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(A short intro to) Deep learning for image reconstruction

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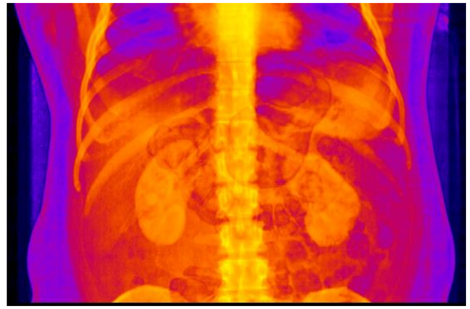
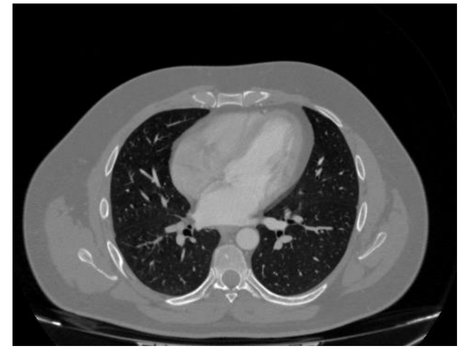
“Natural” vs Reconstructed Images

Nat



Rec



Rec

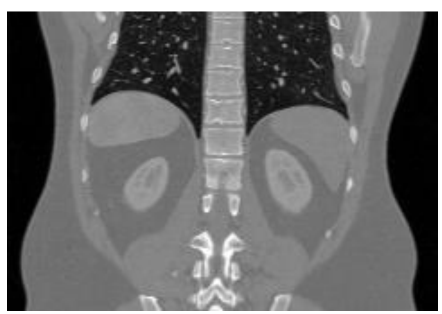
Nat

Nat

Nat

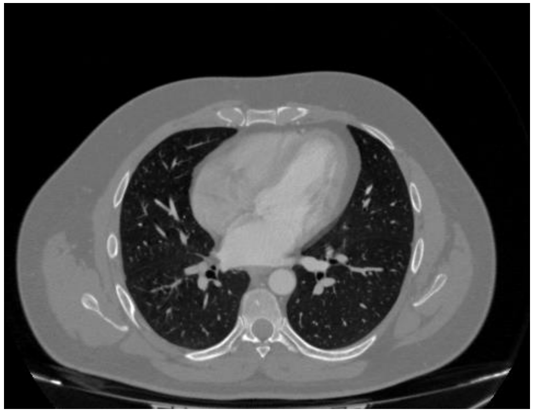
Nat

Rec



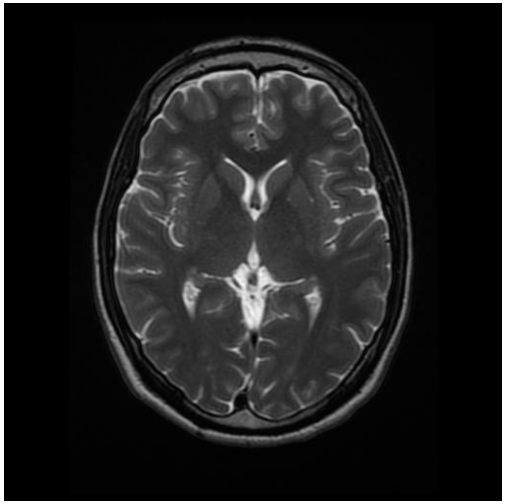
Reconstruction in Medical Imaging

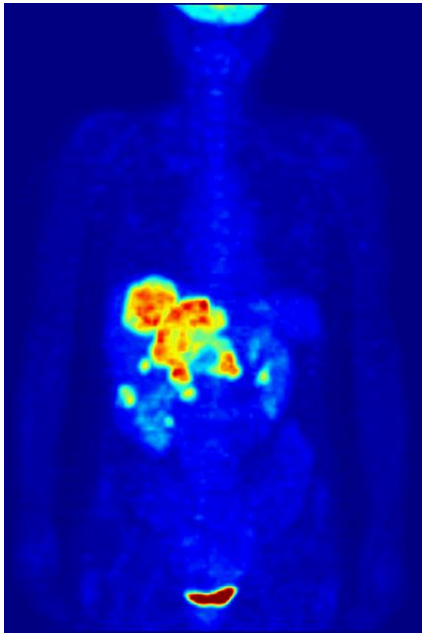
Computerized tomography (CT)



Ultrasound Imaging

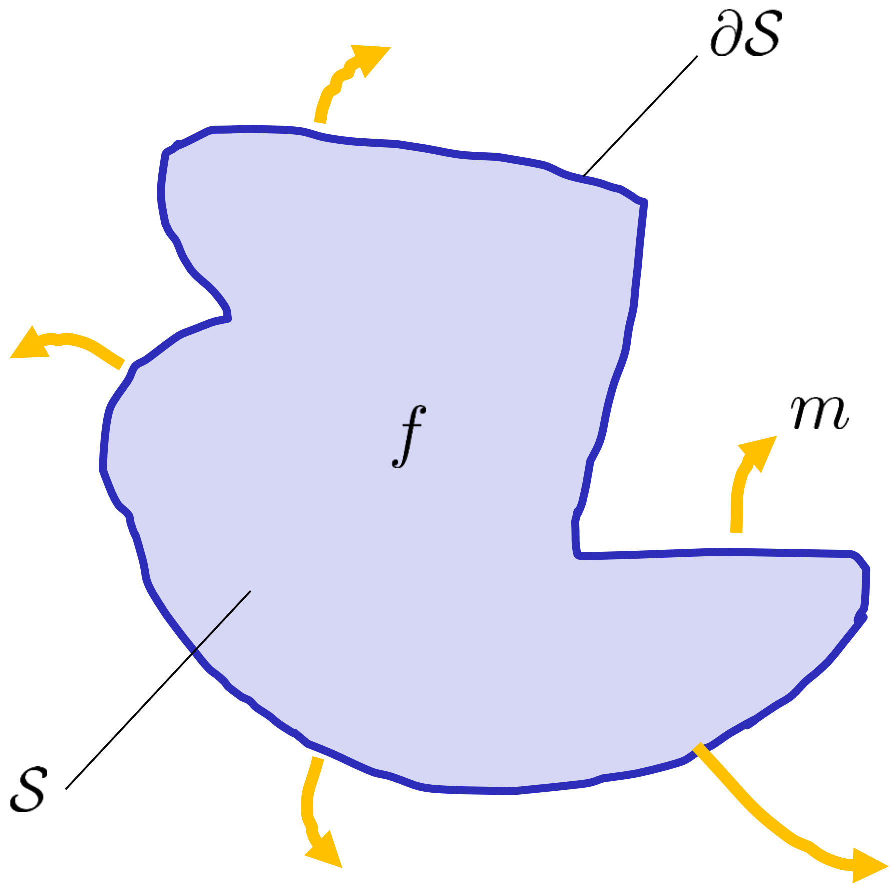
Magnetic Resonance (MRI)





Positron emission tomography (PET)

