Image processing inspired by quantum mechanics: application to image restoration

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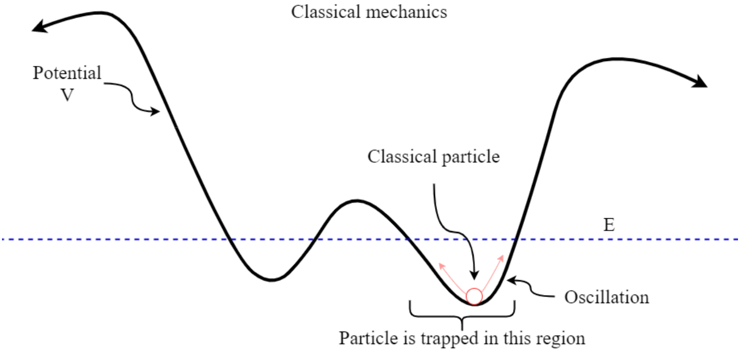
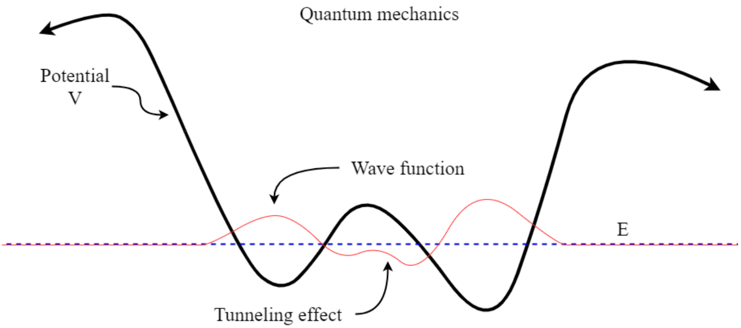
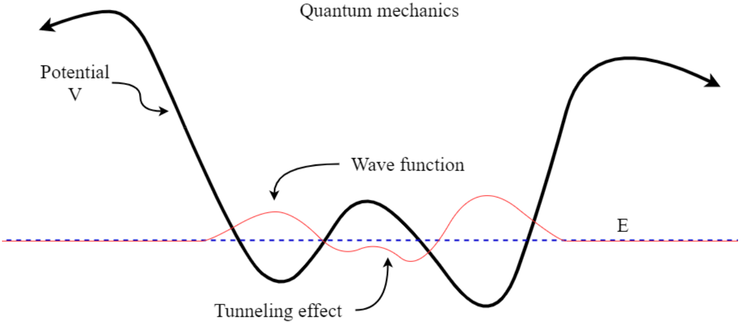
Classical mechanics



Quantum mechanics

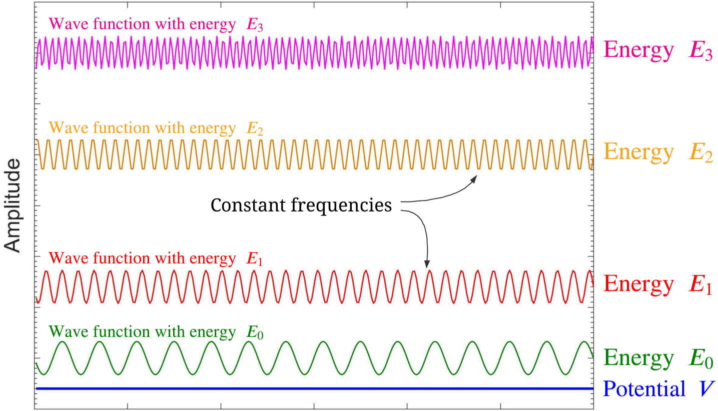


Quantum mechanics



Schrödinger equation: $-\frac{\hbar^2}{2m}\nabla^2\psi = -V(a)\psi + E\psi$

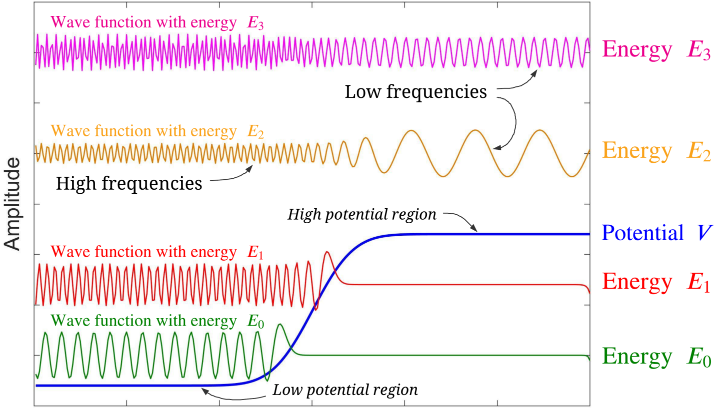
Wave functions



Constant potential

Schrödinger equation: $-\frac{\hbar^2}{2m}\nabla^2\psi = -V(a)\psi + E\psi$

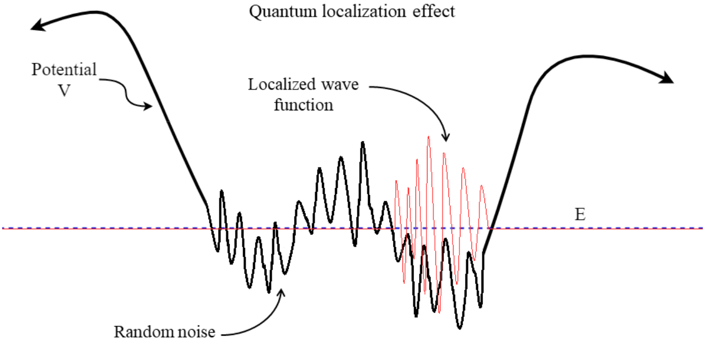
Wave functions



Potential with different heights

Schrödinger equation: $-\frac{\hbar^2}{2m}\nabla^2\psi = -V(a)\psi + E\psi$

Localization effect



Objective

Image processing inspired by quantum mechanics

- ⋆ How to compute wave vectors for images?
- ⋆ Are they useful for image processing tasks?
- ⋆ What kind of image processing tasks can be addressed with such tools?

