

Acoustic nonlinearity parameter of tissue on echo mode: review and evaluation of the different approaches for B/A imaging

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Abstract—The nonlinear B/A parameter induces the distortion of ultrasound waves in tissue. Media with different nonlinear parameter differently distort the ultrasound wave. This paper reviews the techniques that allow to compute this parameter in echo mode imaging. For non homogeneous B/A medium, new formulations are proposed. The simulation results show the capacity of all methods to accurately compute the B/A parameter when diffraction effects of the probe are set off, while only the comparative method works nicely when diffraction is taken into account. First experimental results are also presented, confirming the good performance obtainable with such method.

Index Terms— *Ultrasound nonlinearity, B/A measurement, tissue characterization, RF signal*

I. INTRODUCTION

Nonlinear imaging has multiple medical applications such as tissue characterization, harmonic imaging or contrast imaging. For these methods, the nonlinear properties of the medium have a key role in the generation of harmonics. Two families of methods exist to determine the nonlinear parameter: thermodynamics and finite difference methods [1]. In echo mode, the finite difference methods are effective and the interesting ones are grouped in two categories. First, several methods are based on the increase of the second harmonic during the propagation [2-6]. Secondly, composite signals are transmitted and the nonlinear interactions between the different components are used to determine the nonlinear parameter [7-11].

These methods are often developed and used in media where the nonlinear parameter is constant along the propagation axis. Mathematical extension will be presented to use these methods with non constant nonlinear parameter.

In this study, the first part includes a theoretical background on nonlinear propagation theory and reviews the existing methods that could be implemented in echo mode. New formulations are proposed when the nonlinear parameter is not constant along the propagation axis. Then, the results obtained with a propagation simulator and first experimental images obtained from Radio Frequency (RF) signals are shown.

II. REVIEW OF EXISTING METHODS AND THEIR EXTENSIONS TO ECHO MODE IMAGING

A. Propagation equation

A complete model for nonlinear ultrasound propagation was proposed by Khokhlov Zabolotskaya Kuznetsov (KZK) [12, 13]:

$$\frac{\partial^2 p}{\partial z \partial t} = \frac{\beta}{2\rho_0 c_0^3} \frac{\partial^2 p^2}{\partial t^2} + \frac{\delta}{2c_0^3} \frac{\partial^3 p}{\partial t^3} + \frac{c_0}{2} \left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} \right) \quad (1)$$

where p is the pressure of the wave at the frequency f , z the propagation axis, t the time, x and y the coordinates in the plane perpendicular to the propagation axis, ρ_0 the density of the medium, c_0 the speed of sound, β the nonlinear parameter and δ the attenuation of the medium. The first term on the right side of the equation corresponds to the nonlinear effect. The second one represents the attenuation of tissue during the propagation and the last one represents the diffraction effect of the probe. If the diffraction effects are neglected (case of a plane wave), the KZK equation (1) becomes the Burger equation [1].

Practically, the nonlinear parameter β is directly related to the B/A parameter [2]:

$$\beta = 1 + \frac{B}{2A} \quad (2)$$

in which B and A are defined from the Taylor series of the ultrasound pressure p around p_0 :

$$B = \rho_0^2 \frac{\partial^2 p}{\partial \rho^2} \quad (3)$$

$$A = \rho_0 \frac{\partial p}{\partial \rho} \equiv \rho_0 c_0 \quad (4)$$

B. Second harmonic pressure

1) Second harmonic increase along the propagation axis

For a medium with attenuation and nonlinearity, Zhang *et al.* [3] give the equation describing the increase of the second harmonic p_2 along the propagation axis z :

$$p_2(z) = \frac{\omega p_0^2}{2\rho_0 c_0^3} e^{-\alpha_2 z} \int_0^z \beta(u) e^{(-2\alpha_1 + \alpha_2)u} du \quad (5)$$

where p_0 is the amplitude of the fundamental wave at $z=0$, ω is the angular frequency, α_1 and α_2 are the attenuations of the fundamental and second harmonic waves, respectively. If a theoretical medium is considered without attenuation and with constant nonlinear parameter, (5) becomes the classical formula of Law *et al.* [2]:

$$p_2(z) = \frac{\omega p_0^2 \beta z}{2\rho_0 c_0^3} \quad (6)$$

Equation (5) can be rewritten and derived to obtain the non constant nonlinear parameter in attenuating media along the propagation axis:

$$\beta(z) = \frac{2\rho_0 c_0^3}{\omega p_0^2} \left[\frac{dp_2(z)}{dz} + \alpha_2 p_2(z) \right] e^{2\alpha_1 z} \quad (7)$$