Inverse Problem



Inverse Problem

- **Internal unknowns from external measurements**

$$m = \mathcal{A}(f)$$

External Measurement

Forward (or direct) Model

(Unknown) quantity of Interest

- **Most medical imaging problems are**
  - Linear
  - Corrupted by noise

- **In a discrete setting**

$$\boldsymbol{m} = \boldsymbol{A}\boldsymbol{f} + \boldsymbol{\eta}$$

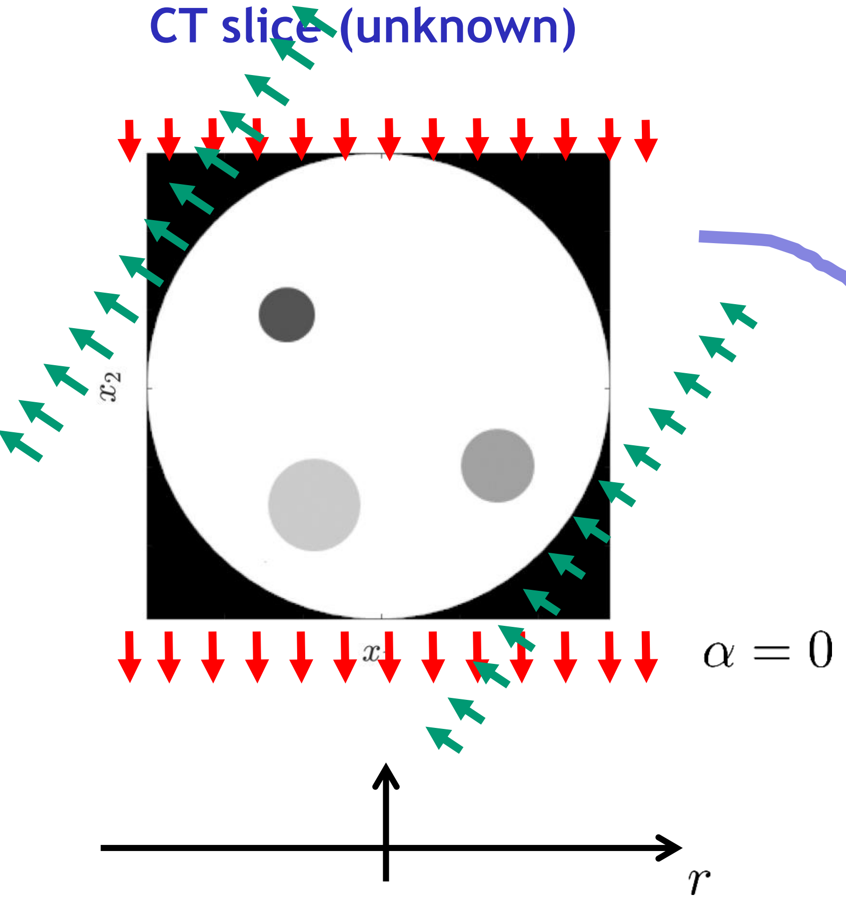
Noise

$\boldsymbol{m} \in \mathbb{R}^M$

$\boldsymbol{A} \in \mathbb{R}^{M \times N}$

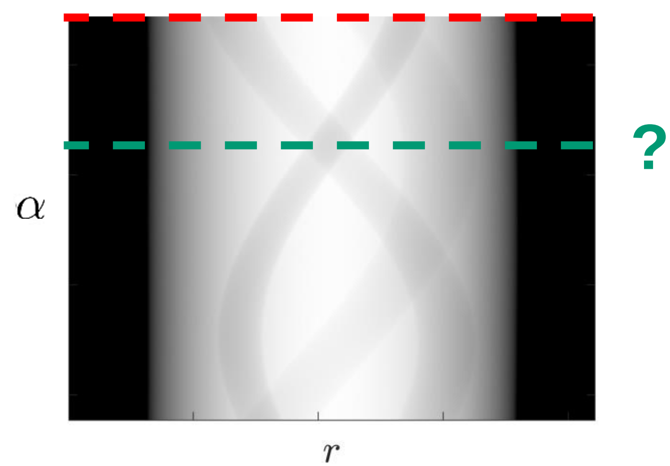
$\boldsymbol{f} \in \mathbb{R}^N$

Computed Tomography (CT)



$$m(\alpha, r) = \int_{\mathcal{L}(\alpha,r)} f(x_1, x_2)\,\mathrm{d}\ell$$

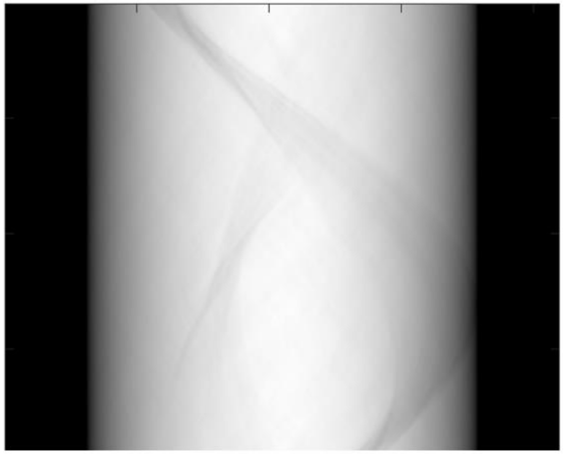
Sinogram (measured)



Computed Tomography (CT)

CT slice (unknown)

Sinogram (measured)



Different Options

- **1. Inversion "by hand"**
  - Model the forward and invert analytically

$$\texttt{derive } \mathcal{R} \texttt{ such that } f = \mathcal{R}(m)$$

- **2. Optimization of handcrafted functionals**
  - Build cost function from prior knowledge about the solution/measurements
  - Minimize the cost function

$$\texttt{find and minimize } \mathcal{C} \texttt{ such that } \mathcal{C}(f;m) \texttt{ is small}$$

- **3. "Learn" to reconstruct**
  - (Probably what you expect from this talk)

$$\texttt{learn } \mathcal{R}_\theta \texttt{ such that } f = \mathcal{R}_\theta(m)$$

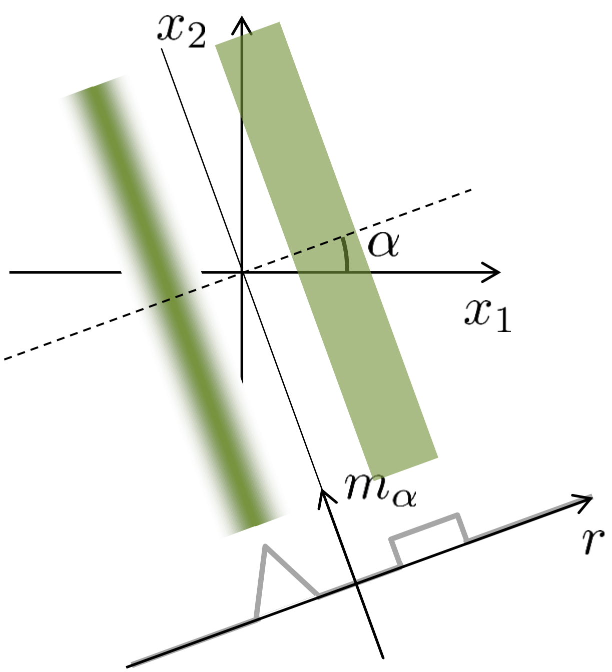
Option #1:
Analytical Methods

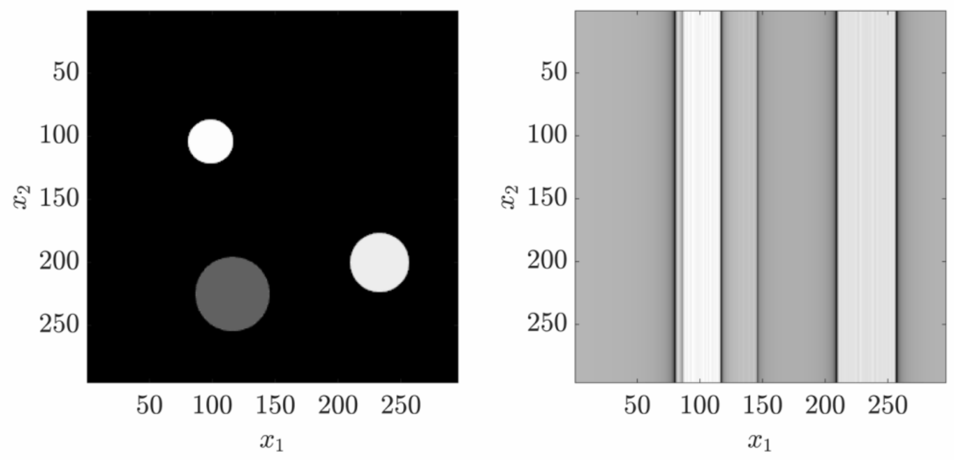
➢ Example with CT (filtered backprojection)

$$f(x_1, x_2) = \int_0^\pi m_\alpha^{\text{filt}}(x_1 \cos\alpha + x_2 \sin\alpha)\, \mathrm{d}\alpha$$

$$\hat{m}_\alpha^{\text{filt}}(\xi) = |\xi|\, \hat{m}_\alpha(\xi)$$

Ramp filter





1. Analytical Methods

- **Pros**
  - Elegant
  - Theoretical guarantees
  - Usually fast implementation
  - What if only few measurements are available?
    - For dose reduction/short scans

- **Cons**
  - Not always possible to derive a solution
  - Influence of noise?
  - What if only few measurements are available?
    - Random or pseudo-random