Outline

Outline

Existing literature

- Seminal (theoretical) work in quantum signal processing[1]
- Image segmentation (tunnel effect)[2][3]
- Pulse-shaped signal analysis[4][5]
- Image denoising[6][7]
- Image processing algorithms adapted to quantum computers

[1] Yonina C Eldar and Alan V Oppenheim. "Quantum signal processing". In: *IEEE Signal Processing Magazine* 19.6 (2002), pp. 12–32.

[2] C. Aytekin, S. Kiranyaz, and M. Gabbouj. "Quantum mechanics in computer vision: Automatic object extraction". In: *IEEE International Conference on Image Processing*. 2013, pp. 2489–2493.

[3] Akram Youssry, Ahmed El-Rafei, and Salwa Elramly. "A quantum mechanics-based framework for image processing and its application to image segmentation". In: *Quantum Information Processing* 14.10 (2015), pp. 3613–3638.

[4] Taous-Meriem Laleg-Kirati, Emmanuelle Crépeau, and Michel Sorine. "Semi-classical signal analysis". In: *Mathematics of Control, Signals, and Systems* 25.1 (2013), pp. 37–61. ISSN: 1435-568X.

[5] Taous-Meriem Laleg-Kirati et al. "Spectral data de-noising using semi-classical signal analysis: application to localized MRS". In: *NMR in Biomedicine* 29.10 (2016), pp. 1477–1485.

[6] Zineb Kaisserli, Taous-Meriem Laleg-Kirati, and Amina Lahmar-Benbernou. "A novel algorithm for image representation using discrete spectrum of the Schrödinger operator". In: *Digital Signal Processing* 40 (2015), pp. 80 –87.

[7] Abderrazak Chahid et al. "A New ROI-Based performance evaluation method for image denoising using the Squared Eigenfunctions of the Schrödinger Operator". In: *2018 40th Annual International Conference of the IEEE Engineering in Medicine and Biology Society (EMBC)*. IEEE. 2018, pp. 5579–5582.

Outline

Hamiltonian operator

- Construct an adaptive basis using the solutions of Schrödinger equation.
- The Schroedinger equation:

$$-\frac{\hbar^2}{2m}\nabla^2\psi = -V(a)\psi + E\psi$$

- Can be rewritten as an eigenvalue problem:

$$\boldsymbol{H}_{\mathcal{QAB}}\psi = E\psi,$$

  where $\boldsymbol{H}_{\mathcal{QAB}} = -\frac{\hbar^2}{2m}\nabla^2 + V$ is the Hamiltonian operator.
- Main idea: replace $V(a)$, the potential of the system, by an image pixels' values.
- The Hamiltonian operator associated to an image[8][9]:

$$\boldsymbol{H}_{\mathcal{QAB}}[i,j] = \begin{cases} \boldsymbol{x}[i] + 4\frac{\hbar^2}{2m} & \textit{for } i = j, \\ -\frac{\hbar^2}{2m} & \textit{for } i = j \pm 1, \\ -\frac{\hbar^2}{2m} & \textit{for } i = j \pm n, \\ 0 & \textit{otherwise}, \end{cases}$$

[8] C. Aytekin, S. Kiranyaz, and M. Gabbouj. "Quantum mechanics in computer vision: Automatic object extraction". In: *IEEE International Conference on Image Processing*. 2013, pp. 2489–2493.

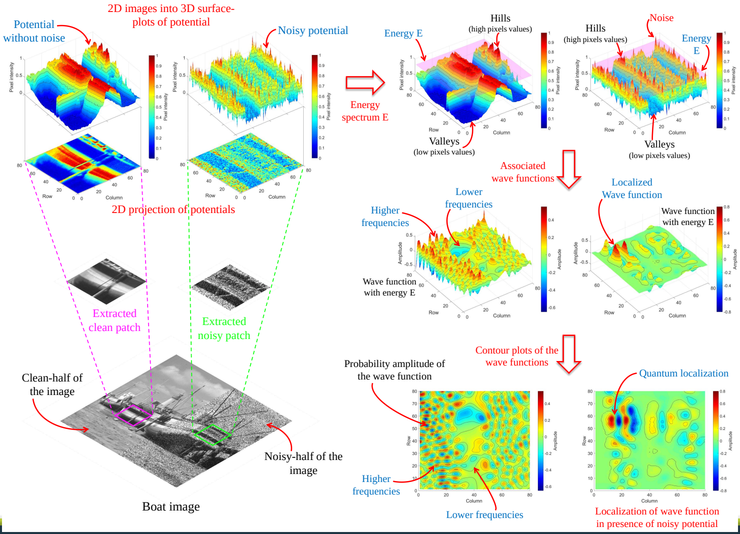
[9] Sayantan Dutta et al. "Quantum Mechanics-Based Signal and Image Representation: Application to Denoising". In: *IEEE Open Journal of Signal Processing* 2 (2021), pp. 190–206.

