CURVATURE-BASED ADAPTIVE REMESHING FOR WAVELET-BASED MULTIRESOLUTION 3D MESHES

A. Gouaillard[1], *A. Gelas*[1,2], *S. Valette*[3], *E. Boix*[1], *and R. Prost*[1]

[1] CREATIS, INSA Lyon, France,
[2]Kanai Laboratory, Keio University, Tokyo, Japan,
[3]Infomatics & Telematics Institute, Thessaloniki, Greece.

ABSTRACT

Most of the meshes coming from a variety of source are over sampled and exhibit highly irregular connectivity that prevent efficient wavelet analysis. Remeshing comes as a solution to sampling and connectivity optimization with respect to the geometry. The main drawback of existing subdivision remeshing approaches is the lack of control either on the number of triangles or on the geometrical error. We propose a new remeshing scheme that directly produces wavelet-based multiresolution meshes which does not suffer from the previous limitations. We provide a finer control on the resulting number of triangles by using an adaptive subdivision scheme. We also show a lowering of the local resulting geometrical error by using a new curvature-based subdivision criterion.

1. INTRODUCTION

Nowadays, polygonal meshes are intensively used as geometric models of 3D objects. Triangular meshes are especially common as triangles are the primitives of most rendering hardware. Models made of billions of polygons are common in the medical field, computer vision field, and many other research areas. At the acquisition time, stress is put on capturing geometry as finely as possible. Thus the acquisition process produces very high resolution, uniformly sampled models. Indeed, fine details of the underlying structures should be captured to be further processed, but at the processing stage, a tradeoff must be done between the accuracy of the model and speed issues. To achieve reasonable running times people use approximations of the model. This is why most of the algorithms that can process meshes come in a progressive form if the mesh is a multiresolution mesh.

Lounsbery et al¡ in [4] generalized the wavelet theory to triangular meshes which have a subdivision connectivity. Unfortunately, most of the existing meshes do not have such connectivity. We must transform them to be able to apply multiresolution techniques. Valette et al. in [8] extended the wavelet theory to arbitrary connectivity meshes. To construct a multiresolution mesh from an arbitrary mesh we need to define a connectivity hierarchy and a geometrical hierarchy. Depending on the application, we focus on building the best connectivity or geometrical hierarchy. In several contexts (modeling, segmentation, compression), only the shape, the geometry, is of interest. In this case, one can remove completely the connectivity information through remeshing and focus on geometry. Remeshing can also directly provide a multiresolution representation of the shape of interest. In general remeshing processes the number of triangles of the original mesh is usually taken as a desired maximum bound. Unfortunately, most existing schemes use regular subdivision. When applying a regular subdivision scheme with a limited number of iterations, one can notice considerable errors in high curvature zones. Since 1994, when Lounsbery in [4] first generalized the wavelet theory to triangular meshes, a lot of work has been done on remeshing polygonal surfaces to multiresolution meshes. Eck et al. [1] provided the first automatic remeshing algorithm. They apply a regular subdivision scheme. Lee et al.[3] developed a fine-to-coarse algorithm. They apply an adaptive subdivision scheme. Kobbelt et al.[2] developed a shrink wrapping approach for the remeshing, whose basic idea is to build a subdivision surface that surrounds the original mesh and then shrink it onto the surface. With this approach they can only remesh genus-0 surfaces. Kim et al. in [5] developped a curvature based adaptive subdivision scheme for wavelet based meshes. Unfortunately, their curvature criterion was not very efficient and the resulting mesh was not conform.

We propose in this paper a new remeshing algorithm which represent an explicit solution, as opposed to [8], to the problem of building a resolution representation of a surface. Same in spirit as [5], we will compute a parameterization of the original mesh and remesh directly in the parameterization domain. It reduces the dimension of the problem from 3 to 2. But, in contrast with [5], we will provide a rigourous analysis of the curvature criterion, and we will

{arnaud.gelas, alexandre.gouaillard, sebastien.valette, eric.boix, remy.prost }@creatis.insa-lyon.fr

build a conform mesh. We will also use the general wavelet definition introduced in [8] to induce directly the wavelet representation of the surface. The paper is organized in 3 main sections. The first one will introduce our algorithm in detail, the second will present the validation of the algorithm on two datasets and we will finally conclude in a last section.

