. *Parallel beam geometry*

The results of Figure 4 were obtained using a 256^2 pixels modified Shepp-Logan phantom where the fifth ellipse's size varies to simulate a beating heart. All results have been computed using 600 projections, equiangularly distributed in a 180° angular range. The ECG-gating window was of width 10%, thus keeping only 60 projections. It was centered alternatively on the point where the "heart" is the biggest ('End Diastole') and on the point where it is the smallest ('End Systole'). The Van Cittert method was used with 3 iterations and initialized with the ungated FBP reconstruction.

The results obtained with Badea's algorithm can appear surprisingly poor compared with those presented in [4]. The difference is due to the fact that a human heart beats about ten times slower than a mouse one, resulting in larger gaps in the angular distribution of the gated projections, which impacts the reconstruction quality.

The following table shows the root mean squared error to the phantom for each method.

	Whole volume		ROI around the heart	
	End systole	End diastole	End systole	End diastole
Ungated	59.80	59.37	11.28	8.73
Badea	64.99	66.16	8.16	9.41
Merging	58.55	58.16	6.82	5.22
Masking	56.81	56.76	6.04	5.06
Van Cittert	54.77	54.61	6.24	5.17

Table I. Root mean squared errors between the original image and the image reconstructed using 60 projections

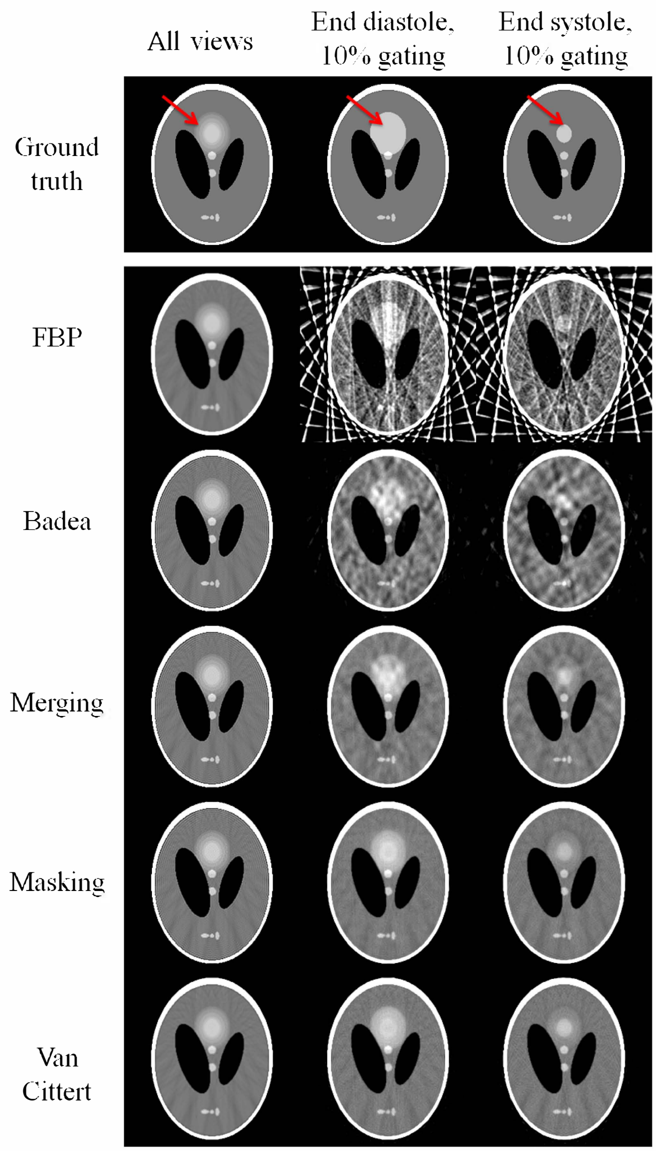

Fig 4. Results in parallel beam geometry. The beating part is pointed out by the arrow. The temporal resolution can be appreciated by how much the heart in the gated reconstruction matches the one in the phantom. Merging generates volumes with a high level of noise, but a good temporal resolution. Masking also reaches a good temporal resolution with a much lower noise level. Van Cittert method shows approximately the same noise patterns as the ungated FBP, with a temporal resolution similar to Merging and Masking.

B. Cone beam geometry

These results were obtained using a 3D software phantom. They have been computed using 300 cone-beam projections, equiangularly distributed in a 240° angular range. The ECG-gating window was of width 20%, thus keeping 60 projections. The iterative FDK method was used with 100 iterations. Only the central slice is shown here.

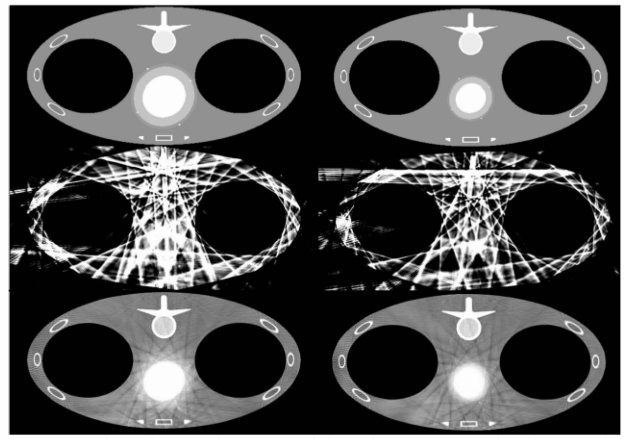

Fig 5. Ground truth, gated FDK and iterative FDK results in cone beam geometry. With 100 iterations, the iterative FDK shows significantly reduced streak artifacts with respect to gated FDK. The temporal resolution has been partially recovered, the heart being slightly smaller in end systole (right) than in end diastole (left).

VII. CONCLUSION

We have proposed two ways to better deal with the border effects when using Fourier-based deconvolution to remove streak artifacts in ECG-gated reconstructions. We have also proposed an iterative deconvolution scheme and have shown how it relates to iterative filtered back projection, and why this approach circumvents both the Fourier-based deconvolution problems (division by zero and border effects) and the PSF's variation problem in fan beam and cone beam.

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