Toy example of Hamiltonian operator for a 4×4 image

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$x(1)+2\frac{\hbar^2}{2m}$	$-\frac{\hbar^2}{2m}$	0	0	$-\frac{\hbar^2}{2m}$	0	0	0	0	0	0	0	0	0	0	0
$-\frac{\hbar^2}{2m}$	$x(2)+3\frac{\hbar^2}{2m}$	$-\frac{\hbar^2}{2m}$	0	0	$-\frac{\hbar^2}{2m}$	0	0	0	0	0	0	0	0	0	0
0	$-\frac{\hbar^2}{2m}$	$x(3)+3\frac{\hbar^2}{2m}$	$-\frac{\hbar^2}{2m}$	0	0	$-\frac{\hbar^2}{2m}$	0	0	0	0	0	0	0	0	0
0	0	$-\frac{\hbar^2}{2m}$	$x(4)+2\frac{\hbar^2}{2m}$	0	0	0	$-\frac{\hbar^2}{2m}$	0	0	0	0	0	0	0	0
$-\frac{\hbar^2}{2m}$	0	0	0	$x(5)+3\frac{\hbar^2}{2m}$	$-\frac{\hbar^2}{2m}$	0	0	$-\frac{\hbar^2}{2m}$	0	0	0	0	0	0	0
0	$-\frac{\hbar^2}{2m}$	0	0	$-\frac{\hbar^2}{2m}$	$x(6)+4\frac{\hbar^2}{2m}$	$-\frac{\hbar^2}{2m}$	0	0	$-\frac{\hbar^2}{2m}$	0	0	0	0	0	0
0	0	$-\frac{\hbar^2}{2m}$	0	0	$-\frac{\hbar^2}{2m}$	$x(7)+4\frac{\hbar^2}{2m}$	$-\frac{\hbar^2}{2m}$	0	0	$-\frac{\hbar^2}{2m}$	0	0	0	0	0
0	0	0	$-\frac{\hbar^2}{2m}$	0	0	$-\frac{\hbar^2}{2m}$	$x(8)+3\frac{\hbar^2}{2m}$	0	0	0	$-\frac{\hbar^2}{2m}$	0	0	0	0
0	0	0	0	$-\frac{\hbar^2}{2m}$	0	0	0	$x(9)+4\frac{\hbar^2}{2m}$	$-\frac{\hbar^2}{2m}$	0	0	$-\frac{\hbar^2}{2m}$	0	0	0
0	0	0	0	0	$-\frac{\hbar^2}{2m}$	0	0	$-\frac{\hbar^2}{2m}$	$x(10)+4\frac{\hbar^2}{2m}$	$-\frac{\hbar^2}{2m}$	0	0	$-\frac{\hbar^2}{2m}$	0	0
0	0	0	0	0	0	$-\frac{\hbar^2}{2m}$	0	0	$-\frac{\hbar^2}{2m}$	$x(11)+4\frac{\hbar^2}{2m}$	$-\frac{\hbar^2}{2m}$	0	0	$-\frac{\hbar^2}{2m}$	0
0	0	0	0	0	0	0	$-\frac{\hbar^2}{2m}$	0	0	$-\frac{\hbar^2}{2m}$	$x(12)+3\frac{\hbar^2}{2m}$	0	0	0	$-\frac{\hbar^2}{2m}$
0	0	0	0	0	0	0	0	$-\frac{\hbar^2}{2m}$	0	0	0	$x(13)+2\frac{\hbar^2}{2m}$	$-\frac{\hbar^2}{2m}$	0	0
0	0	0	0	0	0	0	0	0	$-\frac{\hbar^2}{2m}$	0	0	$-\frac{\hbar^2}{2m}$	$x(14)+3\frac{\hbar^2}{2m}$	$-\frac{\hbar^2}{2m}$	0
0	0	0	0	0	0	0	0	0	0	$-\frac{\hbar^2}{2m}$	0	0	$-\frac{\hbar^2}{2m}$	$x(15)+3\frac{\hbar^2}{2m}$	$-\frac{\hbar^2}{2m}$
0	0	0	0	0	0	0	0	0	0	0	$-\frac{\hbar^2}{2m}$	0	0	$-\frac{\hbar^2}{2m}$	$x(16)+2\frac{\hbar^2}{2m}$

Quantum adaptive transform

- The set of eigenvectors corresponding to the Hamiltonian operator represents the adaptive transform and is denoted as the quantum adaptative basis (QAB).



Outline

Denoising algorithm

Input: x, $\frac{\hbar^2}{2m}$, s, ρ, σ

1 Compute a smooth version of x by Gaussian filtering
2 Construct the Hamiltonian matrix H based on the smoothed version of x
3 Calculate the eigenvectors ψ_i of H
4 Compute the coefficients α_i by projecting x onto the basis formed by ψ_i
5 Threshold the coefficients α_i and recover the denoised signal or image

Output: $\hat{x}$

- Projection on QAB:

$$\hat{\boldsymbol{x}} = \sum_{i=1}^{N^2} \boldsymbol{\alpha}_i \boldsymbol{\psi}_i \tau_i$$

- Soft thresholding:

$$\tau_i = \begin{cases} 1 & \text{for } i \leq s, \\ 1 - \frac{i-s}{\rho} & \text{for } i > s \text{ and for } 1 - \frac{i-s}{\rho} > 0, \\ 0 & \text{otherwise.} \end{cases}$$

where $\boldsymbol{\alpha}_i = \langle \boldsymbol{x}, \boldsymbol{\psi}_i \rangle$ are the coefficients representing the image $\boldsymbol{x}$ in the QAB[10].

