# Freeform Hadamard imaging

## (Ou pourquoi se passer d'images carrées ?)

Nicolas Ducros[1, 2]

[1]CREATIS, Univ Lyon, INSA-Lyon, UCB Lyon 1,CNRS, Inserm, CREATIS UMR 5220, U1206, Lyon, France

[2]IUF, Institut Universitaire de France

*Joint work with J Cohen and L Mahieu-Williame*

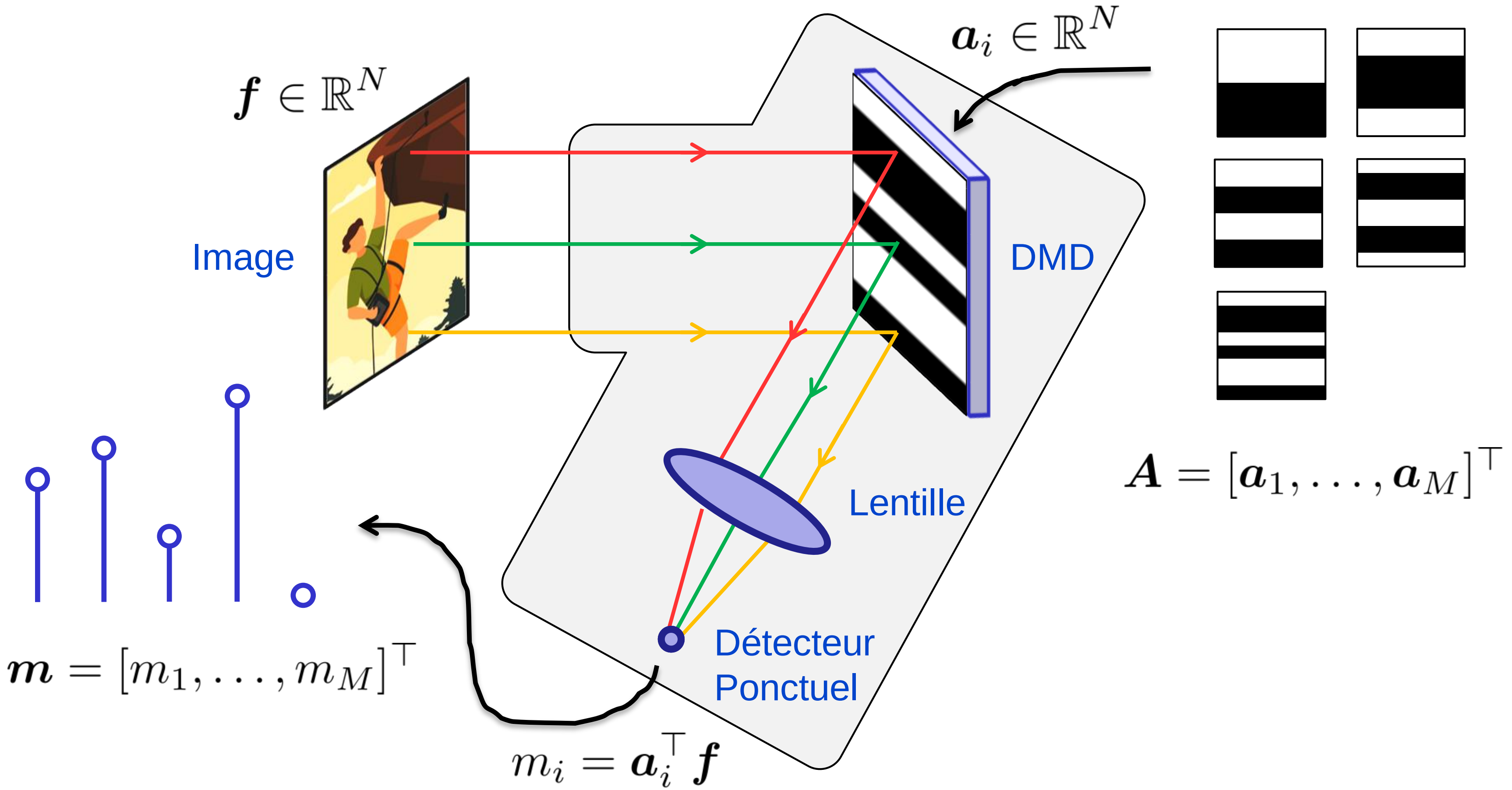
# Single-pixel imaging (Duarte *et al.*, 2008)




