Freeform imaging from Hadamard matrices

Nicolas Ducros[1, 2]

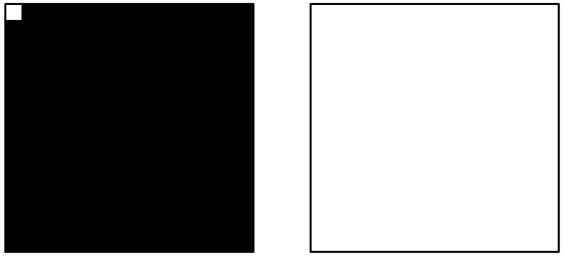
[1]CREATIS, Univ Lyon, INSA-Lyon, UCB Lyon 1,CNRS, Inserm, CREATIS UMR 5220, U1206, Lyon, France

[2]IUF, Institut Universitaire de France

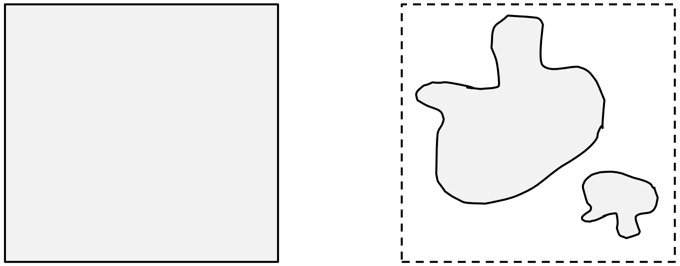
Joint work with J Cohen and L Mahieu-Williame

Two simple ideas

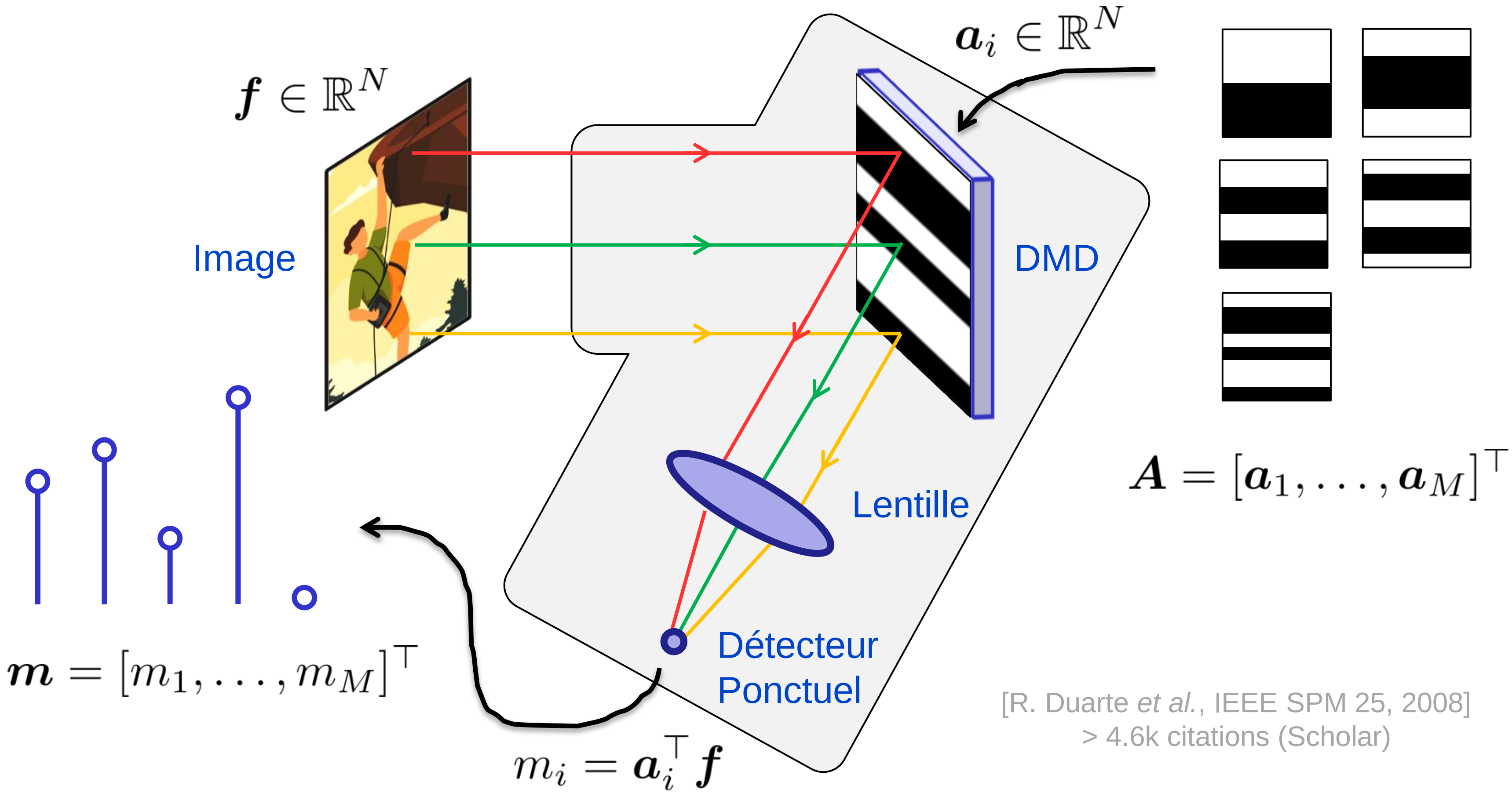
Idea#1: Hadamard modulation to reduce noise



Idea#2: Freeform to capture only relevant information



Single-pixel imaging (Duarte *et al.*, 2008)



[R. Duarte *et al.*, IEEE SPM 25, 2008]
> 4.6k citations (Scholar)

Acquisition

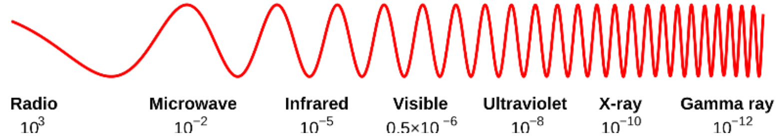
$$\boldsymbol{m} = \boldsymbol{A}\boldsymbol{f}$$