[10] Sayantan Dutta et al. "Quantum Mechanics-Based Signal and Image Representation: Application to Denoising". In: *IEEE Open Journal of Signal Processing* 2 (2021), pp. 190–206.

Denoising results

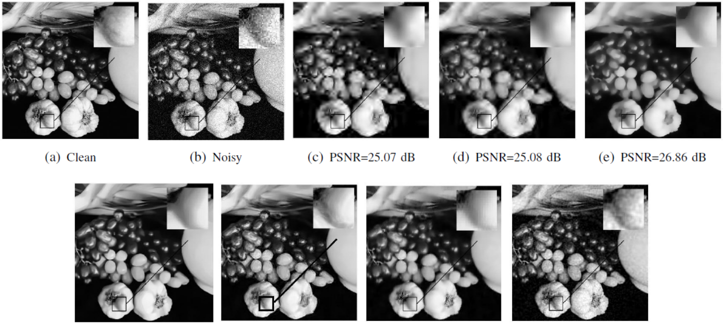
(a) Synthetic image (b) Fruits (c) Moon

- Gaussian, Poisson and speckle noise for a SNR of 15 dB
- Comparison to several methods from the state of the art

Quantitative results

Sample	Method	Gaussian Noise (15dB)			Poisson Noise (15 dB)			Speckle Noise (15dB)		
		SNR (dB)	PSNR (dB)	SSIM	SNR (dB)	PSNR (dB)	SSIM	SNR (dB)	PSNR (dB)	SSIM
Synthetic Image	Wavelet hard	15.01	24.46	0.61	15.01	25.68	0.69	15.01	25.34	0.76
	Wavelet soft	15.71	25.05	0.64	15.61	26.20	0.70	15.49	25.80	0.77
	VST	NA	NA	NA	15.09	25.83	0.69	15.06	25.58	0.76
	TV	15.74	25.07	0.64	15.62	26.23	0.71	15.53	25.78	0.77
	GSP	**20.28**	**28.78**	**0.79**	NA	NA	NA	NA	NA	NA
	NLM	18.70	26.88	0.71	NA	NA	NA	NA	NA	NA
	DL	17.35	26.15	0.71	**17.14**	**27.22**	**0.75**	**17.21**	**27.48**	**0.80**
	Proposed	**23.42**	**31.78**	**0.89**	**23.92**	**32.78**	**0.92**	**25.32**	**33.50**	**0.95**
Fruits	Wavelet hard	18.60	25.07	0.65	18.59	25.53	0.65	18.38	24.86	0.64
	Wavelet soft	18.84	25.08	0.71	18.81	25.29	0.72	18.51	24.50	0.71
	VST	NA	NA	NA	19.37	25.96	**0.76**	19.01	25.61	**0.76**
	TV	20.69	26.86	**0.79**	20.60	26.71	0.75	20.18	26.34	0.74
	GSP	**21.44**	27.43	**0.81**	NA	NA	NA	NA	NA	NA
	NLM	**21.48**	**28.02**	0.77	NA	NA	NA	NA	NA	NA
	DL	21.30	27.37	**0.79**	**20.87**	**27.16**	0.71	**20.39**	**27.08**	0.72
	Proposed	21.39	**28.07**	0.77	**21.93**	**28.31**	**0.79**	**21.83**	**28.29**	**0.82**
Moon	Wavelet hard	22.91	30.02	0.70	21.45	29.90	0.72	21.19	29.07	0.71
	Wavelet soft	23.09	30.98	0.74	22.14	30.51	0.80	21.90	29.79	0.79
	VST	NA	NA	NA	22.58	31.17	**0.85**	22.11	30.01	0.84
	TV	23.35	32.19	0.80	**23.51**	**32.21**	**0.86**	**22.91**	**30.84**	**0.86**
	GSP	23.33	31.22	**0.85**	NA	NA	NA	NA	NA	NA
	NLM	**25.79**	**33.94**	**0.86**	NA	NA	NA	NA	NA	NA
	DL	23.82	32.71	0.81	22.95	31.65	**0.85**	22.32	30.07	0.84
	Proposed	**24.81**	**33.11**	0.83	**24.65**	**33.34**	**0.86**	**23.48**	**31.55**	**0.89**

Denoising results

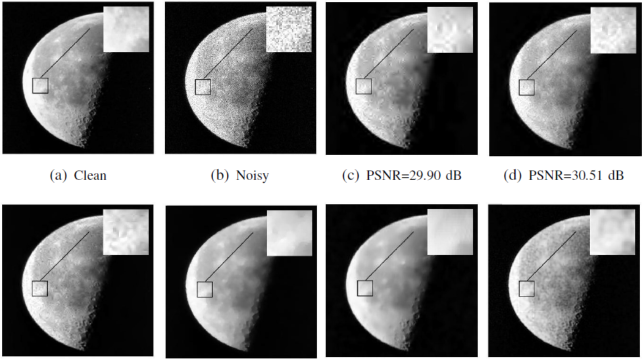


(a) Clean (b) Noisy (c) PSNR=25.07 dB (d) PSNR=25.08 dB (e) PSNR=26.86 dB

(f) PSNR=27.43 dB (g) PSNR=28.02 dB (h) PSNR=27.37 dB (i) PSNR=28.07 dB

- (a) Clean Fruits image, (b) Image corrupted with Gaussian noise corresponding to a SNR of 15 dB.
- Denoising results obtained using: (c) wavelet hard thresholding, (d) wavelet soft thresholding, (e) total variation regularization, (f) graph signal processing, (g) non-local means, (h) dictionary learning and (i) proposed method.