2. CURVATURE-BASED ADAPTIVE REMESHING

2.1. Algorithm layout

The subsequent notations have been chosen to suit our particular application. Parameterization consists in constructing a mapping Λ, a piece-wise linear one to one correspondance between each triangle of the original mesh $\mathcal{M}$, and it's image in a 2D domain $\mathcal{D}$ chosen to be a square. Both $\mathcal{M}$ and $\Lambda(\mathcal{M})$ share the same connectivity. The remeshed surface will be noted $\mathcal{S} = \Lambda^{-1}(\mathcal{S}_\updownarrow)$

1. Curvature indicator is computed and mapped onto $\mathcal{M}$ (see subsection 2.2),
2. Parameterization $\Lambda(\mathcal{M})$ of $\mathcal{M}$ on $\mathcal{D}$ is computed,
3. Direct construction of the base mesh parameterization,
4. Iterative adaptive subdivisions of S_i for $i = 0 \rightarrow l$:
   - Find candidate triangles for subdivision (see subsection 2.3),
   - Optimize candidate triangle list (see subsection 2.4),
   - Make the mesh conform (see subsection 2.5),
5. Compute resulting mesh $\mathcal{S} = \Lambda^{-1}(\mathcal{S}_\updownarrow)$ using barycentric coordinates.

2.2. Curvature Indicator

At each point of a smooth surface $\mathbf{S}$ embedded in $\mathbb{R}^3$ the local bending of the surface is measured by two geometrical invariants i. e. the gaussian curvature $\mathcal{K}$ and the mean curvature $\mathcal{H}$. $\mathcal{K}$ and $\mathcal{H}$ are usually expressed in terms of the principal curvatures κ_{min} and κ_{max}. It should be noticed that taken independently neither $\mathcal{K}$ nor $\mathcal{H}$ capture our intuitive notion of bending of a surface. $\mathcal{K}$ does not distinguish a plane from a cylinder since for both surfaces $\mathcal{K} = 0$ at each point. And $\mathcal{H}$ considers all the members of the family minimal surfaces ($\mathcal{H} = 0$ at each point), among which the plane, the helicoid or the catenoid, as equivalent. Hence when seeking a unique scalar representing both curvatures at a given point of $\mathbf{S}$ one should consider another blending of κ_{min} and κ_{max} than $\mathcal{K}$ nor $\mathcal{H}$. A common punctual curvature indicator is

$$A'_p = \kappa_{min}^2 + \kappa_{max}^2 = 4\mathcal{H}^2 - 2\mathcal{K} \quad (1)$$

where p is any given point of $\mathbf{S}$. In our computational context we modelise smooth surfaces as triangulations, and in order to capture their geometry we need to define discrete curvature equivalents of $\mathcal{K}$ and $\mathcal{H}$. Among all the approaches and techniques encountered in the literature in order to reach this goal, very few effectively prove the convergence of the discrete curvatures to the smooth ones, when the approximating "triangulations tend to the smooth surface". Nevertheless [7] proves this convergence by using the notion of convergence in measure. The authors in [7] use the following well known definitions of the discrete curvatures :

$$\overline{K(v)} = 2\pi - \sum \alpha_i \qquad \overline{H(e)} = \frac{1}{2}\,|e|\,\delta(e) \quad (2)$$

where α_i is the angle of the i-th triangle adjacent to vertex v, $|e|$ the length of edge e and $\delta(e)$ the dihedral angle between the two triangles adjacent to e. $\overline{K(v)}$, the classical angle defect, is carried by the vertices of $\mathcal{M}$ but $\overline{H(e)}$ is carried by the edges. One should notice that $\overline{K(v)}$ and $\overline{H(e)}$ are not punctual curvatures but curvature measures. They are homogenous to the integral of curvatures of "some" smooth surfaces i.e.