$\boldsymbol{A} \in \mathbb{R}^{M \times N}$

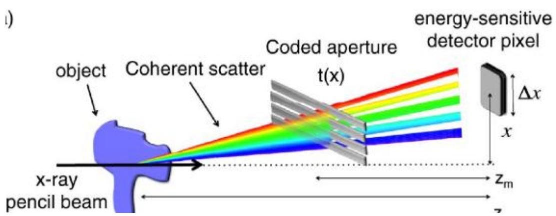
Reconstruction (*M* << *N*)

$$\boldsymbol{f}^* \leftarrow \min_{\boldsymbol{f}} \|\boldsymbol{m} - \boldsymbol{A}\boldsymbol{f}\|_2^2 + \lambda \|\boldsymbol{\Phi}\boldsymbol{f}\|_1$$

Applications

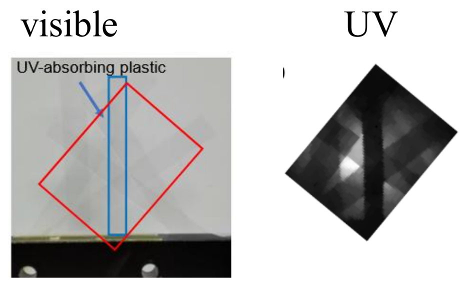


X rays



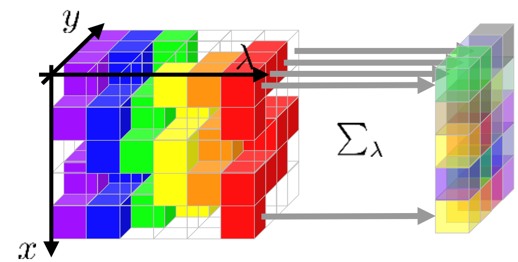
[J. Greenberg *et al.*, Optics letters 39, 2014]

Ultraviolet



[J. T. Ye *et al.*, Appl. Phys. Lett. 123, 2023]

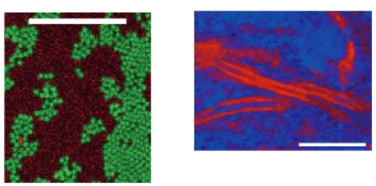
Hyperspectral



[G Beneti Martins et al., Opt. Express, 2023]

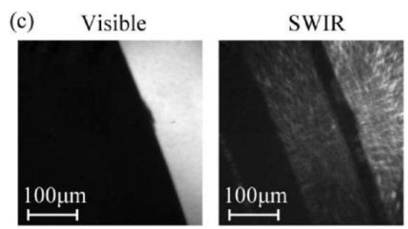
[E Hemsley *et al.*, JOSAA 37 (12), 2020]

Raman imaging



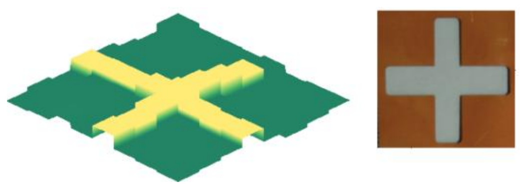
[F Soldevilla *et al.*, Optica 6(3), 2019]
[Scotte *et al.*, Jphys Photonics 5(3), 2023]

Infrared



[N. Radwell *et al.*, Optica 1(5), 2014]

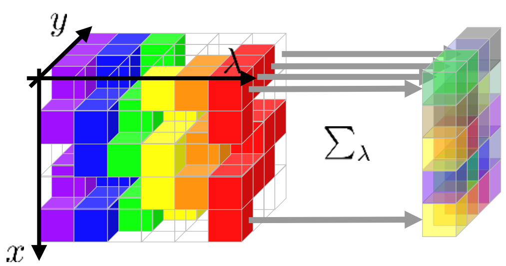
Terahertz imaging



[C. Watts *et al.*, Nature Photonics 8(8), 2014]

Applications (in Lyon)

Hyperspectral

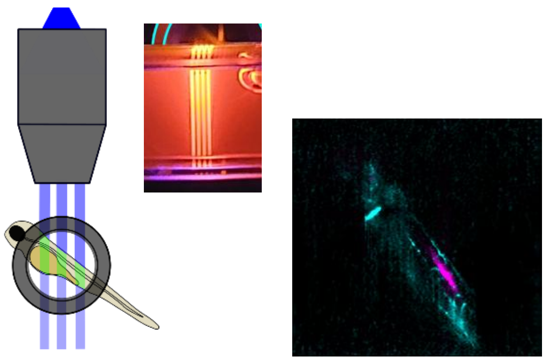


200+ hypercubes in open access

https://pilot-warehouse.creatis.insa-lyon.fr/

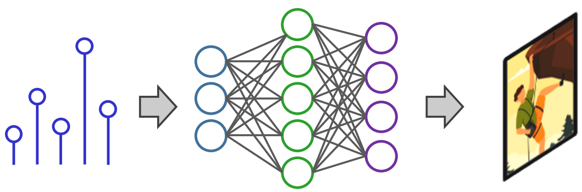
[G Beneti Martins *et al.*, Opt. Express, 2023]

Fluorescence light sheet



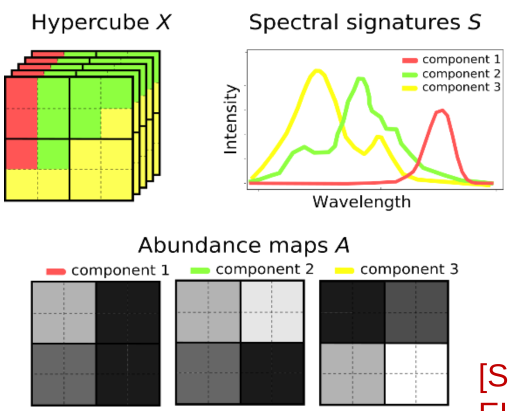
[S Crombez *et al.*, Opt. Express, 2025]