Denoising results



(a) Clean (b) Noisy (c) PSNR=29.90 dB (d) PSNR=30.51 dB

(e) PSNR=31.17 dB (f) PSNR=32.21 dB (g) PSNR=31.65 dB (h) PSNR=33.34 dB

- (a) Clean moon image, (b) Image corrupted with Poisson noise corresponding to a SNR of 15 dB.
- Denoising results obtained using: (c) wavelet hard thresholding, (d) wavelet soft thresholding, (e) variance stabilization transform, (f) total variation regularization, (g) dictionary learning and (h) proposed method.

Pros and cons

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- Good denoising results.
- Good flexibility wrt noise statistics.

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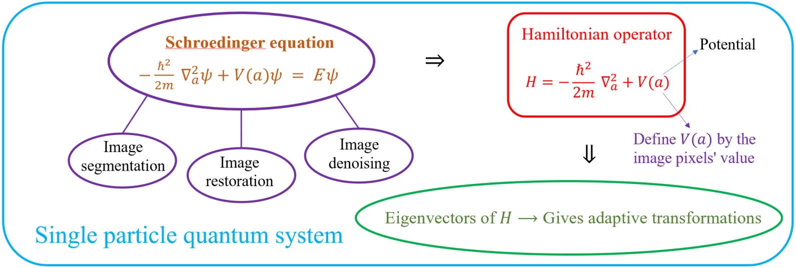
- Limited to small images (Hamiltonian high dimension).
- Block-wise implementation for large images, with blocks processed independently.
- Need to smooth the image before computing the QAB to escape the localization effect.

Outline

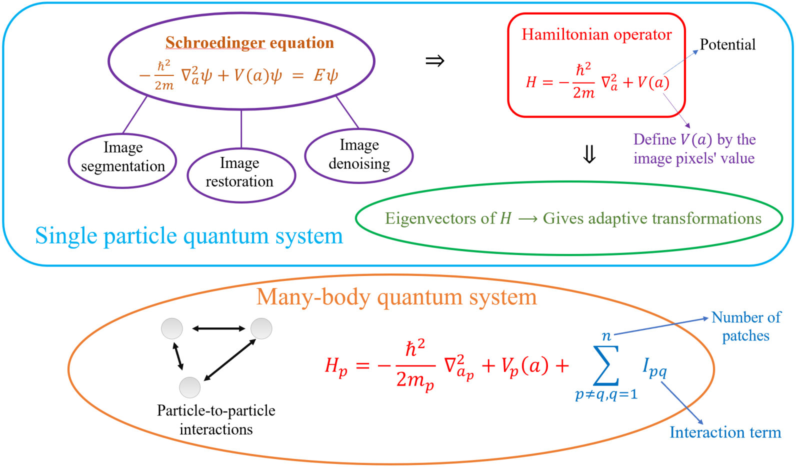
From one particle to multiple particles



[11] C. Aytekin, S. Kiranyaz, and M. Gabbouj. "Quantum mechanics in computer vision: Automatic object extraction". In: *IEEE International Conference on Image Processing*. 2013, pp. 2489–2493.

[12] Sayantan Dutta et al. "Quantum Mechanics-Based Signal and Image Representation: Application to Denoising". In: *IEEE Open Journal of Signal Processing* 2 (2021), pp. 190–206.

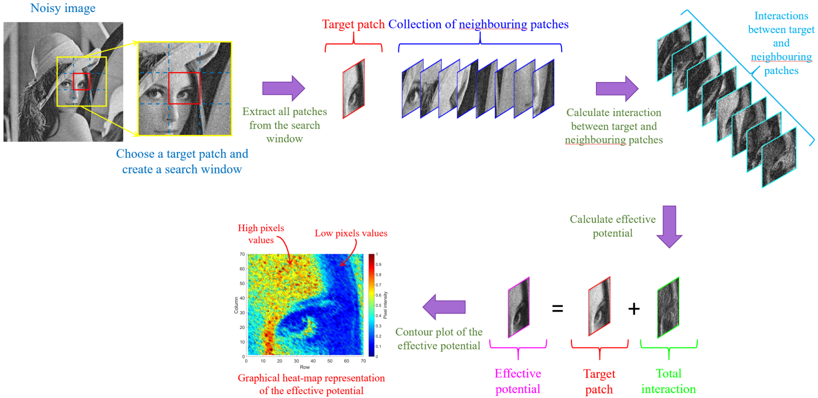
From one particle to multiple particles



[11] C. Aytekin, S. Kiranyaz, and M. Gabbouj. "Quantum mechanics in computer vision: Automatic object extraction". In: *IEEE International Conference on Image Processing*. 2013, pp. 2489–2493.

[12] Sayantan Dutta et al. "Quantum Mechanics-Based Signal and Image Representation: Application to Denoising". In: *IEEE Open Journal of Signal Processing* 2 (2021), pp. 190–206.

From one patch to multiple patches



[13] Sayantan Dutta et al. "Image Denoising Inspired by Quantum Many-Body physics". anglais. In: *IEEE International Conference on Image Processing (ICIP 2021), Anchorage, Alaska, USA, 19/09/2021-22/09/2021*. 2021.

