

ADAPTIVE WAVELETS BASED MULTIREOLUTION MODELING OF IRREGULAR MESHES VIA HARMONIC MAPS

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ABSTRACT

We propose an adaptive wavelets based multiresolution scheme by using harmonic maps for 3D irregular meshes. This approach extends the previous works in [2] and [8], which have been developed for regular triangular mesh subdivision. First, we construct parameterizations of the original mesh that results in a remesh having a subdivision connectivity for the wavelets decomposition. Next, the local subdivision based multiresolution scheme is presented. Our algorithm represents effectively a region of interest or a region having complex and high curvature geometry by using bi-orthogonal wavelets. Through the computer simulation tested on some example meshes, we show that the proposed method is more effective than the previous regular subdivision methods.

1. INTRODUCTION

In computer graphics and geometric modeling, triangular mesh has been known as a very efficient method for representing surface of 3-D objects. As a mesh is generally represented by hundreds of thousands of vertices, a large amount of storage is required, and it takes a long time to render and transmit the data. Obviously, an attractive approach is multiresolution representation [9] which allows a progressive approximation of the surface.

Lounsbery *et al.* [8] proposed a class of wavelets for surfaces subdivision analysis and synthesis, which is applicable to arbitrary topology surfaces. In this method, a multiresolution representation of a mesh consists of a base mesh, together with detail terms, called wavelets coefficients. Unfortunately, this method has been restricted to the application of meshes of which connectivity results from regular 1:4 face splits. Two kinds of remedy can be considered : an irregular subdivision scheme or a remesh of the surface. In the first technique each face of the mesh is subdivided into four, three, two, or remain unchanged according to the original mesh [10]. As a result the face splitting description should be stored and transmitted. In order to apply the Lounsbery multiresolution scheme on meshes without subdivision connectivity [9], Eck *et al.* [2] developed a 'remeshing'

technique which transforms a mesh into another one having 1:4 subdivision connectivity. Lee *et al.* [7] presented a remeshing algorithm with guaranteed error bounds. However, these methods are not adapted to a mesh with complex or high curvature regions, since only regular triangular mesh subdivision is considered. A solution to this problem is proposed in [4, 5]. In [4] a local wavelet decomposition scheme is proposed. The resolution level is variable on the surface. In [5] a local remeshing based on conformal maps allows an adaptive multiresolution scheme where the subdivision level is controlled by the local curvature of the surface.

In this paper, in order to remesh the surface, we propose a new parameterization of irregular 3D surfaces based on harmonic maps. Our parameterization is simple and fast. And a wavelets based multiresolution scheme allows the design of curvature-adaptive local bi-orthogonal wavelets basis. The proposed scheme can represent effectively a region of interest or a region having a complex and high curvature geometry.

In section 2, we present both the regular multiresolution scheme on bi-orthogonal wavelets bases constructed by lifting and the curvature-adaptive wavelets decomposition. In section 3, we consider first the harmonic maps in 3D space and then we propose a new remeshing algorithm suitable for wavelets analysis of the mesh.

2. WAVELETS DECOMPOSITION OF 3D SURFACES

In this approach, we consider bi-orthogonal wavelets bases. Scaling functions $\varphi_i^j(x)$ are defined as hat functions, which have value 1 at vertex i and value 0 at all other vertices of M^j . Wavelets $\psi_i^j(x)$ are constructed using $\varphi_i^{j+1}(x)$ minus a weighted sum of some functions $\varphi_i^j(x)$. In the construction of wavelets, the evaluation of an inner product is the important step. Lounsbery *et al.* proposed a method for exactly computing the inner product of functions defined through regular recursive triangle subdivision of surface [8]. It is defined by pretending that each of the faces of M^j is equilateral.

The inner product is applied to construct the bi-orthogonal wavelets, quite orthogonal to the scaling function by calculating the weights. This construction can be considered as a special case of lifting [9].