$$\overline{K(v)} \approx \iint_V K dA \qquad \overline{H(e)} \approx \iint_V H dA \quad (3)$$

where V is "some" neighborhood of the considered vertex and edge of $\mathcal{M}$. When seeking punctual discrete curvatures the difficulty lies in defining the "proper" neighborhood V, and they are some evidence that the approach of "punctalized" curvature measures [6] is doomed for degenerated (small fatness factor) triangulations. [7] only establishes the convergence in measure of $\overline{K(v)}$ and $\overline{H(e)}$ in the sense

$$\lim_{T_i \to \mathbf{S}} \sum_{v \subset V'} \overline{K(v)} = \iint_{V'} K dA \quad (4)$$

$$\lim_{T_i \to \mathbf{S}} \sum_{e \subset V'} \overline{H(e)} = \iint_{V'} H dA \quad (5)$$

where V' is a fixed neighborhood and $\lim_{T_i \to \mathbf{S}}$ expresses the convergence of the suite of approximating triangulations to $\mathbf{S}$. We are looking for a discrete scalar curvature indicator that captures the local geometrical information contained in a neighborhood V. The classical smooth version (recalled by 1 for the punctual version) is of the form

$$A'_V = \iint_V \kappa_{min}^2 + \kappa_{max}^2 dA = \iint_V 4\mathcal{H}^2 - 2\mathcal{K} dA. \quad (6)$$

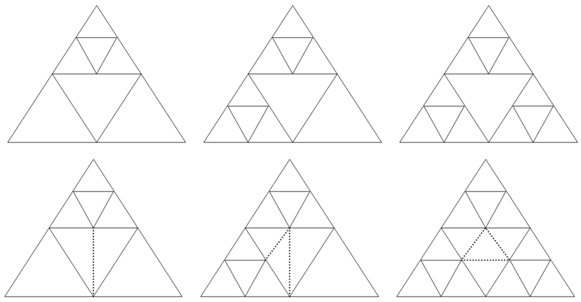

Fig. 1. Conform mesh. The triangles whose neighbour triangles have been subdivided are not conform. Such triangles come in three flavors represented on top row from left to right: one, two or three of the neighbor triangles have been subdivided. The row below shows the possible solutions to make the mesh conform.

Unfortunately we only know a discrete version of $\iint_V HdA$ (as stated by 5) but not of $\iint_V H^2 dA$, since in the general case $\iint_V H^2 dA \neq (\iint_V HdA)^2$. Hopefully,we can approximate this equation by

$$A_V = \left| 4\overline{H(e)} - 2p(v)\overline{K(v)} \right| \quad (7)$$

where $p(v)$ is the perimeter of the 1-ring of the vertex v, that is, a constant homogeneous to a length x thanks to which the measure is independent of scaling. note that when we change the scale of the given surface by a factor x, then H will scale with $1/x$ and K will scale with $1/x^2$. The absolute value is used in order to obtain a positive indicator.

Although we have no equivalent of (5) for a proof of convergence for this approximation of the curvature indicator, we still have some robust theoretical background for believing in it's proper behavior. Particularly we know it won't be sensitive to extremely degenerated triangulations i.e. containing arbitrarily acute triangles.

2.3. Candidate triangles

The curvature indicator has been computed on $\mathcal{M}$ and, as Λ is a bijection, mapped onto $\Lambda(\mathcal{M})$. For a given level i, we tag the triangles which do not respect our curvature indicator criterion ($C_k < \lambda$, where $lambda$ is a threshold). Our subdivision criterion is a function of the curvature indicator:

$$\mathcal{C}_k = \sum_{i=0}^{q_k} A(p_{ik}) \quad (8)$$

for every triangle t_k of the curent $\Lambda(\mathcal{S}_i)$. In this equation p_{ik} are the vertices of $\Lambda(\mathcal{M})$ which are in triangle t_k of $\Lambda(\mathcal{S}_i)$; q_k is the number of p_{ik} contained in the triangle t_k; $A(p_{ik})$ is the curvature measure associated to the p_{ik} vertex. In order to compute this criterion, we have to choose the same parametric domain $\mathcal{D}$, for both parameterization, i.e. $\Lambda(\mathcal{S}_l)$ and $\Lambda(\mathcal{M})$. We compute a mask over the triangle table giving us the candidates for subdivision