Hamiltonian for multiple patch approach

- Hamiltonian for one image patch:

$$H_a = \underbrace{-\frac{\hbar^2}{2m_a}\nabla^2_{y_a} + V(y_a)}_{H_{0_a}} + \underbrace{\sum_{b=1, b\neq a}^{z} I_{ab}}_{H_{I_a}}, \ a = 1, \cdots, z$$

where, I_{ab} is the interaction between the a-th and b-th patches, H_{0_a} is the Hamiltonian in the a-th patch as a single particle system, and H_{I_a} is the total interaction between the a-th patch and its neighbours.

- Effective potential for one image patch:

$$V_a^{effective} = V(y_a) + \sum_{b=1, b\neq a}^{z} I_{ab} = V(y_a) + H_{I_a}$$

- Modelisation of the interactions between patches

$$I_{ab} = p\frac{|\boldsymbol{A}-\boldsymbol{B}|}{D_{ab}^2}$$

Denoising algorithm

- Similar to a non local mean algorithm

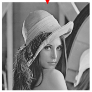
For each image patch

- Compute its effective potential using the interactions with its neighbours.
- Construct the Hamiltonian from the effective potential.
- Compute the QAB by eigendecomposition of the Hamiltonian.
- Denoise the current patch by soft thresholding its projection in the QAB.

Numerical simulations

Three sample images

Lake image



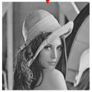
Lena image

House image

Numerical simulations

Three sample images

Lake image



Lena image

House image

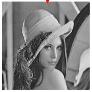
+

Four different noise levels (SNRs of 2, 8, 16 and 22 dB)

Numerical simulations

Three sample images

Lake image



Lena image

House image

+

Four different noise levels (SNRs of 2, 8, 16 and 22 dB)

⇓

Comparison with three state-of-the-art methods

NLM

PGPCA

PLPCA

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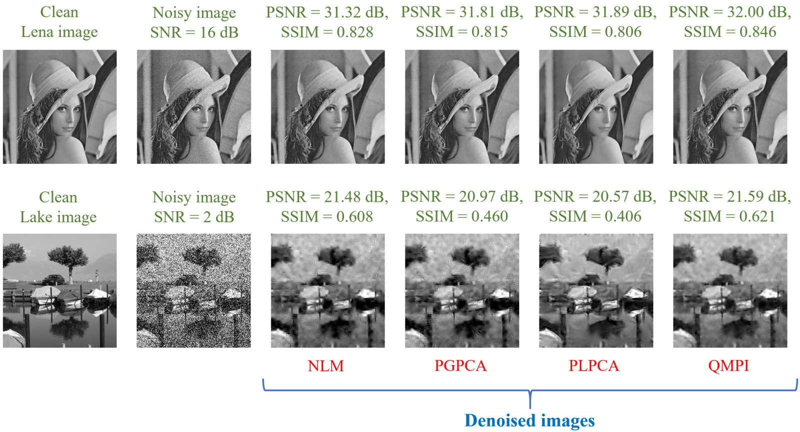
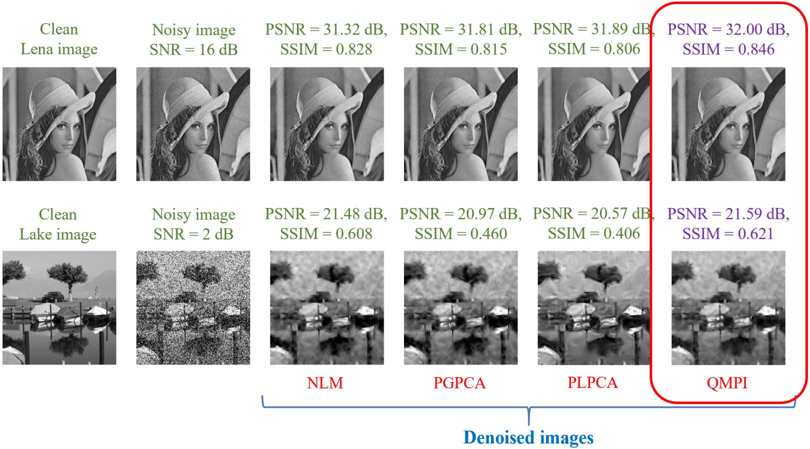
[a] Tolga Tasdizen. "Principal neighborhood dictionaries for nonlocal means image denoising". In: *IEEE Transactions on Image Processing* 18.12 (2009), pp. 2649–2660.

[b] Charles-Alban Deledalle, Joseph Salmon, Arnak S Dalalyan, et al. "Image denoising with patch based PCA: local versus global.". In: *BMVC*. vol. 81. 2011, pp. 425–455.

Results


Clean
Lena image
Noisy image
SNR = 16 dB
PSNR = 31.32 dB,
SSIM = 0.828
PSNR = 31.81 dB,
SSIM = 0.815
PSNR = 31.89 dB,
SSIM = 0.806
PSNR = 32.00 dB,
SSIM = 0.846
Clean
Lake image
Noisy image
SNR = 2 dB
PSNR = 21.48 dB,
SSIM = 0.608
PSNR = 20.97 dB,
SSIM = 0.460
PSNR = 20.57 dB,
SSIM = 0.406
PSNR = 21.59 dB,
SSIM = 0.621
NLM
PGPCA
PLPCA
QMPI
Denoised images

Results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NLM
PGPCA
PLPCA
QMPI
Denoised images

Outline

Denoising can do more than denoise

Plug & play ADMM

- Replace the explicit choice of the regularization function by existing state-of-the-art denoisers[a].