2) Comparative method

The comparative method, also called insertion substitution method, has first been developed by Gong *et al.* [4, 5]. To use it with inhomogeneous medium, eq. (5) has to be used. For the reference medium, where the nonlinear parameter β_0 is supposed constant, (5) is developed to:

$$p_{20}(z) = \frac{\omega p_0^2}{2\rho_0 c_0^3} \beta_0 \frac{e^{-2\alpha_1 z} - e^{-\alpha_2 z}}{\alpha_2 - 2\alpha_1} \quad (8)$$

If the known medium (0) and the unknown one (i) have constant nonlinear parameter, the ratio between second harmonic pressure is linked to the nonlinear parameter as in [6]:

$$\beta_i = \beta_0 \frac{(\rho c^3)_i p_{2i}}{(\rho c^3)_0 p_{20}} \quad (9)$$

In general, the ratio between the two second harmonics gives:

$$\frac{p_{2i}(z)}{p_{20}(z)} = \frac{(\rho c^3)_0}{(\rho c^3)_i} \frac{\alpha_2 - 2\alpha_1}{e^{(-2\alpha_1 + \alpha_2)z} - 1} \frac{\int_0^z \beta_i(u) e^{(-2\alpha_1 + \alpha_2)u} du}{\beta_0} \quad (10)$$

β_i can be isolated and derivated to extract the nonlinear parameter from non constant nonlinear parameter medium:

$$\beta_i(z) = \beta_0 \frac{(\rho c^3)_i}{(\rho c^3)_0} \left[\frac{p_{2i}}{p_{20}} + \frac{d}{dz} \left(\frac{p_{2i}}{p_{20}} \right) \frac{1 - e^{(2\alpha_1 - \alpha_2)z}}{\alpha_2 - 2\alpha_1} \right] \quad (11)$$

C. Composite signals

Differently from the previous method, where a single pulse at a given frequency is transmitted, more complicated transmit waves can be used, e.g., two or more waves at different frequencies and/or different pressure amplitude.

1) Two close high frequencies signals

The use of two close frequencies is proposed by Westerwelt [7] and Nakagawa *et al.* [8]. The technique has also been used with computerized tomography system by Zhang *et al.* [9]. The technique consists in transmitting two close high frequencies f_1 and f_2 . Their nonlinear interaction creates a

pressure wave P_- at the difference frequency $f_2 - f_1$. The formulation proposed by these authors gives:

$$\frac{dP_-(z)}{dz} = \frac{\beta(z)\omega_-}{2\rho_0 c_0^3} P_{f_1}(z) P_{f_2}(z) - \alpha_- P_-(z) \quad (12)$$

The evolution of the nonlinear parameter is direct from (12):

$$\beta(z) = \frac{2\rho_0 c_0^3}{\omega_-} \times \frac{\frac{dP_-(z)}{dz} + \alpha_- P_-(z)}{P_{f_1}(z) P_{f_2}(z)} \quad (13)$$

Equation (13) allows to compute the nonlinear parameter in any media.

2) High pressure low frequency signal modulated by a high frequency low pressure signal

Pasovic *et al.* [10] recently proposed a particular transmission wave that consists in a high pressure low frequency wave modulated by a low pressure high frequency wave. This work experimentally shows the opportunity of measuring the B/A parameter of various tissues, but the theoretical background was not established and did not allow to go further in the use of the method.

3) Pump wave

The use of a pump wave was introduced by Fukukita *et al.* [11] inspired by Ichida *et al.* [14]. The transmitted signal is a classical sum of a pump wave at low frequency and a pulse wave at high frequency. The phase of the pulse is localized at a particular point of the pump wave. Ichida sends the two waves perpendicularly while Fukukita sends them in the same direction. Fukukita added the high frequency pulse in the increase or the decrease of the pump wave. Indeed, due to the nonlinearity of the medium, the high pressure travels faster than the low pressure. As the pump wave is distorted, the pulse wave is compressed or dilated. This effect, linked to the nonlinear parameter, can be measured on the frequency spectrum. First, the ratio spectrum $R(f)$ around the high frequency f_{hf} is computed in an interval defined by df and with the Fourier Transform (FT) of the signal:

$$R(f) = \frac{(FT(\text{increase}))_{[f_{hf}-df; f_{hf}+df]}}{(FT(\text{decrease}))_{[f_{hf}-df; f_{hf}+df]}} \quad (14)$$

From the spectral ratio in (14), the cross over frequency f_x is defined when:

$$\log(R(f)) = 0 \quad (15)$$

Moreover, the slope of the spectral ratio is needed:

$$S_x = \left(\frac{\partial \{ \log[R(f)] \}}{\partial f} \right)_{f=f_x} \quad (16)$$