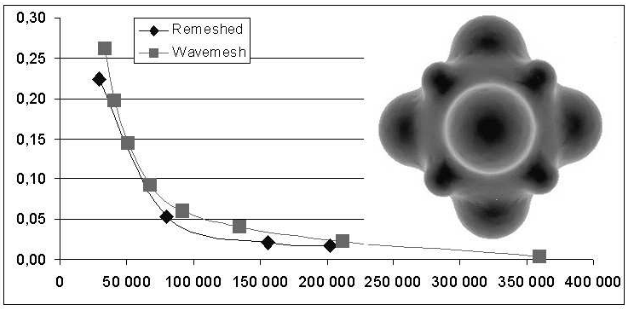

Fig. 2. Rate / Distortion curves of different multiresolution representations of the same synthetic dataset.(Meansquare error in % of the bounding box vs the bit number)

2.4. Optimizing candidate triangle mask

We then compute the area of each group of candidate triangles. If the relative area of one group of candidate triangles is too low, the corresponding triangles are untagged. This is done to avoid too local subdivisions which would not inprove the geometry and which would greatly penalyze connectivity. By defining a threshold on relative area we induce a limit on the sampling of the surface. The resulting candidate triangles are then subdivided regularly.

2.5. Conform mesh and connectivity issues

Unfortunately, on the border of the subdivided areas, the mesh is not conform. We apply non regular subdivision on those triangles according to case table (see fig 2.2). Making the mesh conform is making the connectivity irregular too on the border of previously computed regions. This also explains why we wanted the regularization described in the previous sub section.

3. RESULTS

We applied our remeshing process to the venus head model (courtesy of Stanford) and to a synthetic mesh. The venus head model has an irregular connectivity (vertex valence entropy = 2) but its sampling is already adapted to curvature. The synthetic mesh has been computed using relatively general approaches : marching cube, decimation, laplacian smoothing. It's connectivity is thus highly irregular (vertex valence entropy = 3), and it is still oversampled in some areas.

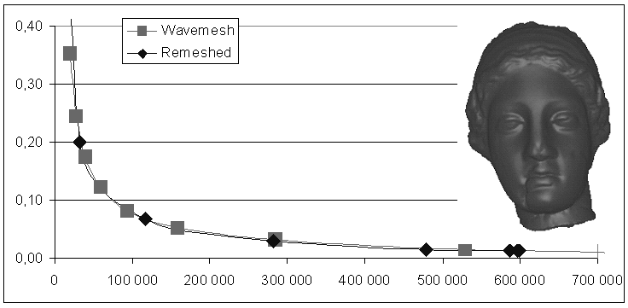

Fig. 3. Rate / Distortion curves of different multiresolution representations of the same Venus Head dataset.(Mean-square error in % of the bounding box vs the bit number)

Two rate vs distortion curves were drawn as shown on figure 3. The first one corresponds to the surface remeshed with our algorithm. The second one correponds to the multiresolution representation of the original surface computed by a state of the art fine-to-coarse compression software : WaveMesh[9]. We can see that the curves are almost identical. It confirms that the quality of our algorithm when processing already good (curvature adapted sampled surfaces) is as good as state-of-the-art compression software. But the depth of the multiresolution representation is much lower. The number of resolutions needed to fully represent the mesh is only 4 with our algorithm against 15 for wavemesh, and 22 for MAPS [3].

As for the synthetic model, the same rate vs distortion curves were drawn on figure 3. One can observe that the quality of the remeshed surface is better than the one computed with Wavemesh. Wavemesh, as all the fine-to-coarse approaches, has to start with the original mesh. This penalyzes the results when the original mesh is oversampled. A second drawback is the connectivity. The connectivity of the original mesh being irregular, encoding is expensive. When remeshing, we reduce the entropy of the connectivity as much as possible (making the mesh conform still decrease the connectivity slightly, but the second step of our algorithm tends to reduce this effect). For example the entropy of the connectivity of venus mesh was reduced from 2 to 1.9 . It was reduced from 3.1 to 1.8 for the synthetic mesh. This has a good and visible impact on the rate / distortion behavior of the resulting mesh.

4. CONCLUSIONS

We proposed here a novel way of building multiresolution representation of 3D surface meshes. Our algorithm adaptively remeshes the original mesh into a multiresolution mesh whose connectivity is adapted to the curvature and has been regularized as much as possible. When applied to irregular and/or oversampled meshes, it builds multiresolution meshes which show better rate/distortion behavior than the one which would have been constructed in a fine-to-coarse fashion by state-of-the-art algorithms.

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