[a] S. V. Venkatakrishnan, C. A. Bouman, and B. Wohlberg. "Plug-and-Play priors for model based reconstruction". In: *2013 IEEE Global Conference on Signal and Information Processing*. 2013, pp. 945–948. DOI: 10.1109/GlobalSIP.2013.6737048.

Regularisation by denoising

- Design a regularization function based on a state-of-the-art denoiser[a].

[a] Y. Romano, M. Elad, and P. Milanfar. "The little engine that could: Regularization by denoising (RED)". In: *SIAM Journal on Imaging Sciences* 10.4 (2017), pp. 1804–1844.

Poisson deconvolution

- The image formation model is governed by the Poisson process $\mathcal{P}(\cdot)$ as

$$\boldsymbol{y} = \mathcal{P}(\boldsymbol{H}\boldsymbol{x}),$$

where $\boldsymbol{y} \in \mathbb{R}^{n^2}$ represents the noisy-blurred observation of the desired image $\boldsymbol{x} \in \mathbb{R}^{n^2}$ and $\boldsymbol{H} \in \mathbb{R}^{n^2 \times n^2}$ is a block circulant with circulant block matrix.

- ADMM to solve the regularized $(g(\boldsymbol{z}))$ deconvolution problem

$$\begin{aligned}
\boldsymbol{x}^{k+1} &= \arg\min_{\boldsymbol{x}} \; f(\boldsymbol{x}) + \frac{\lambda^k}{2} \left\| \boldsymbol{x} - \boldsymbol{z}^k + \boldsymbol{u}^k \right\|_2^2 \\
\boldsymbol{z}^{k+1} &= \arg\min_{\boldsymbol{z}} \; g(\boldsymbol{z}) + \frac{\lambda^k}{2} \left\| \boldsymbol{x}^{k+1} - \boldsymbol{z} + \boldsymbol{u}^k \right\|_2^2 \\
\boldsymbol{u}^{k+1} &= \boldsymbol{u}^k + \boldsymbol{x}^{k+1} - \boldsymbol{z}^{k+1} \\
\lambda^{k+1} &= \gamma \lambda^k
\end{aligned}$$

- The data fidelity term $f(\boldsymbol{x})$ is

$$f(\boldsymbol{x}) = -\boldsymbol{y}^T log(\boldsymbol{H}\boldsymbol{x}) + 1^T \boldsymbol{H}\boldsymbol{x} + \text{constant}$$

Plug & play framework

$$\boxed{z^{k+1} = \arg\min_{z} \left(g(z) + (\lambda^k/2) \left| x^{k+1} + u^k - z \right|^2 \right)}$$

$\Rightarrow$ Associated with a denoising process

$$\Downarrow$$

$$\boxed{z^{k+1} = \mathcal{D}\left(x^{k+1} + u^k \right)}$$

$\Rightarrow$ The 2^{nd} iteration is replaces by a state-of-the-art denoiser[a] (Here $\mathcal{D}$ is the denoiser)

[a] S. V. Venkatakrishnan, C. A. Bouman, and B. Wohlberg. "Plug-and-Play priors for model based reconstruction". In: *2013 IEEE Global Conference on Signal and Information Processing*. 2013, pp. 945–948. DOI: 10.1109/GlobalSIP.2013.6737048.

Plug & play framework for Poisson model

$$\tilde{\boldsymbol{z}}^k = \left(\boldsymbol{x}^{k+1} + \boldsymbol{u}^k\right) \longrightarrow \text{Redefine}$$

$$\Downarrow$$

$$\tilde{\boldsymbol{z}}^k \xrightarrow{\text{Reparameterize}} 2\sqrt{\tilde{\boldsymbol{z}}^k + \frac{3}{8}} \longrightarrow \text{Variance stabilization transform (VST)}^{ab}$$

(gives a Gaussian approximation)

$$\Downarrow$$

$$\boxed{\boldsymbol{z}^{k+1} = \mathcal{D}\left(\tilde{\boldsymbol{z}}^k\right)} \longrightarrow \text{Apply a state-of-the-art denoiser}$$

[a] Lucio Azzari and Alessandro Foi. "Variance stabilization in Poisson image deblurring". In: *2017 IEEE 14th International Symposium on Biomedical Imaging (ISBI 2017)*. IEEE. 2017, pp. 728–731.

[b] Arie Rond, Raja Giryes, and Michael Elad. "Poisson inverse problems by the plug-and-play scheme". In: *Journal of Visual Communication and Image Representation* 41 (2016), pp. 96–108.

Drawbacks

Limitations

- ⋆ These VST-based approaches exhibit inaccuracies while dealing with high-intensity noise[a].
- ⋆ The convolution operation is not invariant under a VST[b].

[a] Joseph Salmon et al. "Poisson noise reduction with non-local PCA". In: *Journal of mathematical imaging and vision* 48.2 (2014), pp. 279–294.

[b] Arie Rond, Raja Giryes, and Michael Elad. "Poisson inverse problems by the plug-and-play scheme". In: *Journal of Visual Communication and Image Representation* 41 (2016), pp. 96–108.