Option #2:
Optimization-Based Methods

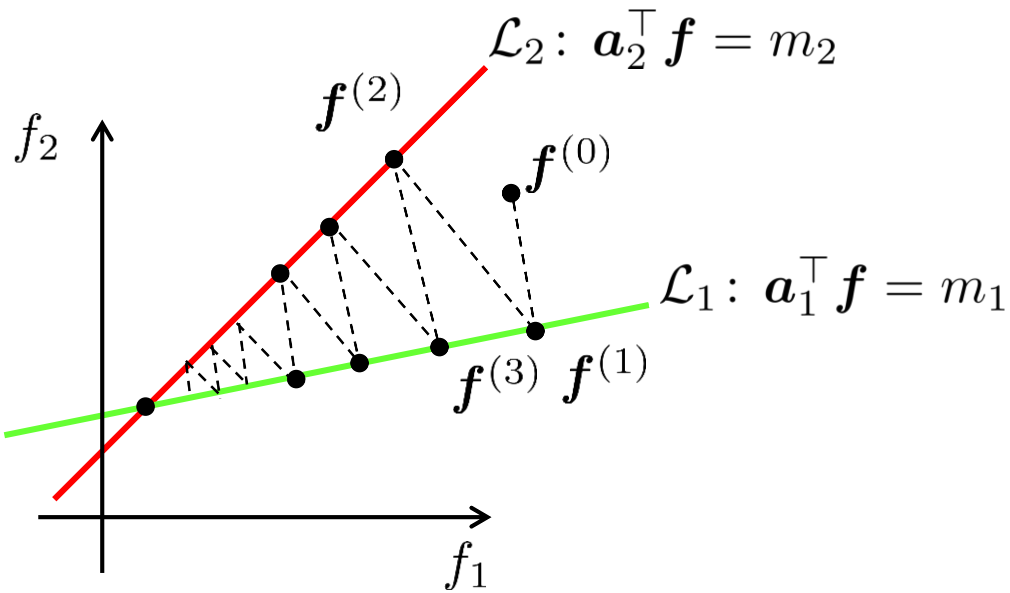
2. Optimization-Based Methods

- **Look for an image with small residuals**

$$\boldsymbol{r} = \boldsymbol{m} - \boldsymbol{A}\boldsymbol{f} \approx \boldsymbol{0}$$

- A simple example:

$$\begin{bmatrix} m_1 \\ m_2 \end{bmatrix} = \begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$



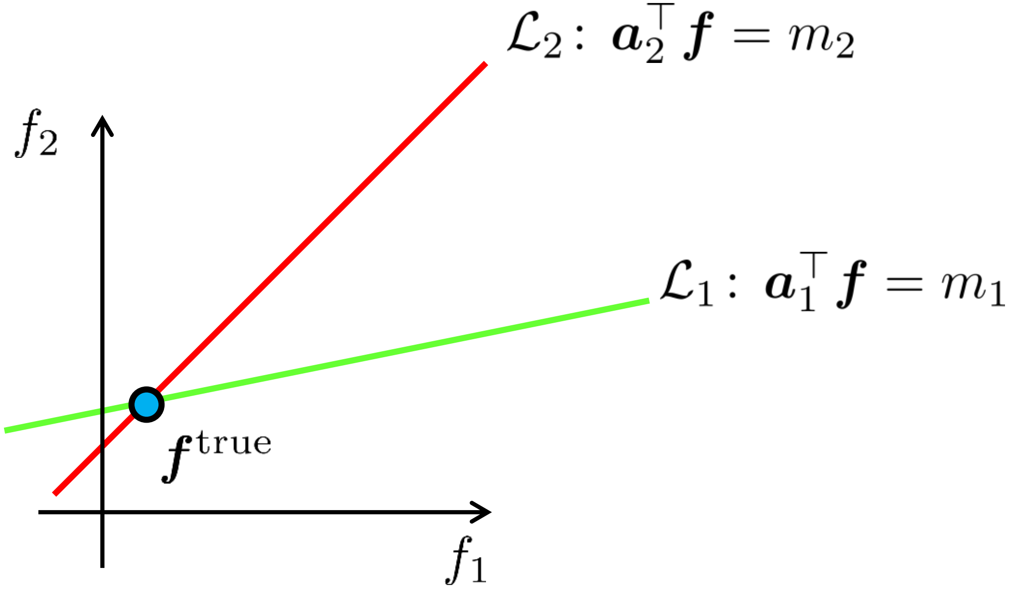
Algebraic reconstruction technique (**ART**) [Gordon R., 1970]

2. Optimization-Based Methods

- **Look for an image with small residuals**

$$\boldsymbol{r} = \boldsymbol{m} - \boldsymbol{A}\boldsymbol{f} \approx \boldsymbol{0}$$

- **Influence of noise**



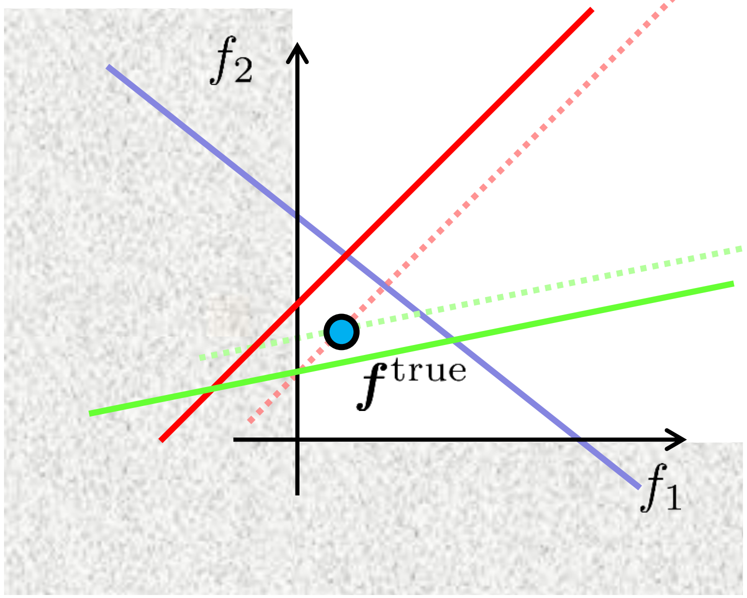
2. Optimization-Based Methods

- **Look for an image with small residuals**

$$\boldsymbol{r} = \boldsymbol{m} - \boldsymbol{A}\boldsymbol{f} \approx \boldsymbol{0}$$

- **Influence of noise**
  - More measurements (i.e., $M > N$)
  - Prior knowledge (e.g., $\boldsymbol{f} > \boldsymbol{0}$)

$$\mathcal{L}_2 : \; \boldsymbol{a}_2^\top \boldsymbol{f} = m_2 + \boxed{\eta_2}$$



$$\mathcal{L}_1 : \; \boldsymbol{a}_1^\top \boldsymbol{f} = m_1 + \boxed{\eta_1}$$

2. Optimization-Based Methods

➢ Typical cost functions

$$\min_{\boldsymbol{f}} \; \mathcal{D}(\boldsymbol{f}, \boldsymbol{m}) + \lambda \Psi(\boldsymbol{f})$$

Data fidelity

Regularization (prior)

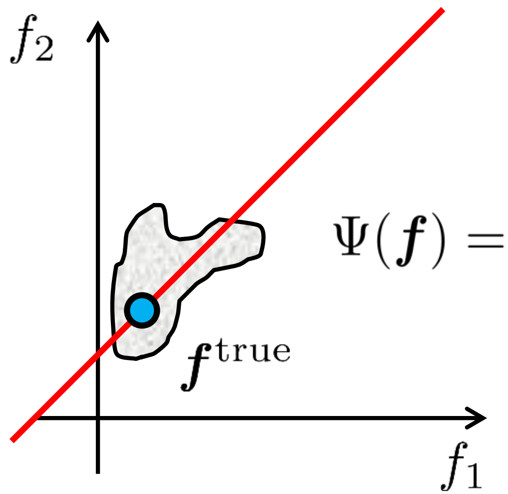
- Data fidelity is related the noise model/measurements confidence. E.g.

$$\mathcal{D}(\boldsymbol{f}; \boldsymbol{m}) = \|\boldsymbol{m} - \boldsymbol{A}\boldsymbol{f}\|_2^2$$

$$\mathcal{D}(\boldsymbol{f}; \boldsymbol{m}) = \mathrm{KL}(\boldsymbol{m}, \boldsymbol{A}\boldsymbol{f})$$

- Regularization convey prior knowledge about the solution.

$$M \ll N$$



$$\Psi(\boldsymbol{f}) = \begin{cases} 0 \text{ if } \boldsymbol{f} \in \mathcal{X} \\ \infty \text{ otherwise} \end{cases}$$

Typical regularizers

- Quadratic/Tikhonov regularization

$$\Psi(\boldsymbol{f}) = \|\boldsymbol{f}\|_2^2$$

… leads to

$$\boldsymbol{f}^* = \boldsymbol{A}^\top(\boldsymbol{A}\boldsymbol{A}^\top + \lambda \boldsymbol{I}_M)^{-1}\boldsymbol{m}$$

