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Local Wavelets Decomposition for 3-D Surfaces

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Abstract

We propose a wavelet based local subdivision of 3-D surfaces, which can be effectively applied to a mesh with complex and high curvature faces. The proposed approach is an extension of Lounsbery et al. works [1], which has been developed only for regular triangular mesh subdivision. For such a purpose, a bi-orthogonal wavelet basis is constructed by defining a local inner product and using the lifting scheme [2]. As only one filter bank is used for the global and local analysis of the surface and also one filter bank for synthesis, this method is very effective. Through the computer simulation tested on some example mesh, we show that the proposed local subdivision is very effective.

1. Introduction

In computer graphics and geometric modeling, triangular mesh has been known as a very efficient technique for representing surface of 3-D objects. As a mesh is generally represented by hundreds of thousands of vertices, a large amount of storage is required, and it takes also a long time to render and transmit the mesh. Obviously, an attractive approach is multiresolution representation [3], which allows a progressive approximation of the surface.

Recently, Lounsbery *et al.* [1] proposed a class of wavelets for surfaces subdivision, which is applicable to arbitrary topology surfaces. In this method, a multiresolution representation of a mesh consists of a base mesh, together with detail terms, called wavelet coefficients. Unfortunately, this method has been restricted to the application of meshes of which subdivision connectivity is a priori known. In order to apply this method on meshes without subdivision connectivity [5], Eck *et al.* [4] developed a 'remeshing' technique, which transforms a mesh into another one having a prior known subdivision connectivity. However, these methods are not adapted to a mesh with complex or high curvature regions, since only regular triangular mesh subdivision is considered, that is, all triangular faces are equally subdivided into four ones.

In this paper, we propose local bi-orthogonal wavelets subdivision method, which can represent effectively a region of interest or a region having complex and high curvature geometry. For such purpose, the previously reported wavelet based surface subdivision method [1] is described and its drawback in representing a mesh having a complex or high curvature regions is discussed, in the next section. The proposed local bi-orthogonal wavelets based subdivision method is presented in section III, where a local inner product is newly defined to obtain local bi-orthogonal wavelets. Some simulation results and a conclusion follow.

2. Wavelet based surface subdivision

Generally, wavelet transform decomposes a signal into a low resolution part called approximation or scaling coefficients and detail parts called wavelet coefficients [6]. A multiresolution scheme is constructed by iterating this transform on the low resolution part. In the case of a 3-D surface, a class of wavelet was originally developed by Lounsebery *et al.* [1] for regular triangular mesh subdivision.

This method starts from a base mesh, denoted as M^0, which is a triangulated polyhedron having the same topology as the data to be approximated. Each triangle of M^0 is refined into four sub-triangles by introducing new vertices at edge midpoints, followed by adding the wavelet coefficients in order to fit with the data. This process is done recursively by two filters: the refining filter P^j and the perturbing filter Q^j. Where the superscript j, $j=0, 1,\ldots,J$, is determined so that M^J and the data differ by no more than a user-specified tolerance [4]. In fact, P^j and Q^j are synthesis filter banks. The filter coefficients in general must vary over the mesh, so the filters are represented by matrices. An analysis filter bank, a low pass filter A^j and a high pass filter B^j, are derived from

$$\left[\frac{A^j}{B^j}\right] = \left[P^j \middle| Q^j\right]^{-1}. \qquad (1)$$

The analysis filter bank is used to construct multiresolution approximation of an input mesh M having subdivision connectivity.

In this approach, scaling functions $\Phi_i^j(x)$ are defined as hat functions, which have value 1 at vertex i and value 0 at all other vertices of M^j. Wavelets $\Psi_i^j(x)$ are constructed as orthogonal functions to $\Phi_i^j(x)$. In the construction of wavelets, the evaluation of an inner product for the scaling functions is the important step. Lounsbery *et al.* proposed a method for exactly computing the inner product of functions defined through regular recursive triangle subdivision of surface [1]. The matrix of inner products is defined as I^j with

$$I^j(i,k) = <\Phi_i^j, \Phi_k^j>. \qquad (2)$$

The inner product is applied to construct the bi-orthogonal wavelets. It is defined by pretending that each of the faces of M^j is equilateral. This construction can be considered as a special case of lifting [2].

As this method considers only regular subdivision whereby all triangular faces of the mesh are equally subdivided into four ones, it is not suitable for a mesh having complex or high curvature regions.