DL-based reconstruction



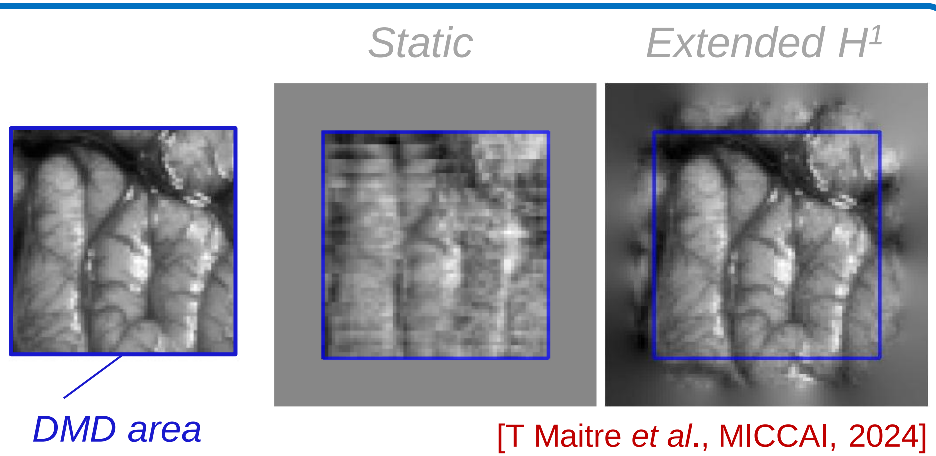
[JFJ. Abascal *et al.*, Opt. Express, 2025]

Spectral unmixing



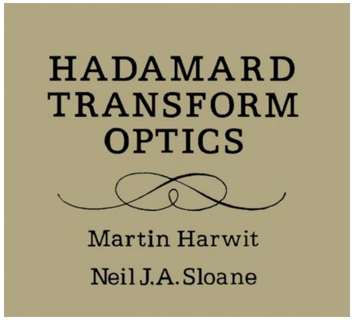
[S Hariga *et al.*, EUSIPCO, 2024]

Motion-compensated imaging

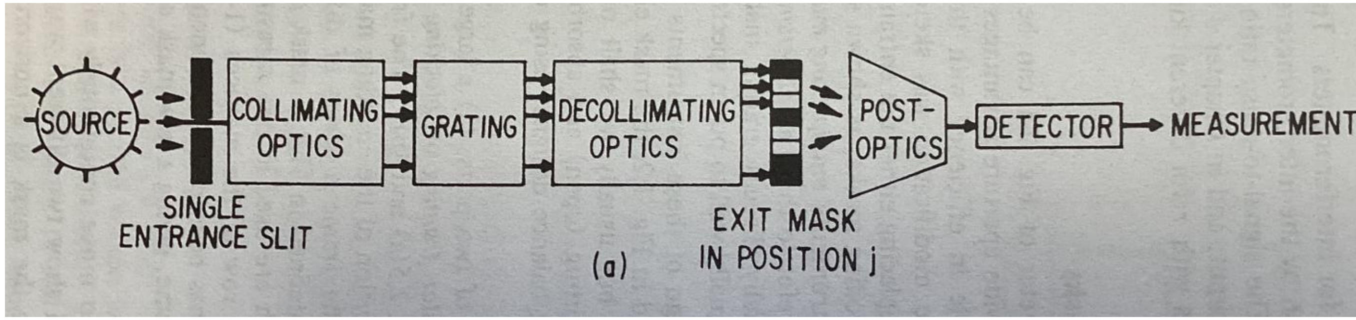


[T Maitre *et al.*, MICCAI, 2024]

Hadamard spectroscopy (from the 70's)



[M. Harwit and N Sloane, Academic Press, 1979]



'(...) conventional spectrometer is modified by using a ***mask to encode the light*** *at the output'*

Hadamard

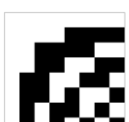
$$\boldsymbol{A} = \boldsymbol{H} \in \{-1, 1\}^{N \times N}$$

L2 reconstruction

$$\boldsymbol{f}^* = \frac{1}{N}\boldsymbol{H}^\top \boldsymbol{m}$$

Fellgett's advantage

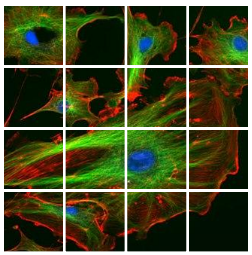
$$\mathrm{var}\left(f_n^*\right) = \frac{1}{N}\sigma^2 < \sigma^2$$



$$\boldsymbol{H}^\top \boldsymbol{H} = N \boldsymbol{I}_N$$

Freeform Imaging

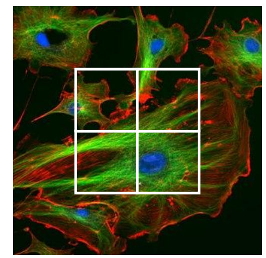
Motivation and definition



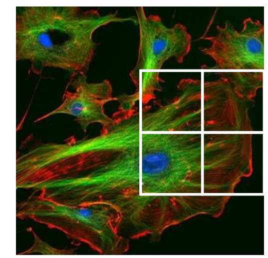
4x4 image

Given a time budget

Fewer pixels = reduced noise

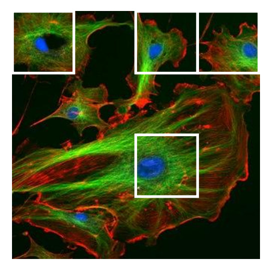


2x2 image



2x2 image

Why not?



4-pixel image

→ **Freeform imaging**

= Capturing an arbitrary pixel subset within the FOV.