Finally, Fukukita expressed the nonlinear parameter as:

$$\beta(z) = \frac{\rho_0 c_0^3 \sigma_0^2 S_x(z)}{2z \omega_h p_0 f_x(z)} \quad (17)$$

with σ_0^2 the variance of the pulse duration time. Equation (17) is valid in constant nonlinear parameter medium. Indeed, the spectral ratio slope is defined by:

$$S_x(z) = \frac{2\omega_h f_x(z)}{\sigma_0^2} \times \tau \quad (18)$$

and in a non constant nonlinear medium, τ is expressed as:

$$\tau = p_0 \int_0^z \frac{\beta(u)}{\rho_0 c_0} du \quad (19)$$

For a constant nonlinear parameter medium, (19) and (18) gives (17), but, in general case, β is obtained by:

$$\beta(z) = \frac{\rho_0 c_0^3 \sigma_0^2}{2\omega_h p_0} \times \frac{d}{dz} \left(\frac{S_x(z)}{f_x(z)} \right) \quad (20)$$

III. RESULTS

A. Simulation parameters

The simulations have been based on a KZK simulator [15]. The simulated medium has a sound velocity of 1500 m.s^{-1} and a density of 1000 kg.m^{-3} . The B/A values of the medium evolve along the 150 mm of the propagation. B/A parameter for the medium will be defined by (Fig. 1):

$$\frac{B}{A} = \begin{cases} 3 & \text{for } 0 \leq z \leq 4 \text{ cm} \\ 3 + 5(z-4) & \text{for } 4 < z \leq 5 \\ 8 & \text{for } 5 < z \leq 9 \text{ cm} \\ 8 - 5(z-9) & \text{for } 9 < z \leq 10 \\ 3 & \text{for } 10 \text{ cm} < z \end{cases} \quad (21)$$

The probe used in the simulator is based on the characteristics of a real linear array with 128 elements and transmit focus set at 71 mm. The frequency of the transmit signal can be set between 1 and 6 MHz.

The simulator is composed of the three characteristic terms of the KZK equation. The absorption, the nonlinearity and the diffraction effect of the probe can be set. At each point of the z axis, the pressure profile is obtained. From it, the fundamental or second harmonic components of the pressure are extracted.

Two different cases have been simulated. First, the diffraction effect of the probe was turned off and the transducer elements transmit an ideal wave which did not interact with neighbour's transmission. Second the diffraction effect of the probe was considered and the transducer elements transmit a wave which is propagated in the entire space and interactions with the other elements transmissions are taken into account.

B. Simulations without diffraction

In a first approximation, the diffraction effects are set off and the Burger equation is used in simulations. Results of simulation of presented methods are shown in the same Fig. 1.

The maximum error on the B/A parameter computed from the four methods along the propagation distance is only 2.5%.

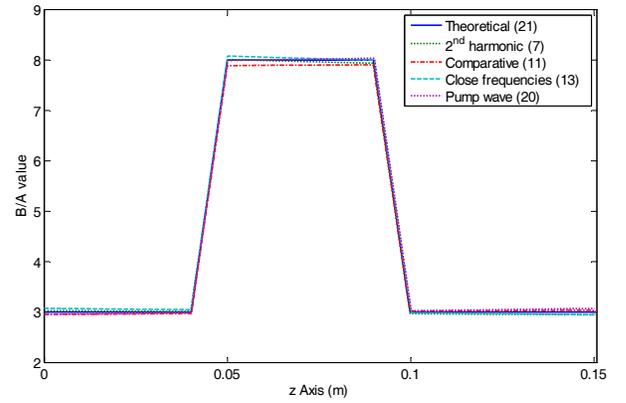


Fig. 1. Theoretical and simulated B/A evolution in function of the propagation distance. The results proposed by the different methods are quite effective, following the particular evolution of the B/A parameter

C. Simulation with diffraction effect of the probe

Now the diffraction effect of the probe is set on. Focussing effects in the fundamental and second harmonic (amplitude $\times 20$) components are shown in Fig. 2.

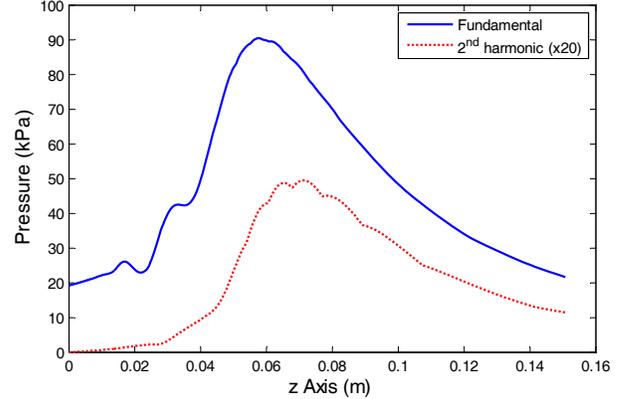


Fig. 2. Pressure evolution of the fundamental and the second harmonic in function of the propagation distance when the transmit beam was focused at 71 mm for $f=3\text{MHz}$.