- Sparsity-promoting

$$\Psi(\boldsymbol{f}) = \|\boldsymbol{\Phi}\boldsymbol{f}\|_1$$

… requires iterative algorithms

$$\boldsymbol{z}^{(k)} = \boldsymbol{f}^{(k-1)} - \eta \boxed{\boldsymbol{A}^\top(\boldsymbol{A}\boldsymbol{f}^{(k-1)} - \boldsymbol{m})}$$

$$\boldsymbol{f}^{(k)} = \boxed{\mathtt{prox}_{\lambda\Psi}}(\boldsymbol{z}^{(k)})$$

gradient of data fidelity

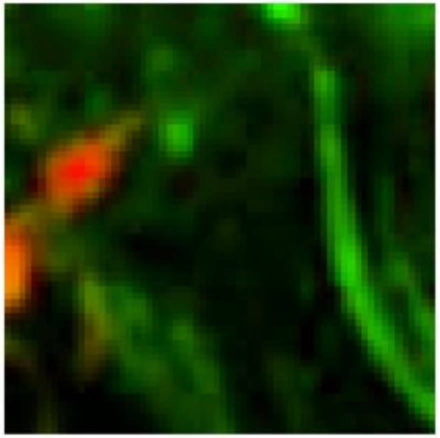
proximal operator of regularizer

2. Optimization-Based Methods

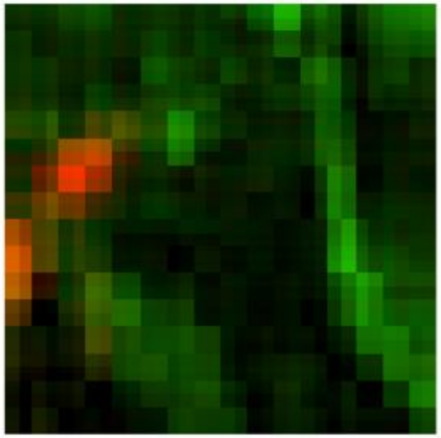
➢ Illustrative results

$N = 64 \times 64$ image
$M = 333$ measurements
$N / M \approx 8\%$

Ground-Truth



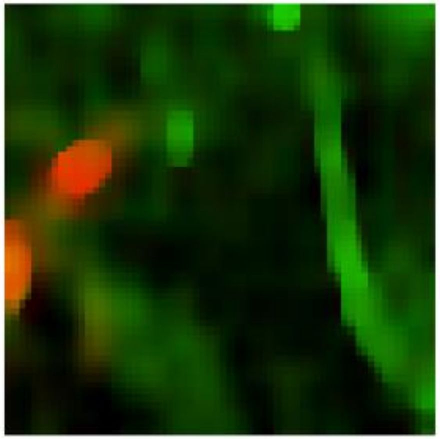
$\Psi(\boldsymbol{f}) = \|\boldsymbol{f}\|_2^2$



++ Analytical solution (fast computation)

- - Image quality

$\Psi(\boldsymbol{f}) = \|\nabla \boldsymbol{f}\|_1$



- - Iterative algorithms (time consuming)

++ Image quality

Option #3:
Learning-Based Methods

3. Learning-Based Methods

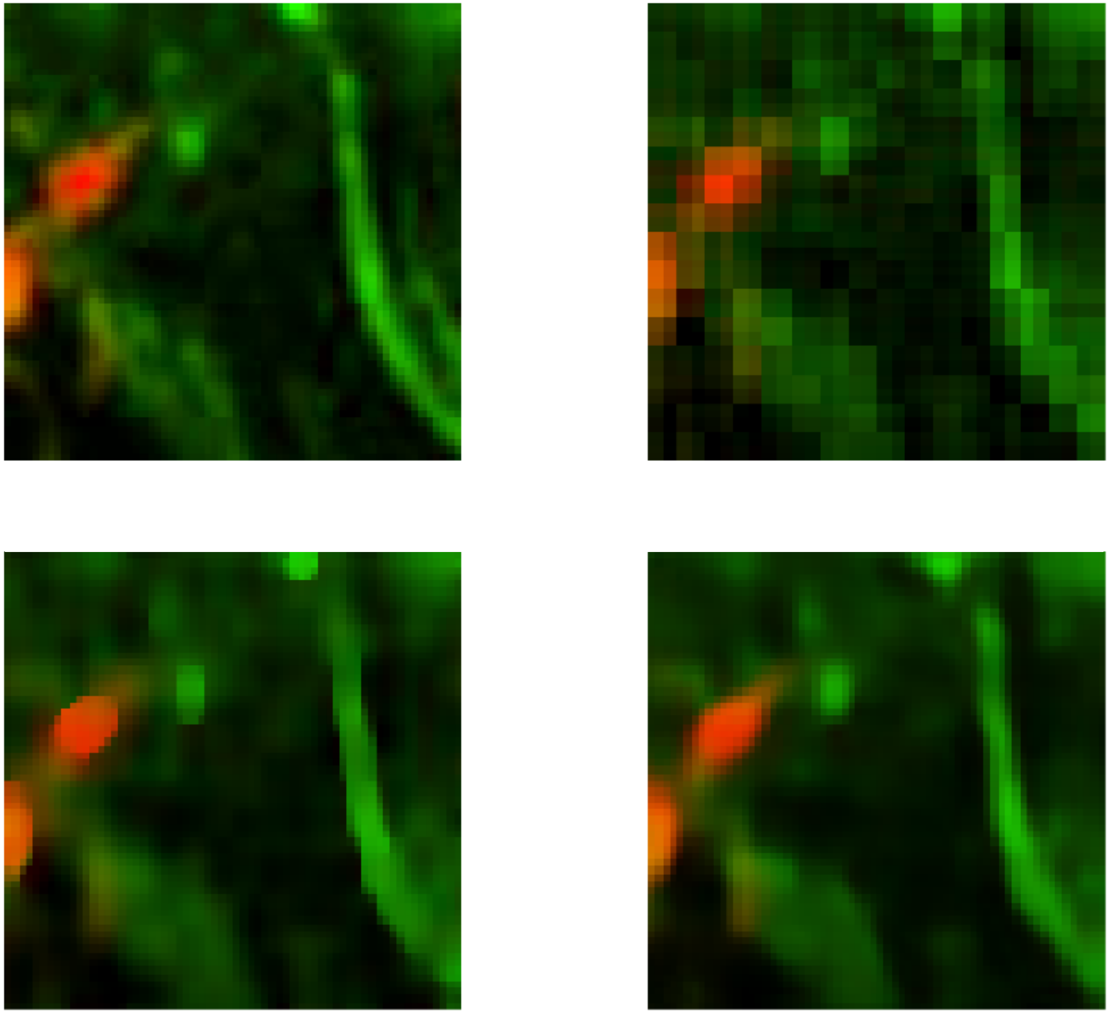
➢ Optimization- vs learning-based methods

N = 64 × 64 image
M = 333 measurements
N / M ≈ 8%

Ground-Truth

$\Psi(\boldsymbol{f}) = \|\boldsymbol{f}\|_2^2$

(pinv)



$\Psi(\boldsymbol{f}) = \|\nabla \boldsymbol{f}\|_1$

+2.5 dB (wrt pinv)

CNN

+3.5 dB (wrt pinv)

[N. Ducros et al., IEEE ISBI, 2019]

3. Learning-Based Methods

- **Our dream is to find**

$$\mathcal{R}^* : \mathbb{R}^M \mapsto \mathbb{R}^N \text{ such that } \mathcal{R}^*(\boldsymbol{m}) = \boldsymbol{f}^{\text{true}}$$

… able to reconstruct well any image, i.e., something like

$$\mathcal{R}^* \in \arg\min_{\mathcal{R}} \frac{1}{L} \sum_{\ell} \|\mathcal{R}(\boldsymbol{m}^\ell) - \boldsymbol{f}^\ell\|_2^2$$