3. Local subdivision of surfaces

In sharp contrast with the previous methods, region of interest with a complex and high curvature surface could be effectively represented by a local subdivision. To locally subdivide the surface, the faces of M^j should be considered as non-equilateral. Thus the inner product of scaling functions used in [1] should be modified. Applying the Lounsbery recurrence relation to irregular subdivision, the following equation is obtained.

$$\begin{aligned} I_L^j &= \int_{x \in M^0} (\Phi_L^j(x))^T \Phi_L^j(x)\, dx \\ &= \int_{x \in M^0} (P_L^j)^T (\Phi_L^{j+1}(x))^T \Phi_L^{j+1}(x)\, P_L^j\, dx \\ &= (P_L^j)^T I_L^{j+1}\, P_L^j\,. \end{aligned} \qquad (3\text{-a})$$

Here, I_L^{j+1} should be evaluated from I_L^j. In contrast with the regular decomposition, where each of the entries $I^j(i,k)$ in I^j has one or more corresponding entries $I^{j+1}(i',k')$ in I^j, up to a factor of ¼ ; that is $¼(I^j(i,k)) = I^{j+1}(i',k')$, due to the equilateral subdivision, the irregular decomposition result to non-equilateral subdivision and the equation (3-a) is undetermined. To overcome this difficulty we add the following conditions at each level of resolution:

- Each diagonal element of I^j should be the sum of the other elements of the corresponding row,

$$I_L^j(m,m) = \sum_{i=1}^{m-1} I_L^j(m,i) + \sum_{i=m+1}^{n} I_L^j(m,i). \qquad (3\text{-b})$$

- An absolute scale is chosen so that the sum of the elements of I^0 becomes unity.

The above additional conditions provide the solution necessary to turn the homogeneous system into non-homogeneous one. As a result, the local inner product can be computed exactly. As a region of interest of a mesh is subdivided recursively into four equilateral subtriangles at the incremental resolution, except on the border of the region, the inner product permits to define a local bi-orthogonal wavelet basis. Expressed in matrix form, it follows

$$\Psi_{L\_orth}^j(x) = \Psi_L^j(x) - \Phi_L^j(x)\alpha_L^j, \qquad (4)$$

where the coefficients α_L^j can be determined by requiring that $\Psi_{L\_orth}^j(x)$ be as orthogonal as possible to $\Phi_L^j(x)$.

The evaluation of the inner product using (3-a) and (3-b) is costly in term of computations. Fortunately, the matrix I^{j+1} can be evaluated without solving the equation (3-a) and (3-b), since the subdivided triangle generates new vertices that have always the valences (the number of edges associated to each vertex) 4, or 6. It allows to categorize all possible local inner products in several cases, according to different valences and resolution of the vertices. For example, in the case that a base mesh is an octahedron, if all triangular faces of the base mesh are subdivided from the resolution 0 to j and a face or several faces of the obtained mesh M^j are only subdivided from j to $j+k$, the inner product between the new vertices 'p_{n1}, p_{n2}' having valences six is $I_L^{j+k}(p_{n1}, p_{n2}) = \dfrac{2}{4^{j+k} \times 4}$. Such equations dramatically reduce the computation cost of the local inner product. In the case that the vertices of a base mesh have the same valences 'V' and a face of the base mesh is subdivided from the resolution 0 to j, the results are summarized in Table 1.

4. Results

To evaluate the performance of the proposed local subdivision, several meshes are selected and tested. Two of them, one synthetic and another real medical data.

In the Figure 1, Figure 1(a) to (f) show the approximated meshes by applying analysis filters to Figure 1(a). Figure 1(a) is the deformed mesh in order to fit with the original data for the mesh, which is obtained by subdividing the region (1) of Figure 1(d) from $j = 2$ to the resolution $j = 5$. Figure 1(d) is obtained by modifying the geometry for the region (1) of the sphere at the resolution $j = 2$. Figure 1(f) is the base mesh, an octahedron.

In the Figure 2 experiment, the initial high resolution mesh of the heart has irregular connectivity with the mesh obtained from a base mesh by recursive 4 to 1 splitting (Figure 2(a)). In order to obtain a mesh having the subdivision connectivity for regions-of-interest ROI (1) and ROI (2) of the initial mesh, we apply a local remeshing based on Lee *et al.* approach [7] (Figure 2(b)). Then we consider the local multiresolution analysis of both ROIs (Figure 2(c)-(e)).

5. Conclusions

A bi-orthogonal wavelets based local subdivision has been presented. Due to the exact computation of the local inner product, a multiresolution representation can be accomplished for 3-D surfaces. This approach allows the best approximation for a part of the complex mesh, or for regions with high curvature.

6. References

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Table 1. The inner product at the resolution j, in the case that the vertices of a base mesh have the same valence 'V' and a face of the base mesh is subdivided from the resolution 0 to j.

Cases	**Inner product at the resolution j**
Outside vertices 'v_a, v_b' of the local subdivision region	Same values with the inner product at the resolution $j = 0$
A vertex 'v_c' of the base mesh composing the local subdivision faces	$I_L^j(v_c, v_c) = \frac{1}{V}(\frac{2}{4^j} + V - 2)$
A vertex 'v_d' having the valence 6 and generated at the resolution j	$I_L^j(v_d, v_d) = \frac{12}{4^j \times V}$
A vertex 'v_e' having the valence 4 and generated at the resolution j	$I_L^j(v_e, v_e) = \frac{3}{4^j \times V}$
A vertex 'v_f' having the valence 6 and any vertex 'v_g'	$I_L^j(v_f, v_g) = \frac{2}{4^j \times V}$
Vertices 'v_h, v_i' having the valences 4 and generated at the resolution j	$I_L^j(v_h, v_i) = \frac{2}{4^j \times V}$
A vertex 'v_j' having the valence 4 and generated at the resolution j, and a vertex 'v_k' generated at the resolution $j-1$	$I_L^j(v_j, v_k) = \frac{1}{4^j \times V}$