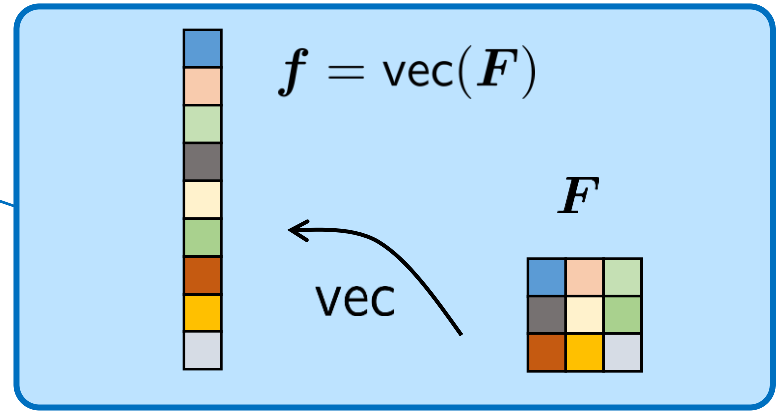
https://commons.wikimedia.org/wiki/File:FluorescentCells.jpg

How can freeform imaging be achieved?

$$\boldsymbol{m} = \boldsymbol{A}\boldsymbol{f}$$

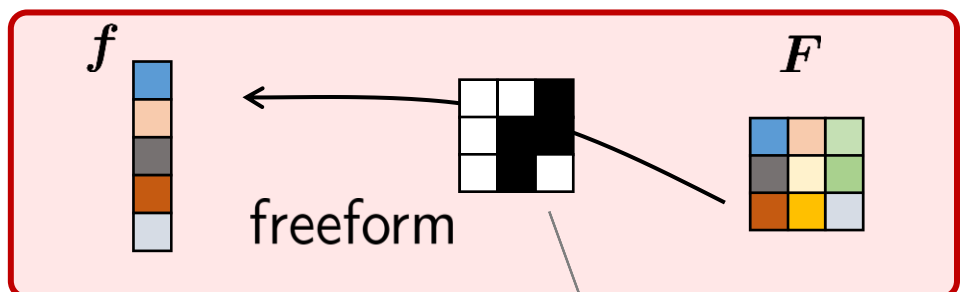
P: Total number of pixel within the full FOV

Full (*P* pixels) Here: *P* = 9 pixels



N: Number of pixels within the freeform region

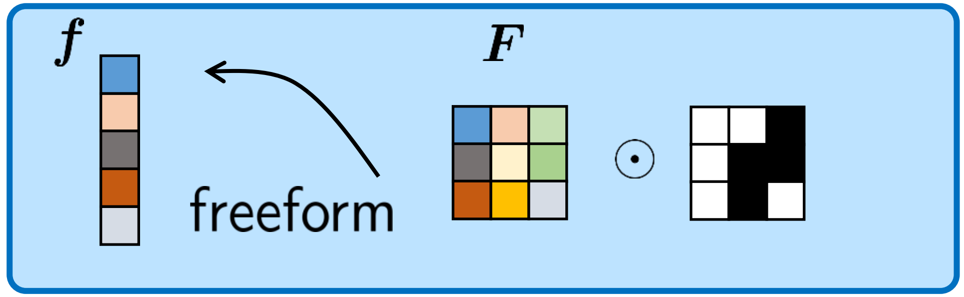
Freeform (*N* pixels) Here: *N* = 5 pixels



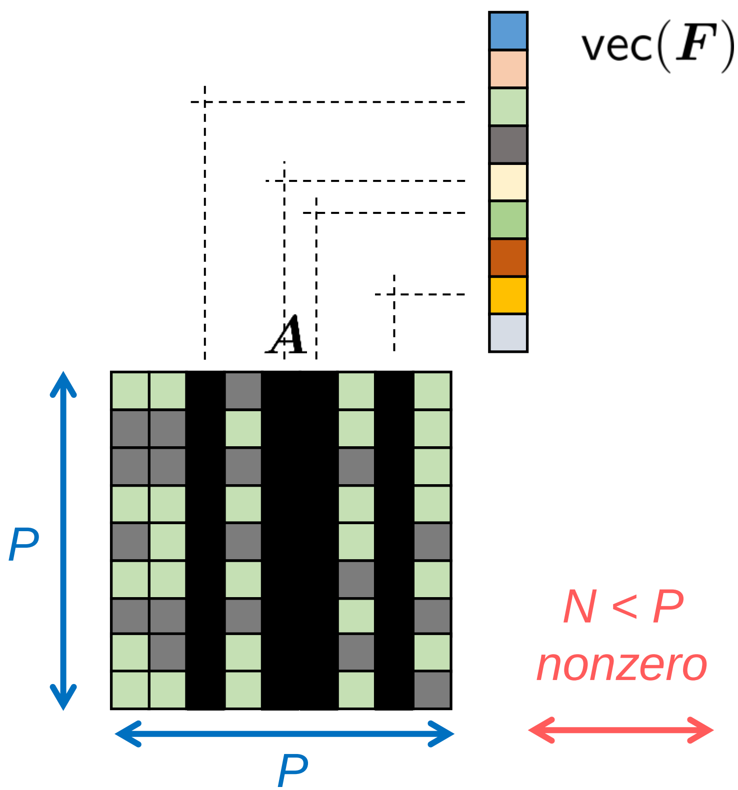
Mask indicating the pixels within the freeform region (white = in; black = out)

How can freeform imaging be achieved?

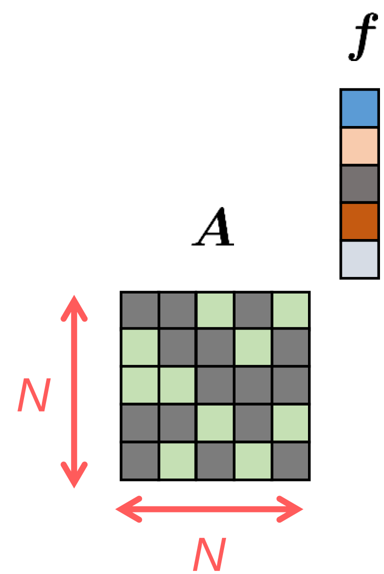
$$m = Af$$



1. Masked matrices



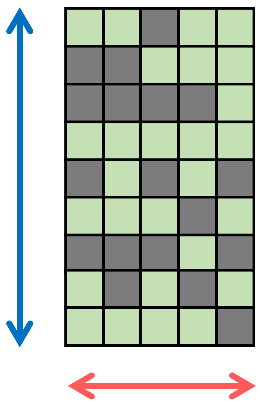
2. Non imaging matrices



Fellgett's advantage in freeform imaging?