Minimum mean square error (MMSE) estimator

… Often intractable

- **We have to reduce the dimension of the solution space**
  - E.g.,

$$\mathcal{R}(\boldsymbol{m}) = \boldsymbol{W}\boldsymbol{m} + \boldsymbol{b},$$

Linear MMSE estimator

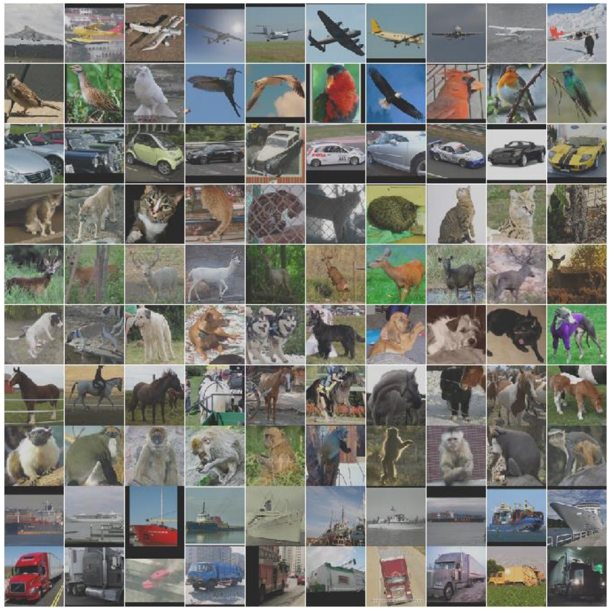
- **Linear MMSE**

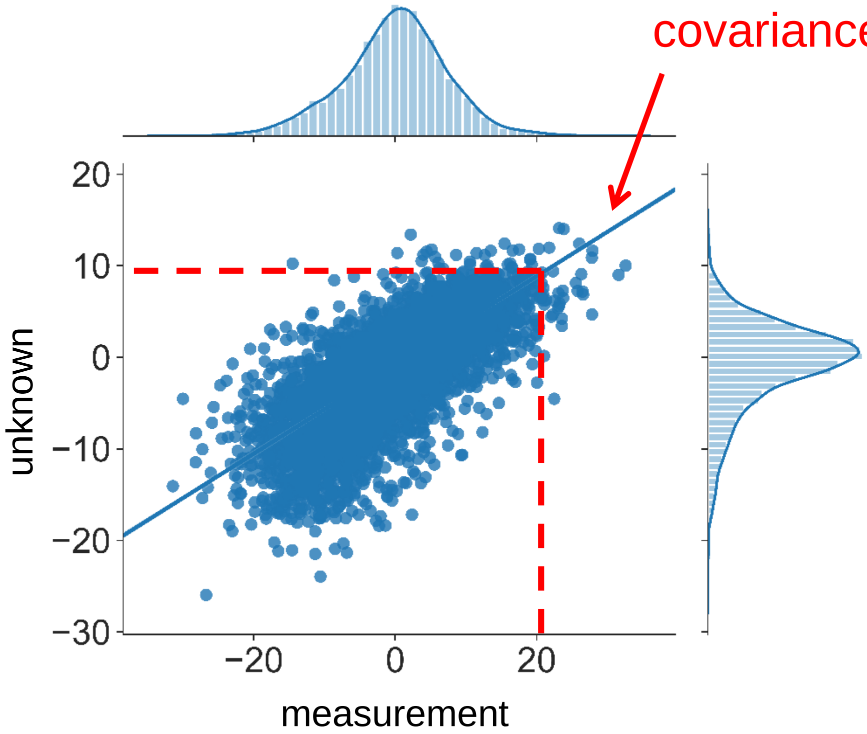
$$\boldsymbol{f}^* = \bar{\boldsymbol{f}} + \boldsymbol{\Sigma}_{f,m}\boldsymbol{\Sigma}_m^{-1}(\boldsymbol{m} - \bar{\boldsymbol{m}})$$

Covariance between measurements and unknowns

Covariance of measurements

slope → covariance





3. Learning-Based Methods

- **Learning approaches *only* reduce the dimension of the solution space to a family of *non linear* mappings**

$$\boldsymbol{\theta}^* \in \arg\min_{\boldsymbol{\theta}} \frac{1}{L} \sum_{\ell} \|\mathcal{R}(\boldsymbol{\theta}; \boldsymbol{m}^{\ell}) - \boldsymbol{f}^{\ell}\|_2^2$$

- Training phase
  - Image-measurement pairs $\{\boldsymbol{f}^{(\ell)}; \boldsymbol{m}^{(\ell)})\}_{1\leq\ell\leq L}$
  - Loss (e.g., MSE)
  - Optimization machinery (i.e., PyTorch or TensorFlow)

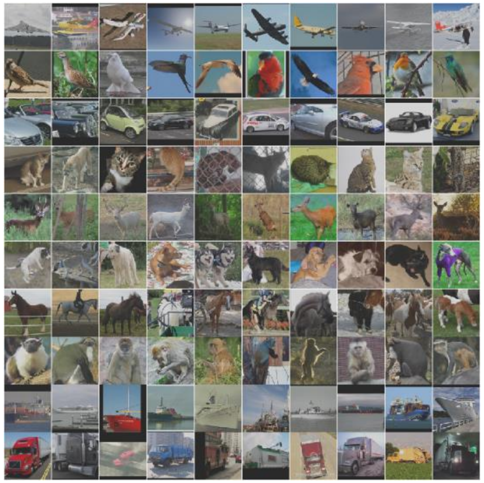
D.P. Kingma and J.L Ba, ICRL, 2015 (> 215k citations)

A. Paszke *et al.*, NEURIPS, 2019 (> 22k citations)

- Reconstruction phase

$$\boldsymbol{f}^* = \mathcal{R}_{\boldsymbol{\theta}^*}(\boldsymbol{m})$$

STL-10 dataset

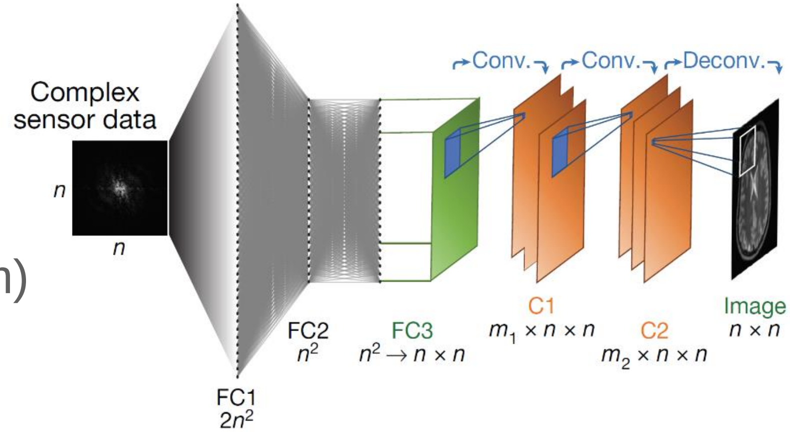


3. Learning-Based Methods

- **Pros**
  - Reconstruction performance
    - Empirically excellent (i.e., almost always outperform optimization-based approaches)
  - Computation times
    - Training phase is slow, i.e., several hours or days
    - Inference is fast, i.e., tens or hundreds of milliseconds

- **Cons**
  - No reconstruction guarantees (*mathematicians don't like it*)
  - Black box (*radiologists don't like it*)
  - Practical issue
    - How to choose the model?

"Automap"
(> 6.10^9 param)



[B. Zhu et al., Nature Letters, 2018] (> 1.5k citations)