Original (256x256) | 1300 mes., x50 | 6500 mes, x10

[R. Duarte *et al.*, IEEE SPM 25, 2008] > 4.6k citations (Scholar)

# Single-pixel imaging (Duarte *et al.*, 2008)




**Acquisition**

$$\boldsymbol{m} = \boldsymbol{A}\boldsymbol{f}$$

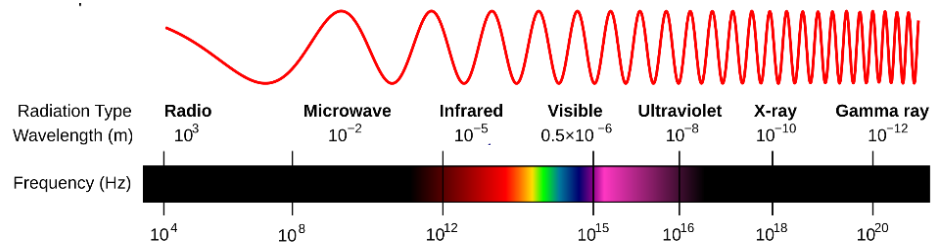
$\boldsymbol{A} \in \mathbb{R}^{M \times N}$

**Reconstruction (*M* < *N*)**

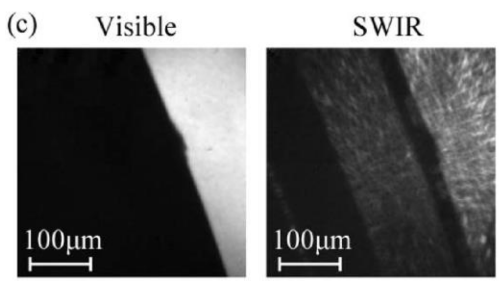
$$\boldsymbol{f}^* \leftarrow \min_{\boldsymbol{f}} \|\boldsymbol{m} - \boldsymbol{A}\boldsymbol{f}\|_2^2 + \lambda \|\boldsymbol{\Phi}\boldsymbol{f}\|_1$$

# Applications



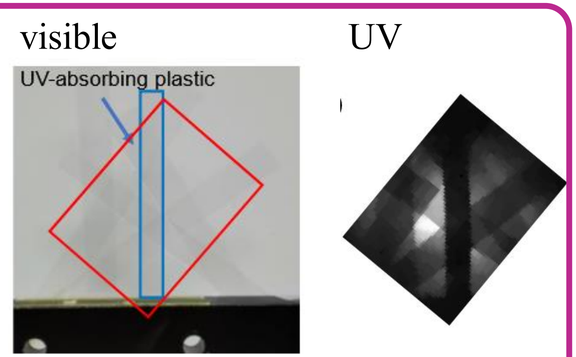



## Infrared



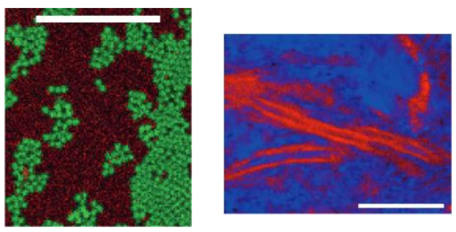

[N. Radwell *et al.*, Optica 1(5), 2014]

## Ultraviolet




[J. T. Ye *et al.*, Appl. Phys. Lett. 123, 2023]