Column-decimated Hadamard imaging

Acquisition matrix *M* ≥ *Q*



M measurements
for *Q* pixels

$$\boldsymbol{A}^\top \boldsymbol{A} = K \boldsymbol{I}_Q$$

Mean Squared Error* (MSE)

Reference flux
(depends only on experimental parameters)

$$\mathbb{M}\text{se}\,(\mathbf{f}^*) \approx \frac{1}{t}[2Q\bar{f} + 4M\,f_{\text{ref}}]$$

M* ≥ *Q

Number of pixels in the freeform region

Number of measurements

Image mean flux
(only depends on the scene)

*Assumes:

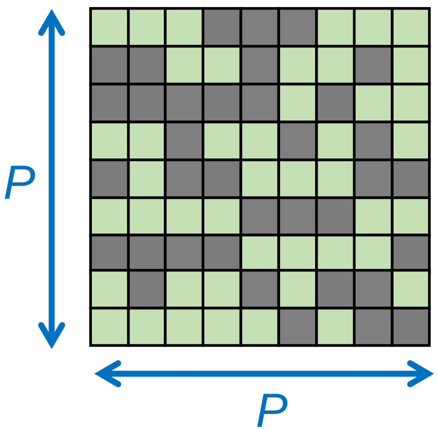
- Poisson-Gaussian noise [EMVA standard, 2021]
- Hadamard = negative part/S matrix
- Pseudoinverse reconstruction

Freeform MSE

#px in freeform region — #measurements

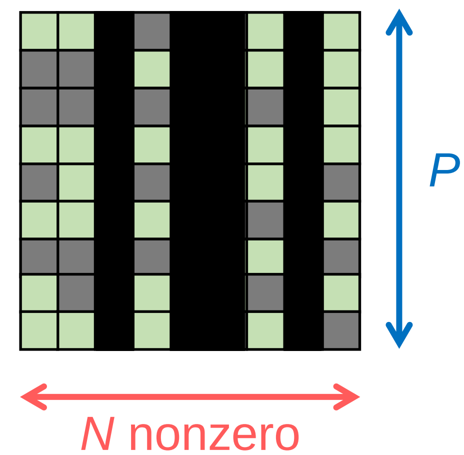
$$\mathbb{M}\mathrm{se}\,(\mathbf{f}^*) \approx \frac{1}{t}[2Q\bar{f} + 4M f_{\mathrm{ref}}], \quad M \geq Q$$

Full



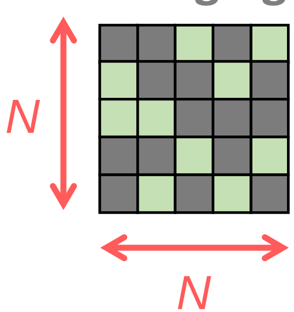
$$\mathbb{M}\mathrm{se}\,(\mathbf{f}^*) \approx \frac{1}{t}[2P\bar{f} + 4P f_{\mathrm{ref}}],$$

Masked



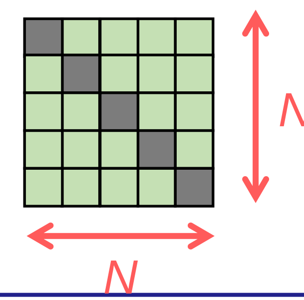
$$\mathbb{M}\mathrm{se}\,(\mathbf{f}^*) \approx \frac{1}{t}[2N\bar{f} + 4P f_{\mathrm{ref}}],$$

Non imaging



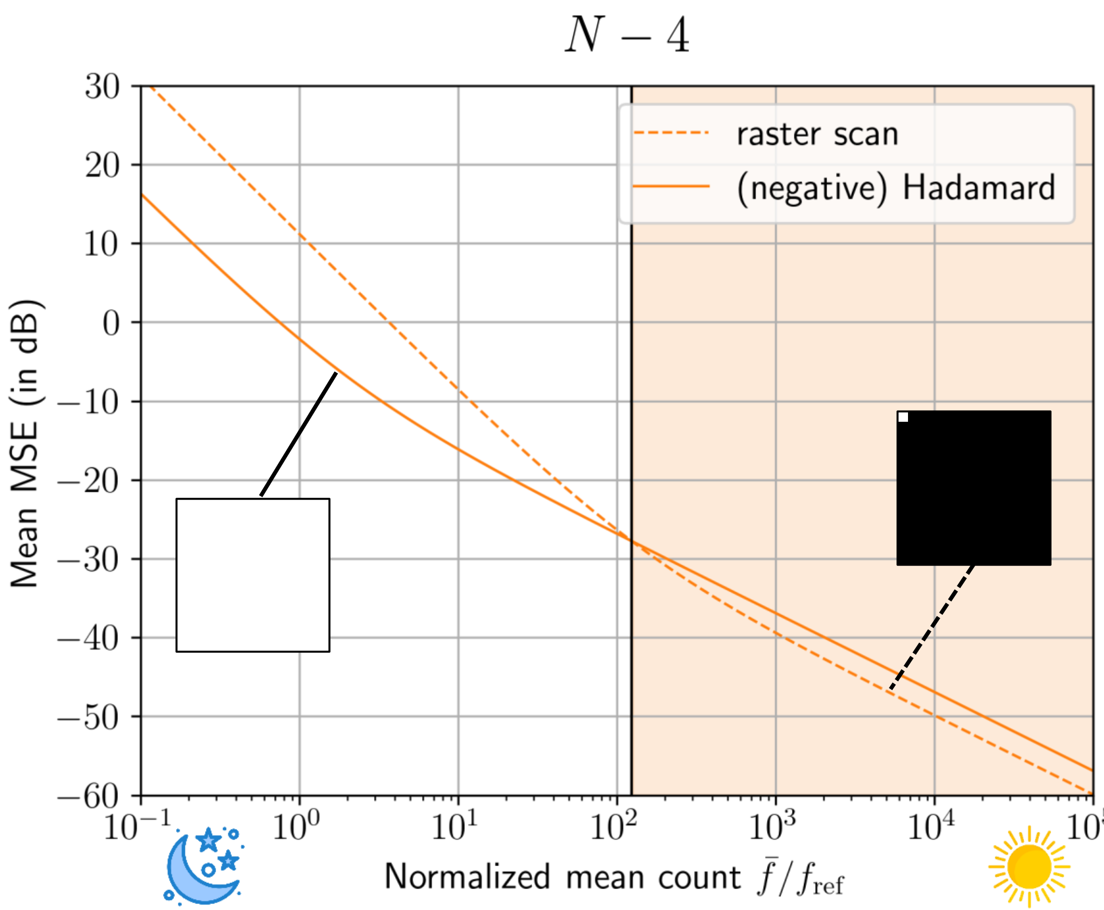
$$\mathbb{M}\mathrm{se}\,(\mathbf{f}^*) \approx \frac{1}{t}[2N\bar{f} + 4N f_{\mathrm{ref}}],$$