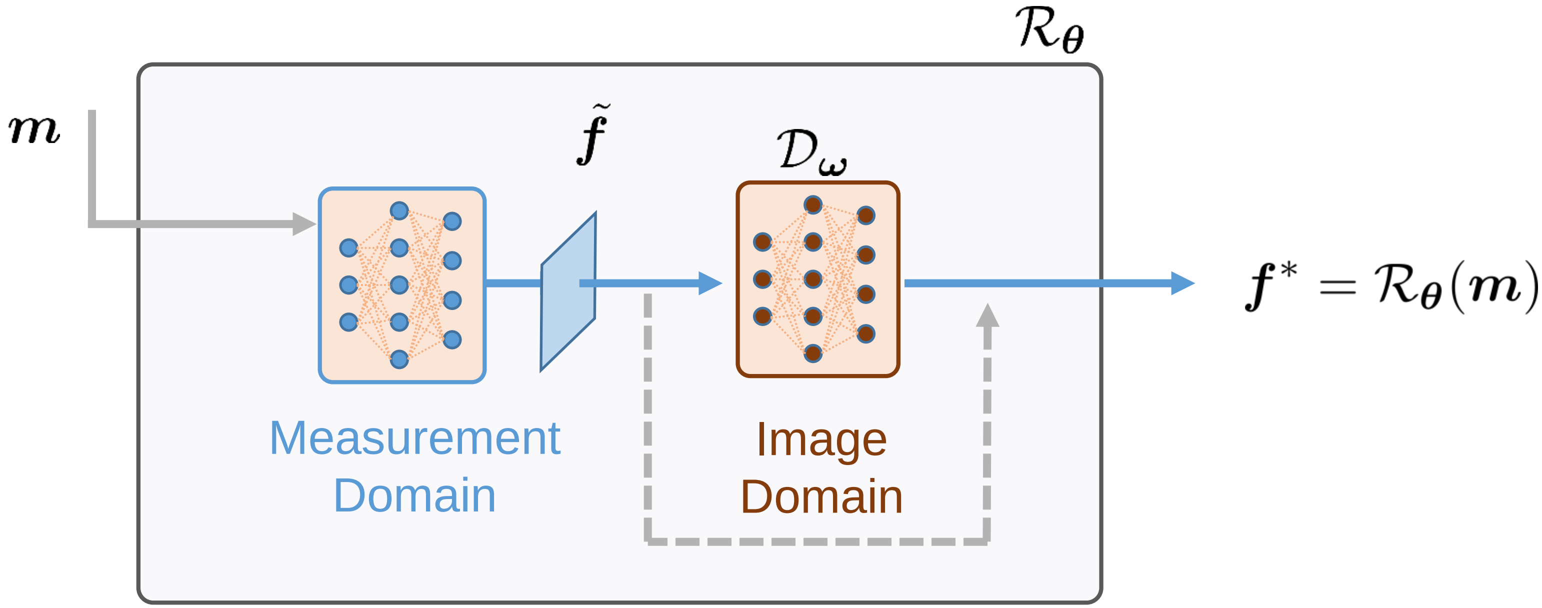
3.1. Post-Processing

- **Two-step methods**

$$\tilde{\boldsymbol{f}} = \tilde{\boldsymbol{A}}^{-1}\boldsymbol{m}$$

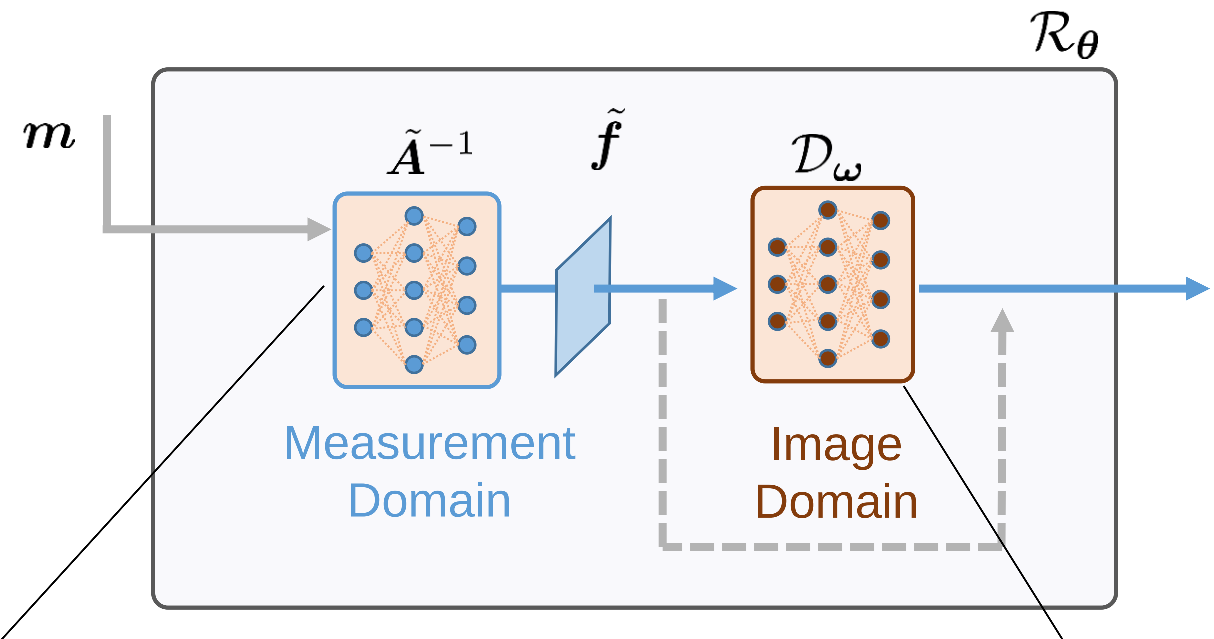
$$\boldsymbol{f}^* = \mathcal{D}_{\boldsymbol{\omega}}(\tilde{\boldsymbol{f}}) + \tilde{\boldsymbol{f}}$$

where $\tilde{\boldsymbol{A}^{-1}}$ is an approximate inverse of the forward, i.e., $\tilde{\boldsymbol{A}}^{-1}\boldsymbol{A}\boldsymbol{f} \approx \boldsymbol{f}$



3.1. Post-Processing

➢ **Equivalent to a neural network with a frozen layer**



network parameters

$$\boldsymbol{f}^* = \mathcal{R}_{\boldsymbol{\theta}}(\boldsymbol{m})$$

E.g., $\boldsymbol{\theta} = \boldsymbol{\omega}$.

```
Atilde = nn.Linear(..., bias=False, ...)
Atilde.weight.requires_grad = False
```

$$\tilde{\boldsymbol{A}}^{-1} = \boldsymbol{A}^\top$$

$$\tilde{\boldsymbol{A}}^{-1} = \boldsymbol{A}^\dagger \text{ (E.g., } \boldsymbol{A}^\top(\boldsymbol{A}\boldsymbol{A}^\top)^{-1} \text{ for full row rank } A)$$

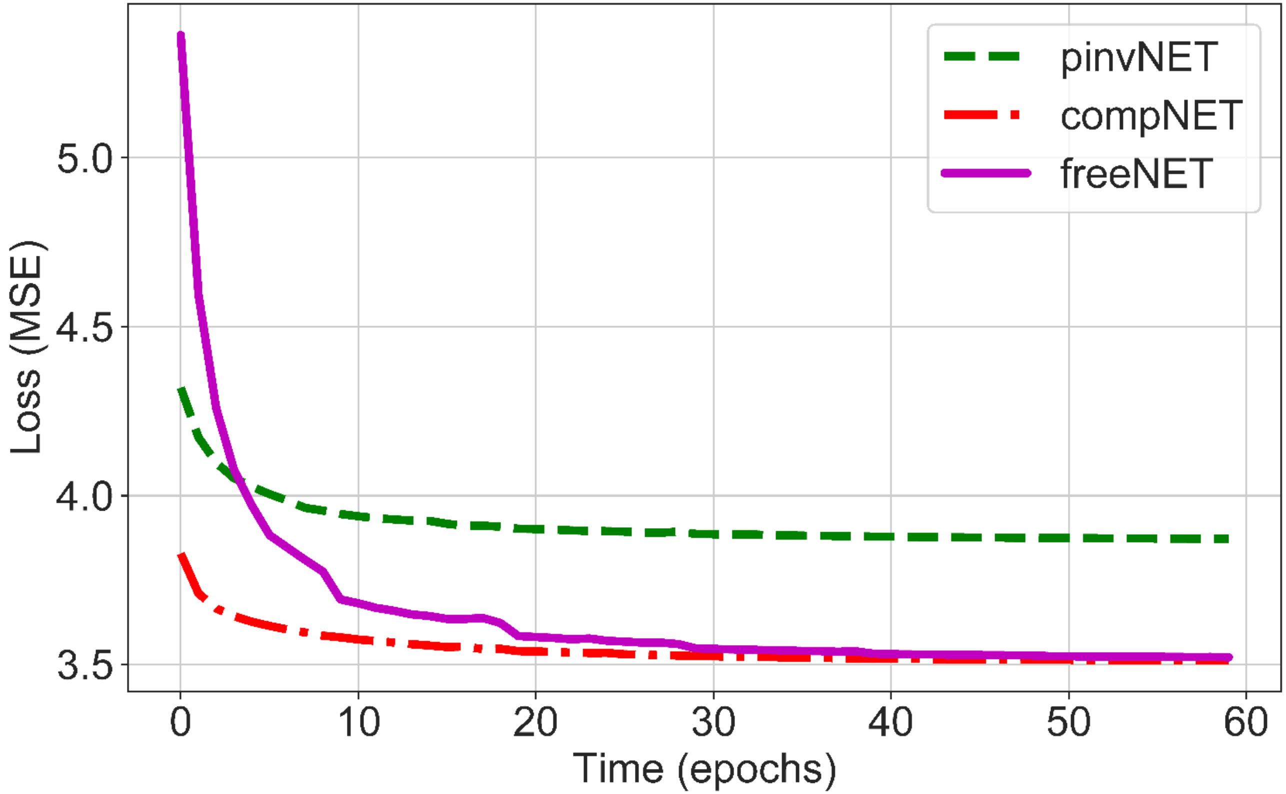
$$\tilde{\boldsymbol{A}}^{-1} = \boldsymbol{A}^\top(\boldsymbol{A}\boldsymbol{A}^\top + \boldsymbol{I})^{-1}$$

```
D = nn.Module(...)
requires_grad = True
```