Starting from a base mesh, the recursive subdivision leads to a collection of refinable scaling functions and hence a sequence of nested linear spaces, as required by multiresolution analysis. This means that the surfaces subdivision in the wavelet theory generates two filters for each resolution level: the refining filter \mathbf{P}^j and the perturbing filter \mathbf{Q}^j depending both of scaling function and wavelets. In fact, \mathbf{P}^j and \mathbf{Q}^j are synthesis filter banks. The filter coefficients vary over the mesh, so the filters are represented by matrices. Since this process is also invertible, an analysis filter bank, a low pass filter \mathbf{A}^j and a high pass filter \mathbf{B}^j , are derived from [4, 8]:

$$\begin{bmatrix} \mathbf{A}^j \\ \mathbf{B}^j \end{bmatrix} = \begin{bmatrix} \mathbf{P}^j & \mathbf{Q}^j \end{bmatrix}^{-1} \quad (1)$$

The analysis filter bank is used to construct multiresolution approximation of an input mesh M having a subdivision connectivity.

This theory is based on the regular subdivision. As this method considers only regular subdivision whereby all triangular faces of the mesh are equally subdivided into four ones, it is not suitable for our local subdivision. A technique we have proposed in [5], allows to reconstruct the local bi-orthogonal wavelets for the base mesh which is subdivided locally according to the mean curvature and the region density [4, 5].

3. PARAMETERIZATIONS ON IRREGULAR 3D SURFACES

3.1. Harmonic maps

The proposed parameterizations of irregular 3D surfaces start by constructing the harmonic maps. The idea to use the harmonic maps is partly inspired by the works of Eck *et al.* [2] and Zhang [11]. In the work [2], the initial mesh is partitioned into connected surface patches without hole and each such patch is mapped onto an equilateral triangle by using the harmonic maps. The harmonic maps on those equilateral triangles are then used to resample the triangular mesh with a specified connectivity among the resampled points. Zhang [11] uses the harmonic maps to conduct surface matching.

Our application of harmonic maps is used for the first step of the parameterization of irregular 3D surfaces onto 2D space. The mapped image has the same connectivity as the original mesh and allows to obtain the simple parameterizations for a wavelet decomposable mesh.

Now we examine briefly the theory of the harmonic maps. The harmonic maps are the mathematical tools

which studies the mapping between two manifolds from an energy point of view. In the special case in which $M \subset R^3$ is a surface of disc topology and $P \subset R^2$ is a convex region, the following problem has a unique solution: Fix a homeomorphism b between the boundary of M and the boundary of P . Then there is a unique harmonic map $h: M \rightarrow P$ that agrees with b on the boundary of M and minimizes the metric distortion of M [2, 3, 11]. Eck *et al.* find that the metric distortion energy $E(h)$ associated with a harmonic map h can be interpreted as the energy of a configuration of springs placed on the edges of M :

$$E(h) = \frac{1}{2} \sum_{\{i,j\} \in \text{edges}(M)} k_{ij} \|h(i) - h(j)\|^2 \quad (2)$$

where each spring constant k_{ij} is a simple function of the lengths of nearby edges in the original mesh M . Thus the harmonic maps h on M can be computed by solving a sparse linear least-squares system.

The harmonic map is infinitely differentiable on each face of M , maintains an embedding property, and also attempts to preserve aspect ratio of triangles [2, 11].