Raster Scan



$$\mathbb{M}\mathrm{se}\,(\mathbf{f}^*) \approx \frac{1}{t}[N\bar{f} + N^2 f_{\mathrm{ref}}],$$

Low flux increases Fellgett's advantage

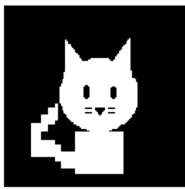


→ **Hadamard** multiplexing is more effective in **low-light** conditions.

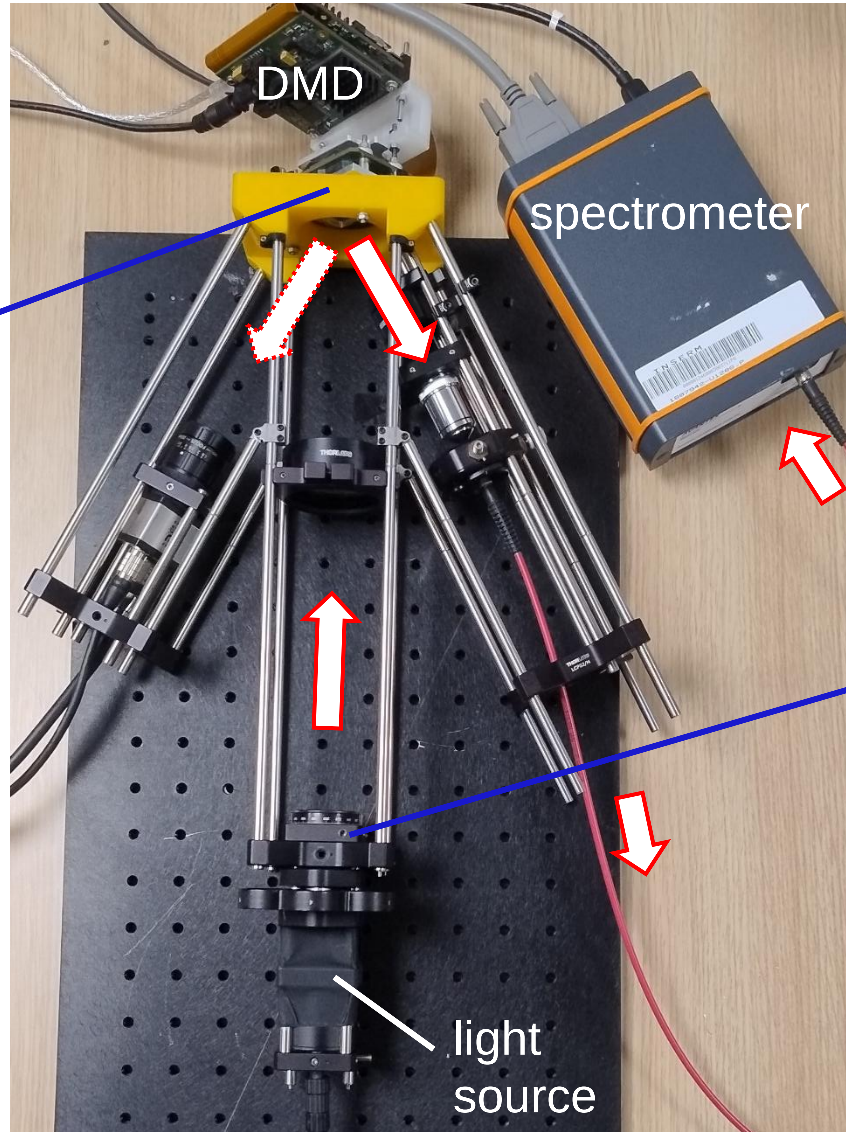
Experimental verification

Acquisition device

Freeform mask



total #px — $\frac{P}{N} = 4$ — #px in the freeform region

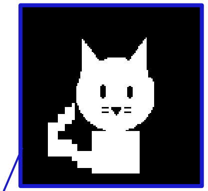


Siemens star resolution target

Freeform image at high flux

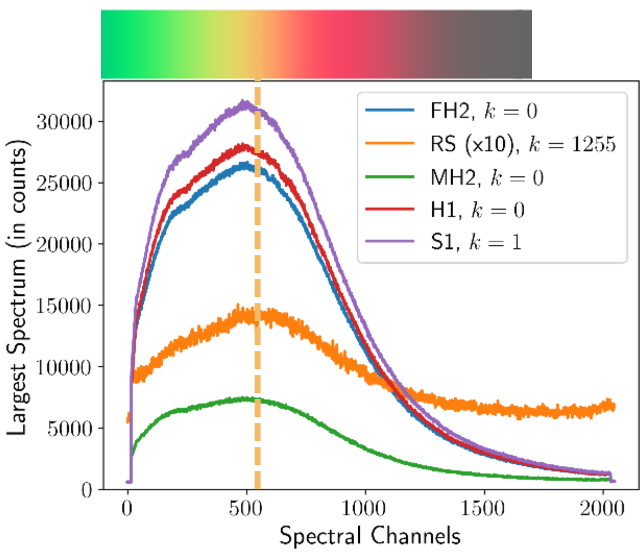
Siemens star resolution target

Freeform region