3.3. Training

- **STL-10 (training: ~100k images; test: 8k images)**

$$\sum_{\ell \in \mathcal{I}_{\text{test}}} \|\boldsymbol{f}^{(\ell)} - \mathcal{R}_{\boldsymbol{\theta}}(\boldsymbol{m}^{(\ell)})\|^2$$



$$\tilde{\boldsymbol{A}}^{-1} = \boldsymbol{A}^{\dagger}$$
$$\tilde{\boldsymbol{A}}^{-1} = \boldsymbol{A}^{\top}(\boldsymbol{A}\boldsymbol{A}^{\top} + \boldsymbol{\Sigma})^{-1}$$
$$\tilde{\boldsymbol{A}}^{-1} = \texttt{Linear}$$

1.3M vs 8k trainable parameters

[N. Ducros et al., IEEE ISBI, 2019]

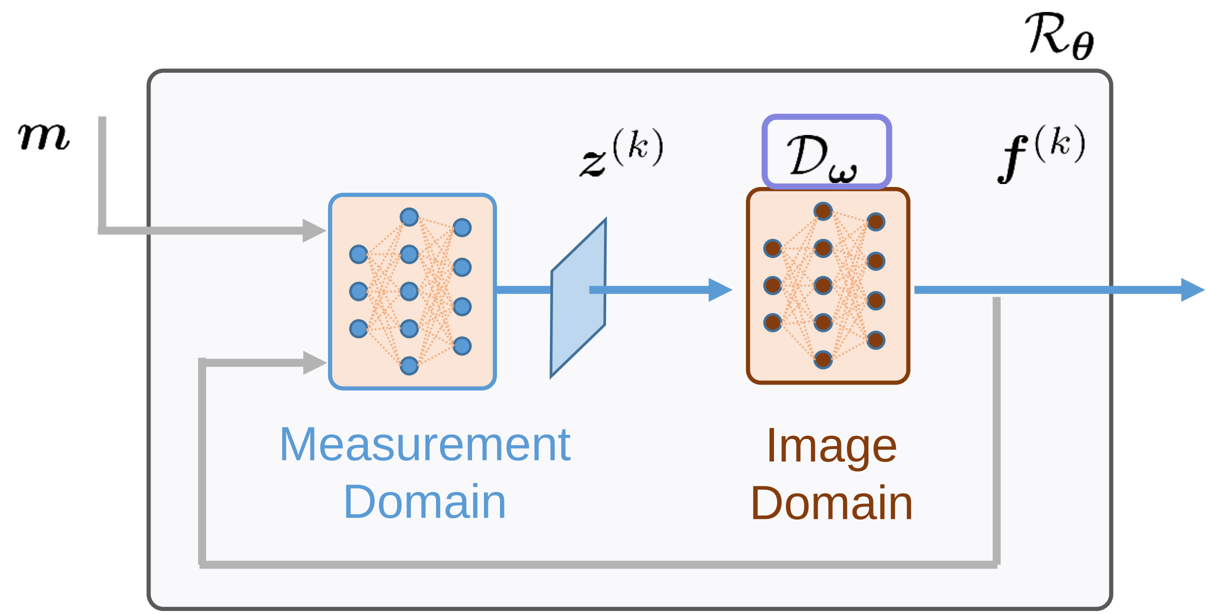
3.2. Unrolling (a.k.a. Unfolding)

➢ **Iterative methods**

$$\boldsymbol{z}^{(k)} = \boldsymbol{f}^{(k-1)} - \eta \boxed{\boldsymbol{A}^\top (\boldsymbol{A}\boldsymbol{f}^{(k-1)} - \boldsymbol{m})}$$
$$\boldsymbol{f}^{(k)} = \boxed{\texttt{prox}_{\lambda\Psi}}(\boldsymbol{z}^{(k)})$$

data fidelity

proximal operator



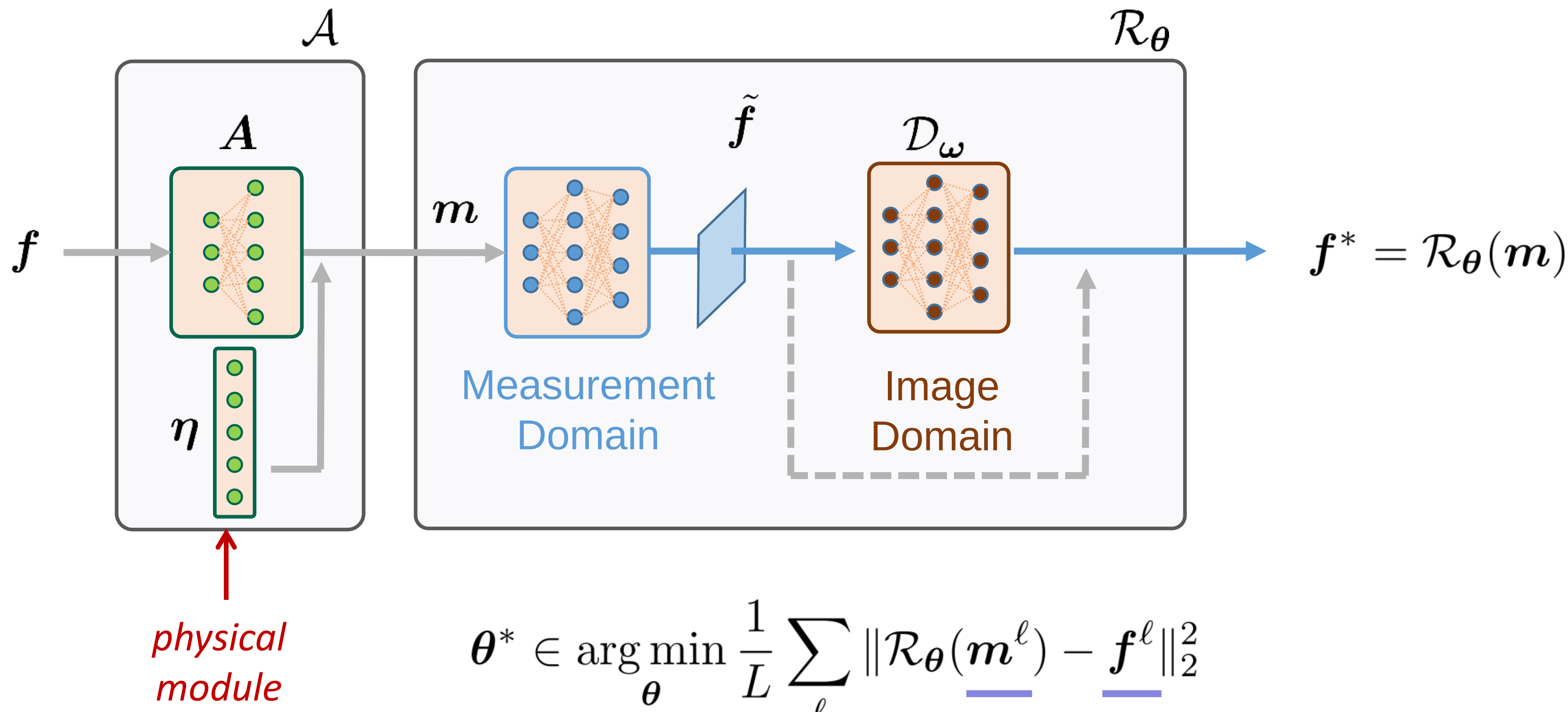
network parameters

$$\boldsymbol{f}^* = \mathcal{R}_{\boldsymbol{\theta}}(\boldsymbol{m})$$
$$= \boldsymbol{f}^{(K)}$$

Parameters can be <u>shared</u> across iterations <u>or not</u>

E.g., $\boldsymbol{\theta} = \boldsymbol{\omega}$,
or $\boldsymbol{\theta} = [\boldsymbol{\omega}^{(1)}, \ldots, \boldsymbol{\omega}^{(K)}]$.