3.2. A new remeshing algorithm

First of all, we evaluate the centroid coordinate of the original mesh M and map M into a new coordinate system with the centroid coordinate as the origin. Then, M is partitioned into two local meshes M_1 and M_2 . We restrict here the partition with two parts. Note that nothing opposes to the generalization to N parts ($N \geq 2$). By a boundary mapping, each local mesh is mapped onto the convex image with central vertex v_{ci} as a center. Here, v_{ci} is determined in the middle of each local mesh i and the radius r_i for our boundary mapping is obtained by computing the mean geodesic distances between v_{ci} and each boundary vertex of the local mesh [6]. Then the interior vertices are mapped into the convex images by using harmonic maps. As a result, the obtained images form meshed circles but have the same connectivity with the original mesh. Next, we construct the base mesh M^0 as an octahedron. This base mesh M^0 is constructed so that the distance between the origin and each vertex has the same radius r_i with the mapped image of the local original mesh. Then we apply the local and global subdivision algorithm proposed in [5] to M^0 . After that M^0 is subdivided according to the local curvature c_i of the original vertices and the density of regions [5]. In order to avoid isolated subdivided or non-subdivided triangles we perform a spatial regularization of subdivision

levels. This subdivided base mesh is partitioned for having the same parts with the number of local original meshes.

Successively, the projection onto 2D space is realized and a deformation is build to form a circle. Now we construct the parameterizations with mapped original local meshes : interior and boundary parameterizations. These parameterizations are derived by evaluating the barycentric coordinates. Boundary parameterizations allow to complete the fusion of the different local remeshes.

Finally we obtain a remesh having the subdivision connectivity suited for wavelets decomposition.

4. RESULTS

To evaluate the performance of both the parameterizations and local subdivision based multiresolution analysis, two example meshes are selected and tested. They are real medical data.

Figures 1-(a) and 2-(a) show the original meshes of left lung and heart with irregular connectivity. In order to construct a mesh having the subdivision connectivity, we apply the proposed new parameterization algorithm. Figures 1-(b) (c) represent the harmonic maps of the local left lung meshes M_1 and M_2 . Figures 1-(d) and 2-(b) depict the remeshes of the left lung and heart meshes to the resolution $j = 4, 5$. Finally, we can obtain the approximated meshes in the figures 1-(e) at the resolution $j = 4$, and in the figures 2-(c), (d), (e) at the resolution $j = 2, 3, 4$. These results clearly prove the effectiveness of the proposed approach. A usual objective performance criteria is the L^2 remeshing error [1]. However L^2 is not appropriate to evaluate the local error in regions of high curvature then we only consider the visual quality .

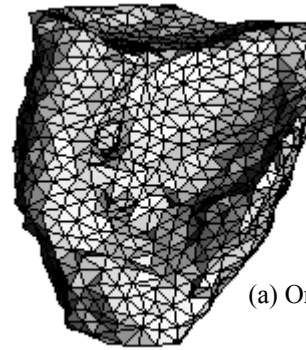
5. CONCLUSIONS

We have presented the new parameterizations method and a multiresolution representation for irregular 3D surfaces. This approach is simple and the execution runtimes are reasonable. Wavelets based multiresolution scheme can represent effectively a region of interest or a region having a complex and high curvature geometry by taking automatically the appropriate resolution level.

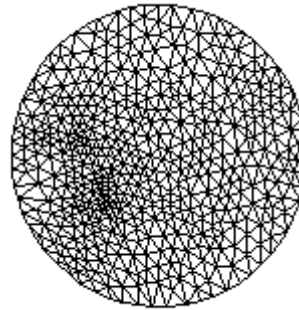
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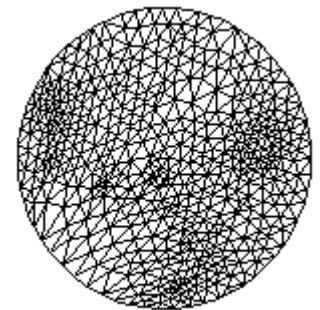
This work is in the scope of the scientific topics of the PRC-GDR ISIS research group of the French National Center for Scientific Research (CNRS).