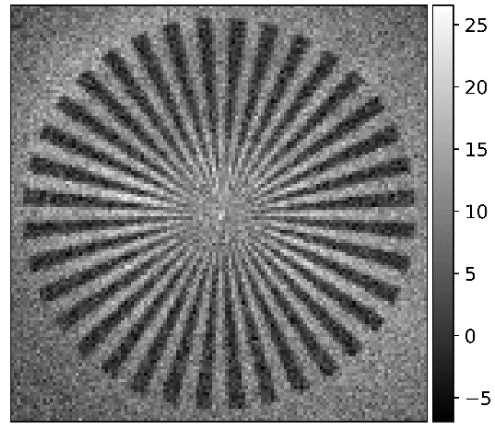


DMD area

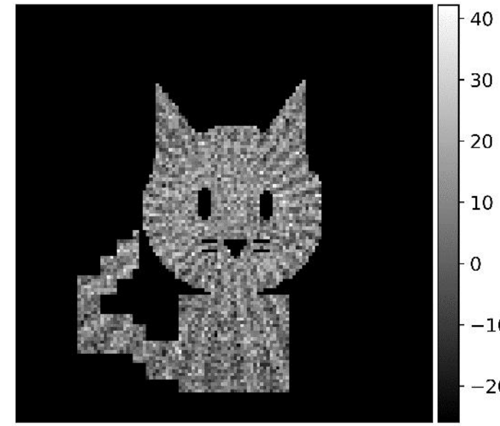
581 nm



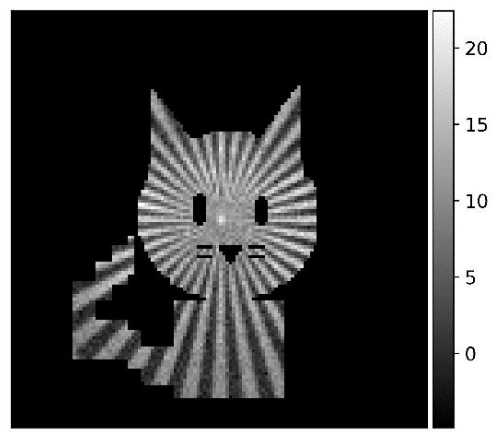
Full



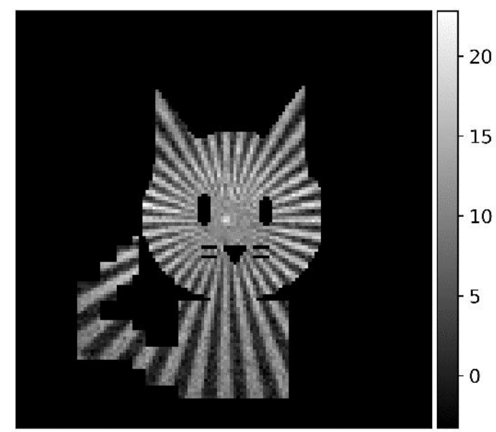
Raster Scan



Masked



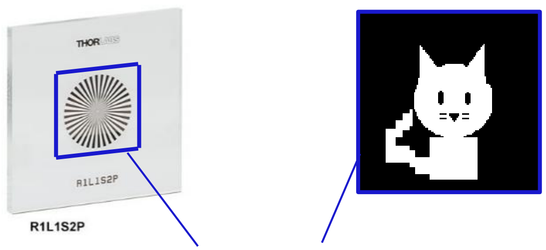
Non imaging



Freeform image at low flux

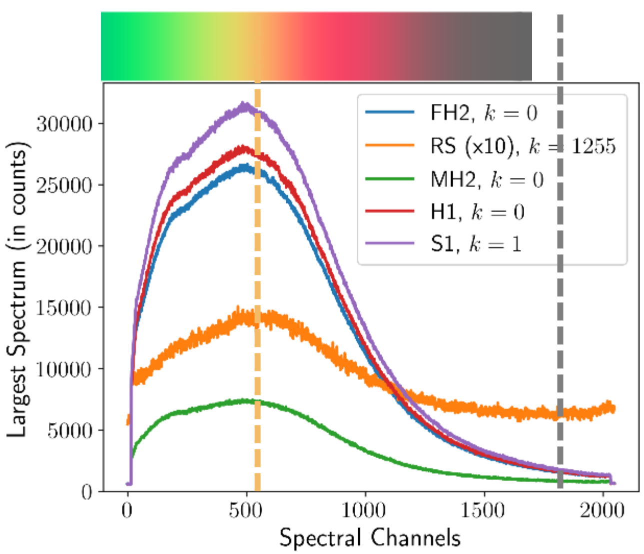
Siemens star resolution target

Freeform region

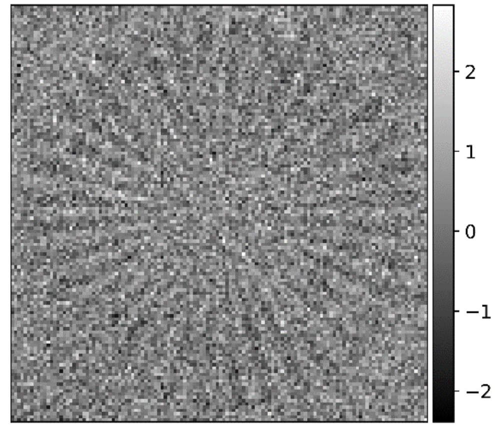


DMD area

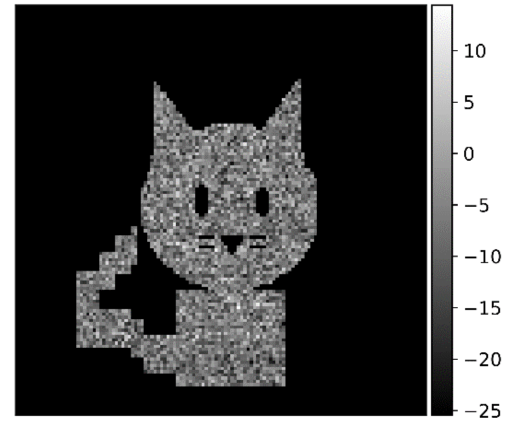
726 nm



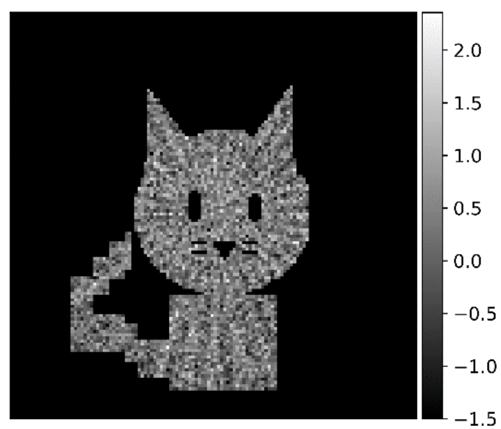
Full



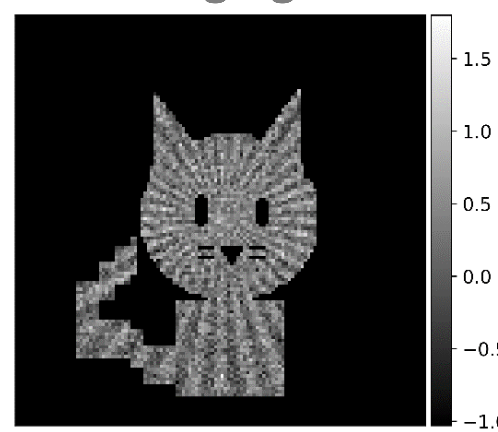
Raster Scan