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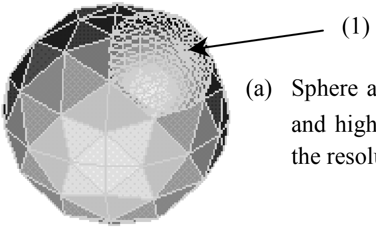
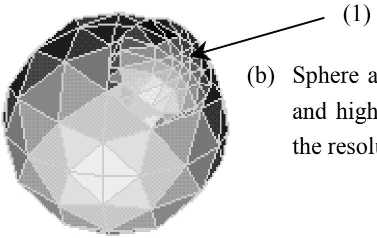
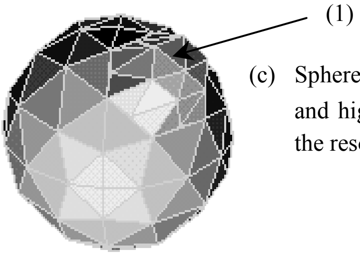


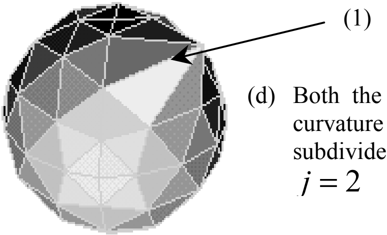
(a) Sphere at the resolution $j = 2$ and high curvature region (1) at the resolution $j = 5$



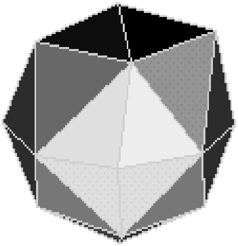
(b) Sphere at the resolution $j = 2$ and high curvature region (1) at the resolution $j = 4$



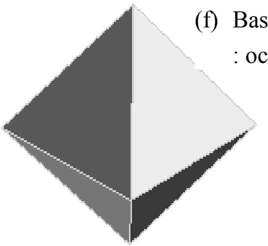
(c) Sphere at the resolution $j = 2$ and high curvature region (1) at the resolution $j = 3$



(d) Both the sphere and the high curvature region (1) are subdivided at the resolution $j = 2$

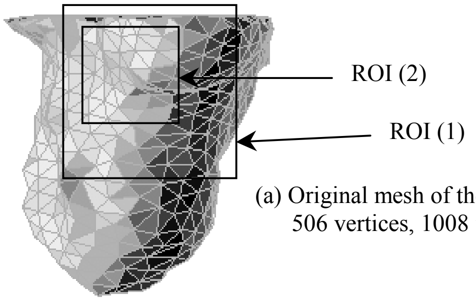


(e) Sphere at the resolution $j = 1$

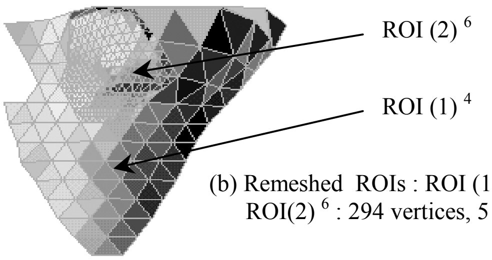


(f) Base mesh M^0 (resolution $j = 0$) : octahedron

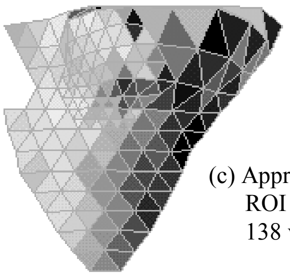
Figure 1. The local wavelets decomposition of the synthetic data.



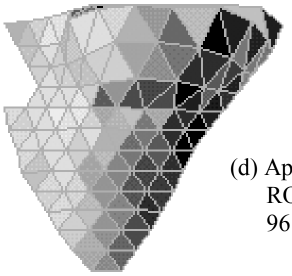
(a) Original mesh of the heart : 506 vertices, 1008 faces



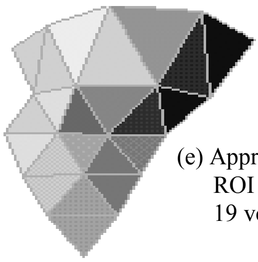
(b) Remeshed ROIs : ROI (1) ^{4} and ROI(2) ^{6} : 294 vertices, 513 faces



(c) Approximation of the Fig.2(b), ROI (1) ^{4} and ROI(2) ^{5} : 138 vertices, 225 faces



(d) Approximation of the Fig.2(c), ROI (1) ^{4} and ROI(2) ^{4} : 96 vertices, 153 faces



(e) Approximation of the Fig.2(d), ROI (1) ^{3} and ROI(2) ^{3} : 19 vertices, 22 faces

Figure 2. The local wavelets decomposition of real medical data with the local resolution j, ROI (k) ^{j}, k = 1,2.