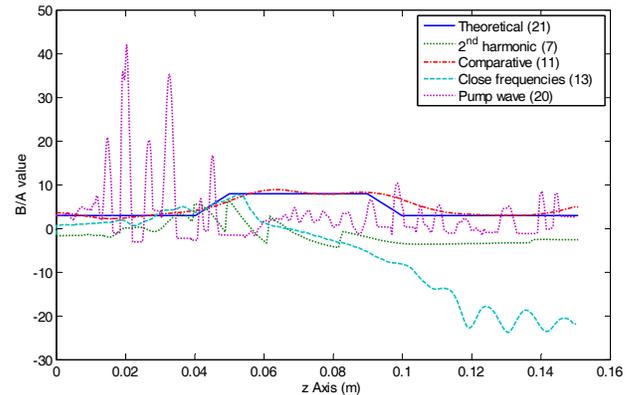


Fig. 3. B/A evolution as a function of the propagation distance when diffraction effect is considered. Only the comparative method leads to satisfying results.

If the B/A parameter is extracted according to the same formulas used above, the results are no more as good as without diffraction, as shown in Fig. 3. Only the comparative method can be considered acceptable with diffraction phenomenon.

D. Experimental results

Experimental RF signals have been acquired by an Ultrasonix SONIX RP scanner to test the comparative method. An elastography phantom (CIRS, Elasticity QA Phantom 049) was used to simulate two different media (interior and exterior of the inclusion). The reference medium with constant B/A parameter has been defined on the left of the phantom in a homogeneous region. Fig. 4 compares the images obtained through the classic harmonic method and the insertion comparative method. The latter is more homogeneous and does not depend of the probe focalization depth. Moreover, the investigation depth is increased. The comparative method at the first order was use here. Indeed, the derivative part was neglected in a first approximation.

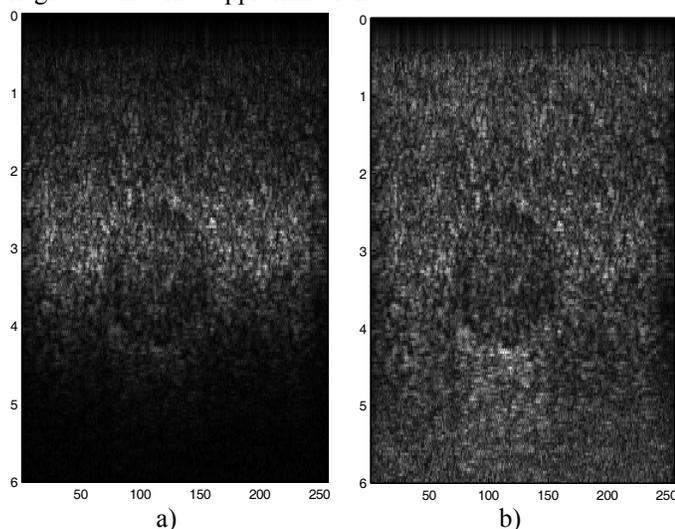


Fig. 4. Comparison of the classic harmonic image (a) and the insertion comparative image (b). The background in (b) is more homogeneous and do not depend of the focalization depth of the probe.

IV. DISCUSSION AND CONCLUSIONS

The review of the B/A imaging on echo mode images highlights several methods capable of extracting the nonlinear parameter of tissues. However, they can be further developed and particularly when diffraction effects are considered. Another difficulty comes from the transmission of composite signals on ultrasound scanners. When diffraction is considered, the behavior of the methods is very different and only the comparative method works nicely. The diffraction seems not being correctly described by the theoretical background. Some authors have tried to decrease the diffraction effect using special transducers [5] but, to our knowledge, no work exists on a real ultrasound probe. However, this effect can be controlled when the method is based on the ratio of two similar measures obtained with the same diffraction scheme.

The first result on RF data shows the interest of imaging P_{2i}/P_{20} . The focalization effect is decreased and the exploration depth is increased. A first approximation of a relative value of the B/A parameter is also obtained. Even so, the final calculus of the B/A parameter is difficult because it involves a derivative part which can reach large value due to the speckle noise in ultrasound image.

This study shows that the diffraction effect of the probe is the main issue that must be studied. As the methods are effective without the diffraction effect of the probe, the reduction of this effect could allow to use these methods on experimental RF signals. Transmission of plane waves could also be an alternative solution to limit the diffraction effect but in this case, the energy transmitted and the non linear effect are quite low.

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