- **With a "physical" module: no need for meas/image pairs**



$$\boldsymbol{\theta}^* \in \arg\min_{\boldsymbol{\theta}} \frac{1}{L} \sum_{\ell} \|\mathcal{R}_{\boldsymbol{\theta}}(\boldsymbol{m}^{\ell}) - \boldsymbol{f}^{\ell}\|_2^2$$

$$\mathcal{A}(\boldsymbol{f}) = \boldsymbol{A}\boldsymbol{f} + \boldsymbol{\eta}$$

$$\boldsymbol{\theta}^* \in \arg\min_{\boldsymbol{\theta}} \frac{1}{L} \sum_{\ell} \|(\mathcal{R}_{\boldsymbol{\theta}} \circ \mathcal{A})(\boldsymbol{f}^{\ell}) - \boldsymbol{f}^{\ell}\|_2^2$$

3.4. Plug-and-Play

- **Idea: use denoisers (e.g., BM3D) in place of proximal operators**

$$\boldsymbol{z}^{(k)} = \boldsymbol{f}^{(k-1)} - \eta \boldsymbol{A}^\top (\boldsymbol{A}\boldsymbol{f}^{(k-1)} - \boldsymbol{m})$$

$$\boldsymbol{f}^{(k)} = \underbrace{\boxed{\mathtt{prox}_{\lambda\Psi}}}_{\text{proximal operator}}(\boldsymbol{z}^{(k)}) \quad \Longrightarrow \quad \boldsymbol{f}^{(k)} = \underbrace{\boxed{\mathtt{Denoi}}}_{\text{Denoiser}}(\boldsymbol{z}^{(k)}; \sigma)$$

Noise level

- **The denoiser can be data-driven. E.g.** $\mathtt{Denoi} = \mathtt{CNN}_{\boldsymbol{\theta}^*}$ **with**

$$\boldsymbol{\theta}^* \in \arg\min_{\boldsymbol{\theta}} \frac{1}{L} \sum_{\ell} \|\mathtt{CNN}_{\boldsymbol{\theta}}(\boldsymbol{f}^\ell + \underline{\sigma\boldsymbol{\epsilon}}; \sigma) - \boldsymbol{f}^\ell\|_2^2$$

Gaussian noise with variance σ^2 $\quad \boldsymbol{\epsilon} \sim \mathcal{N}(0,1)$

3.4. Plug-and-Play

- **Idea: use denoisers in place of proximal operators**

$$
\begin{aligned}
\boldsymbol{z}^{(k)} &= \boldsymbol{f}^{(k-1)} - \eta \boldsymbol{A}^{\top}(\boldsymbol{A}\boldsymbol{f}^{(k-1)} - \boldsymbol{m}) \\
\boldsymbol{f}^{(k)} &= \boxed{\texttt{Denoi}}(\boldsymbol{z}^{(k)}; \sigma)
\end{aligned}
$$

- **Pros**
  - Training is independent of the forward model
    - Flexibility
    - Applies to any inverse problem
  - Adapt to varying noise levels via hyperparameter
  - Convergence can be guaranteed

- **Cons**
  - Manual tuning of hyperparameter
  - Many iterations required compared to supervised methods (E.g., K = 100—1,000 vs K = 1—10)
    - Longer reconstruction times
    - Higher memory requirement
  - Convergence is not always guaranteed (e.g., Lipschitz constant of denoiser)
  - Underlying prior not always known

3.5. Untrained/unsupervised

- **Deep Generative Models**

$$f = \mathcal{R}_{\theta}(z)$$

Random vector

$$z^* \leftarrow \min_{z} \|A\mathcal{R}_{\theta}(z) - m\|_2^2$$

$$f^* = \mathcal{R}_{\theta}(z^*)$$

- **Pros**
  - Only requires measurements from a single acquisition
  - Theoretical guarantees (based on compressed sensing, e.g., considering Gaussian random matrices)
- **Cons**
  - Long and challenging reconstruction
  - Training of DGM is challenging (lots of data/long times)
  - Stability issues (arbitrary forward models, out-of-distribution images, etc.)

3.5. Untrained/unsupervised

- **Deep Image Priors**

$$f = \mathcal{R}_{\theta}(z)$$

Fixed random vector

$$\theta^* \leftarrow \min_{\theta} \|A\mathcal{R}_{\theta}(z) - m\|_2^2$$
$$f^* = \mathcal{R}_{\theta^*}(z)$$

Note: Minimization must be stopped before convergence (tends to noise otherwise)

- **Pros**
  - Only requires measurements from a single acquisition
  - The reconstruction quality is surprisingly good

- **Cons**
  - Long reconstruction times
  - No guarantees

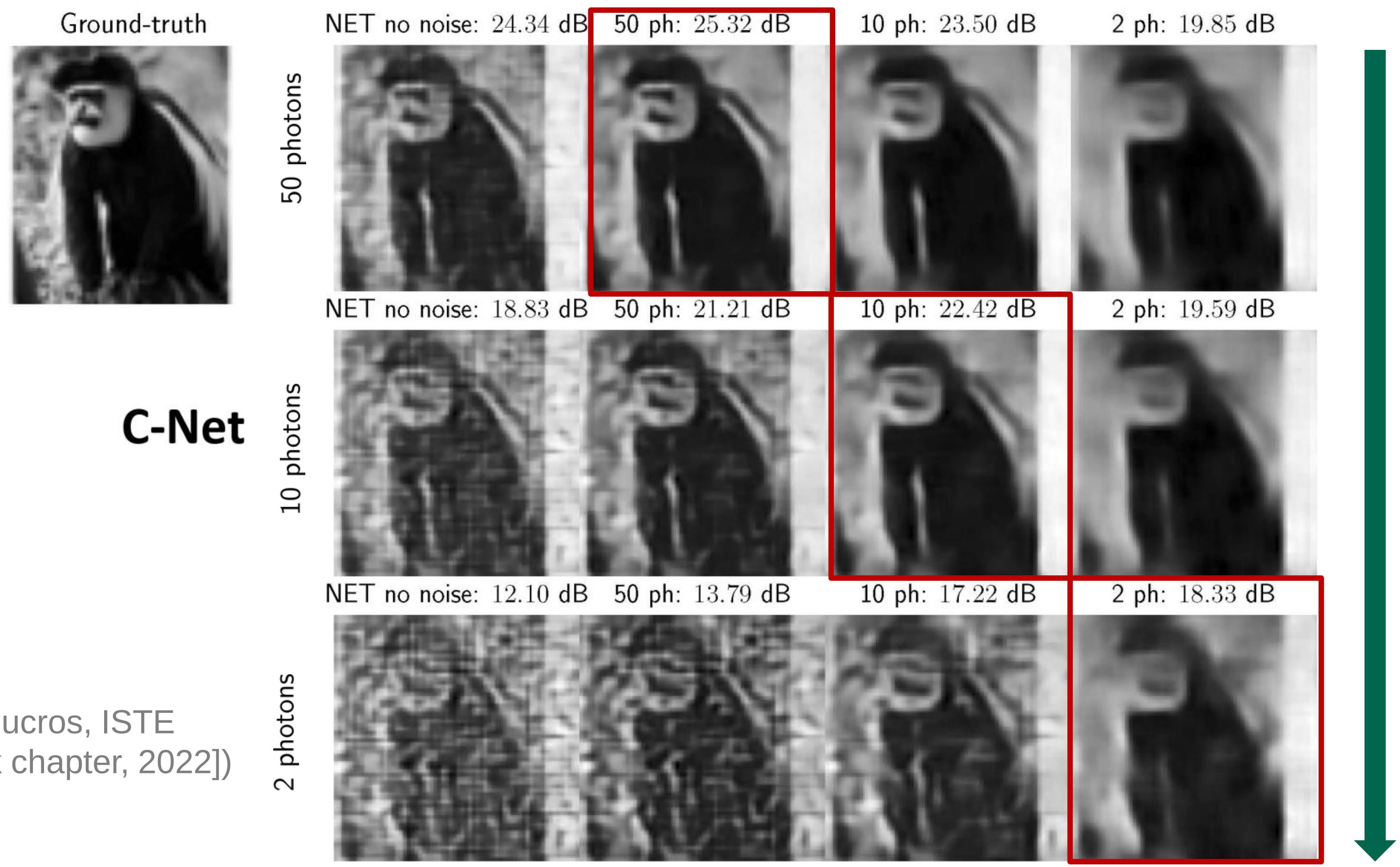
Conclusions

Benchmark

	Memory Requirement	Recon-struction Time (inference)	Training	Hyperparam/ Comment
Supervised	Low to intermediate	1—10	$\boldsymbol{A} \quad \{\boldsymbol{f}^\ell\}$	No adaptation (forward, noise)
PnP	Intermediate to high	100—1,000	$\{\boldsymbol{f}^\ell\}$	Noise level
Untrained	Usually low	> 1,000	—	Number of iterations

Noise robustness

Increasing training noise



Increasing test noise

[N. Ducros, ISTE Book chapter, 2022])

Conclusions

- **Data-driven DL-based approaches for image reconstruction are**
  - Powerful!
  - No longer black boxes
  - Supervised, PnP, based on generative models, untrained, etc.

- **Supervised vs PnP methods**
  - Supervised methods usually require fewer parameters
  - Supervised methods performs very well
  - PnP methods adapts to different
    - Imaging modalities (i.e., forward models)
    - Noise levels

- **Warning**
  - Handling noise is still an issue.
    - Evaluate the robustness to noise level deviations
    - Train with noise (supervised)
    - Tune hyperparameters (PnP)

→ Hands-on session on Friday at 2 pm! ←