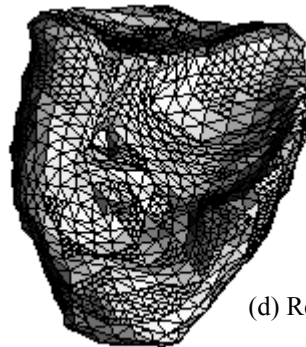
(a) Original mesh : left lung



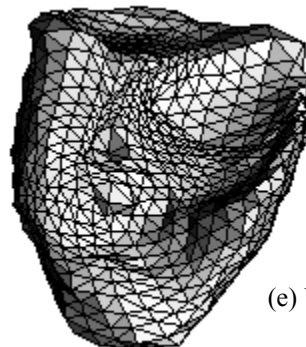
(b) Harmonic maps of M_1



(c) Harmonic maps of M_2

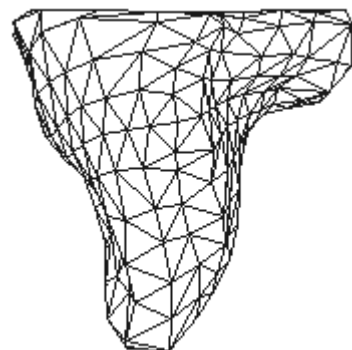
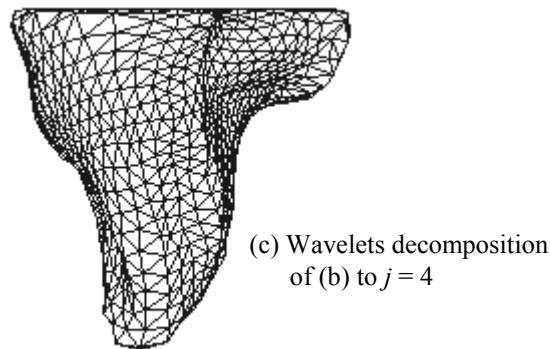
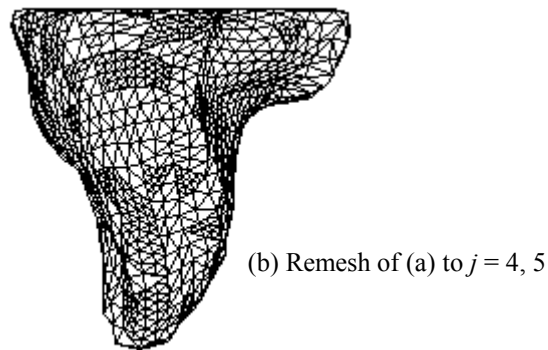
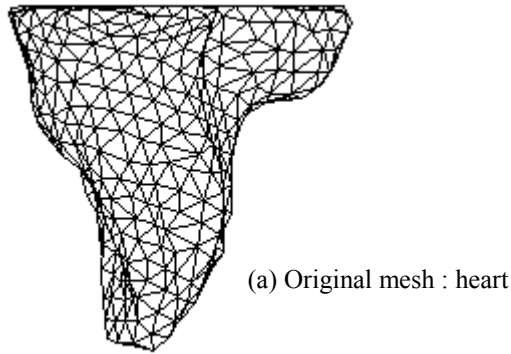


(d) Remesh of (a) to $j = 4, 5$



(e) Wavelets decomposition of (d) to $j = 4$

Figure 1. The remeshing and wavelets decomposition of the left lung mesh



(d) Wavelets decomposition of (c) to $j = 3$

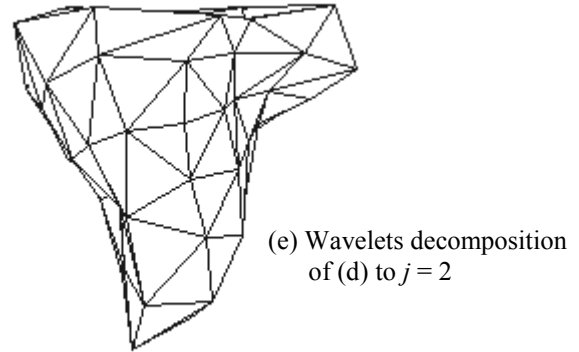


Figure 2. The remeshing and wavelets decomposition of the heart mesh

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