Plug in our denoiser without any VST

The PnP-ADMM scheme with proper parameterization reads as

$$
\begin{aligned}
\boldsymbol{x}^{k+1} &= \arg\min_{\boldsymbol{x}} \Big(-\boldsymbol{y}^T log(\boldsymbol{H}\boldsymbol{x}) + 1^T \boldsymbol{H}\boldsymbol{x} + (\lambda^k/2) \Big| \boldsymbol{x} - \boldsymbol{z}^k + \boldsymbol{u}^k \Big|^2 \Big) \\
\boldsymbol{z}^{k+1} &= \mathcal{D}_{\mathcal{QAB}} \Big(\boldsymbol{x}^{k+1} + \boldsymbol{u}^k \Big) \\
\boldsymbol{u}^{k+1} &= \boldsymbol{u}^k + \boldsymbol{x}^{k+1} - \boldsymbol{z}^{k+1}
\end{aligned}
$$

- Denoising process executes the time-consuming task of computations of all the projection coefficients of the noisy image onto the adaptive basis.
- But very few coefficients actually participate in the restoration process.
- Using the orthogonal matching pursuit (OMP) algorithm computes a sparse approximation of the most significant coefficients. This reduces the computational time significantly[14].

[14] Sayantan Dutta et al. "Plug-and-Play Quantum Adaptive Denoiser for Deconvolving Poisson Noisy Images". In: *IEEE Access* (2021).

Numerical simulations

- A detailed survey has been performed through three sample images: one synthetic image and two cropped versions of standard images.
- All the images are distorted with a Gaussian blurring kernel of size 4×4 and standard deviation $\sigma = 3$ with with three different Poisson noise levels corresponding to SNRs of 20, 15 and 10 dB.
- We provide a comparison with a state-of-the-art PnP-ADMM methods[a] (P^4IP) and a standard total variation-based ADMM deconvolution algorithm[b] (TV-ADMM) adapted to Poisson observations.

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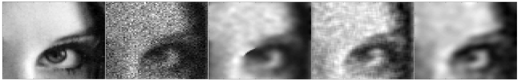
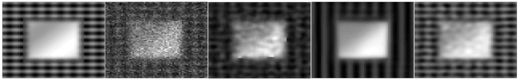
[a] Arie Rond, Raja Giryes, and Michael Elad. "Poisson inverse problems by the plug-and-play scheme". In: *Journal of Visual Communication and Image Representation* 41 (2016), pp. 96–108.

[b] François de Vieilleville et al. "Alternating direction method of multipliers applied to 3d light sheet fluorescence microscopy image deblurring using gpu hardware". In: *2011 Annual International Conference of the IEEE Engineering in Medicine and Biology Society*. IEEE. 2011, pp. 4872–4875.

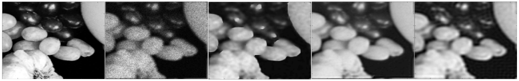
Results



(d) Lena image corrupted with 10 dB Poisson noise.



(e) Synthetic image corrupted with 15 dB Poisson noise.



(f) Fruits image corrupted with 20 dB Poisson noise.

Figure: In each row, the first, second, third, fourth and fifth images are accordingly clean, blurred noisy, deblurred result by TV-ADMM, deblurred result by P^4IP and deblurred result by the proposed Algorithm.

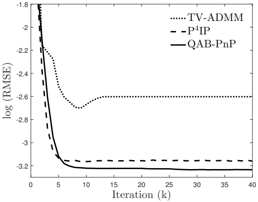


Figure: Logarithmic RMSE as a function of the number of iterations.

Quantitative data

TABLE III
QUANTITATIVE DATA (AVERAGE OVER 200 NOISE REALIZATIONS)

Sample	Method	Poisson Noise		
		SNR = 20dB	SNR = 15dB	SNR = 10dB
Synthetic	TV-ADMM	26.46±0.10 0.66±0.01	24.80±0.34 0.58±0.01	22.52±1.55 0.52±0.02
	P^4IP	23.90±1.37 0.74±0.06	20.91±2.18 0.59±0.11	18.96±3.34 0.48±0.18
	QAB-PnP	**29.86**±**0.12** **0.92**±0.00	**27.18**±0.43 **0.86**±0.01	**24.23**±1.34 **0.74**±0.03
Lena	TV-ADMM	27.37±0.31 0.74±0.01	24.52±0.65 0.66±0.01	19.97±1.32 0.52±0.02
	P^4IP	27.32±0.44 **0.81**±0.01	24.87±2.76 **0.76**±0.07	18.67±4.83 0.55±0.16
	QAB-PnP	**28.97**±0.19 **0.81**±0.00	**27.04**±0.44 0.75±0.01	**20.18**±**3.39** **0.65**±0.08
Fruits	TV-ADMM	20.51±0.38 0.57±0.01	19.02±0.23 0.55±0.01	**17.54**±0.93 0.51±0.01
	P^4IP	20.42±1.79 0.59±0.04	17.22±4.62 0.52±0.11	14.35±3.85 **0.53**±0.04
	QAB-PnP	**21.37**±0.94 **0.62**±0.01	**19.35**±0.96 **0.57**±0.02	17.28±3.55 0.51±0.12

*The best values are highlighted in bold.

Outline

Conclusion

Perspectives

- ⋆ Any relationship with graph image processing?
- ⋆ Consider the Schrödinger equation in the frequency domain.
- ⋆ Look at other (fascinating) quantum mechanics principles.
- ⋆ Machine learning-based algorithms.
- ⋆ Quantum computing.

Codes available

- ⋆ `https://github.com/SayantanDutta95`

Thank You
For Your Attention