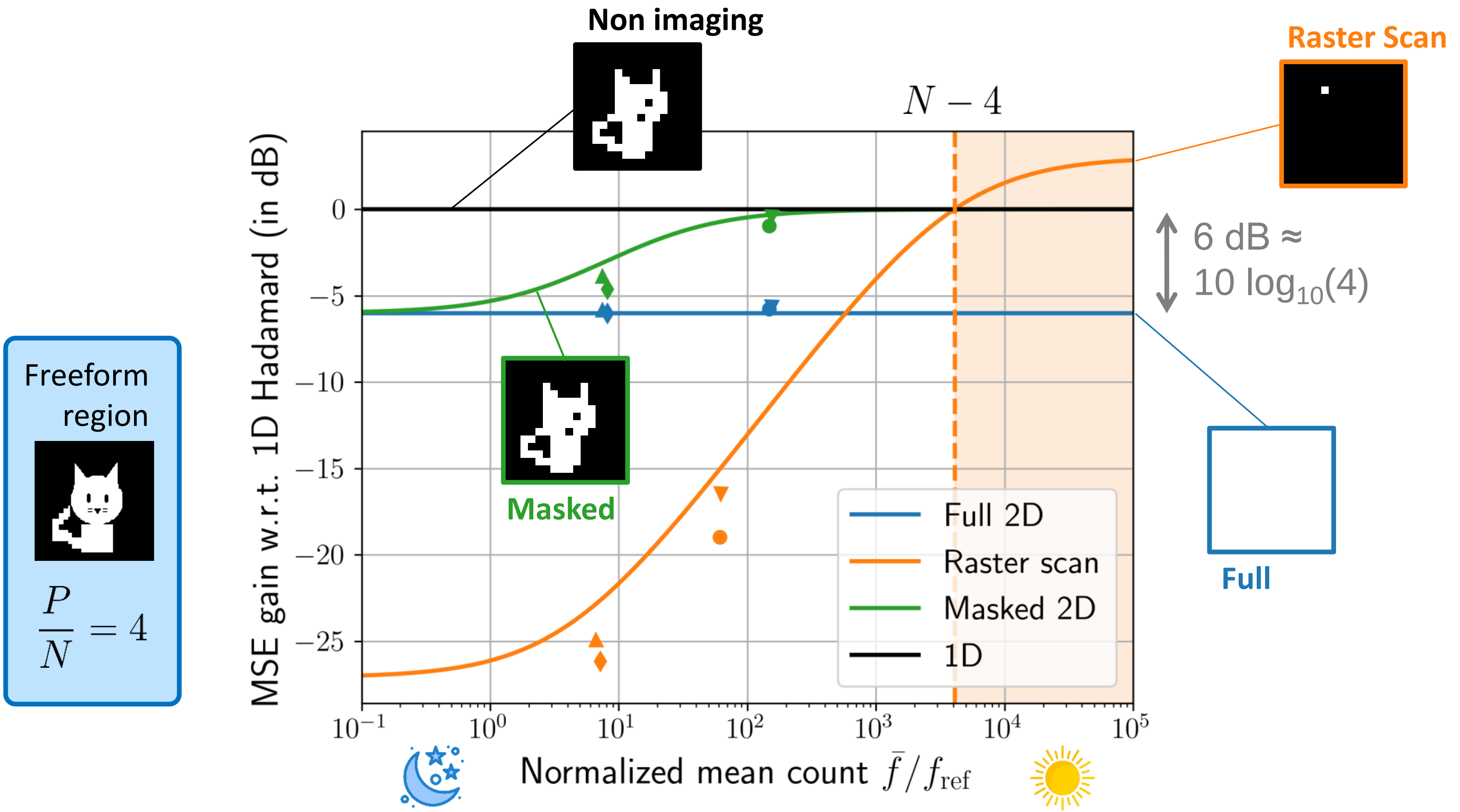
Masked



Non imaging

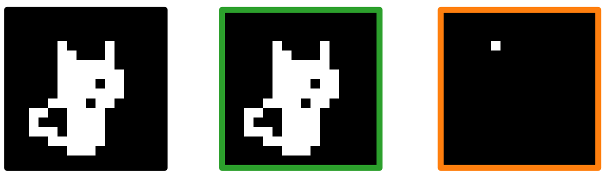


MSE: theory vs experiment



Note: The figure above was obtained using a split Hadamard matrix, not a negative Hadamard matrix, as the acquisition matrix.

1. Freeform imaging is **easy**



Non Imaging — Masked — Raster Scan

2. Freeform imaging **reduces noise**

3. Hadamard freeform imaging **outperforms raster scan** in low/intermediate light conditions

"Freeform Hadamard imaging: Back to the roots of computational optics"
https://hal.science/hal-05337760v1

https://github.com/openspyrit/spyrit-examples/tree/design/2025_freeform