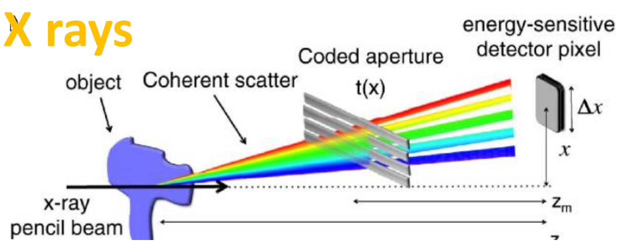
## Raman imaging



[F Soldevilla *et al.*, Optica 6(3), 2019]

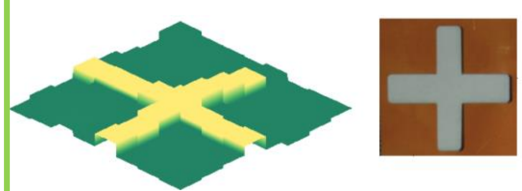
[Scotte *et al.*, Jphys Photonics 5(3), 2023]

## X rays




[J. Greenberg *et al.*, Optics letters 39, 2014]

## Terahertz imaging



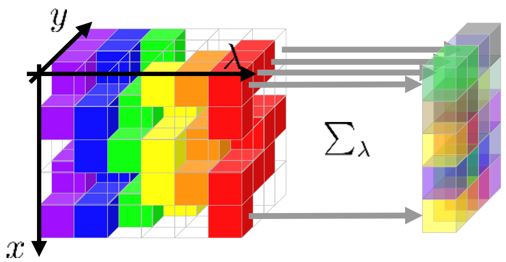
[C. Watts *et al.*, Nature Photonics 8(8), 2014]

## Hyperspectral imaging

[E Hemsley *et al.*, JOSAA 37 (12), 2020]

# Applications (in Lyon)

## Hyperspectral

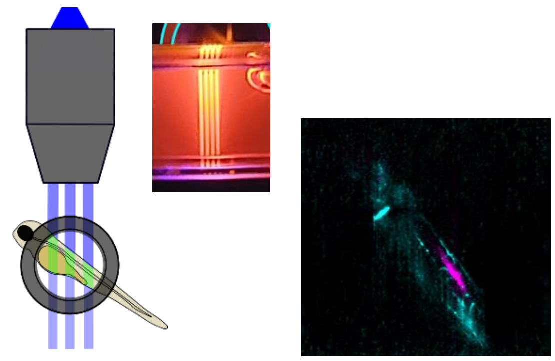



200+ hypercubes in open access

https://pilot-warehouse.creatis.insa-lyon.fr/

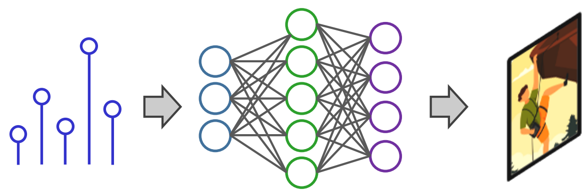
[G Beneti Martins *et al.*, Opt. Express, 2023]

## Fluorescence light sheet



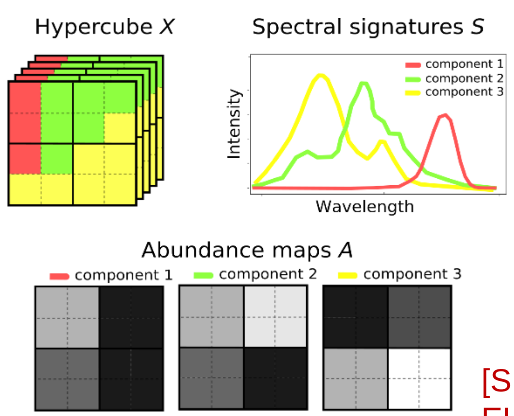
[S Crombez *et al.*, Opt. Express, 2025]

## DL-based reconstruction



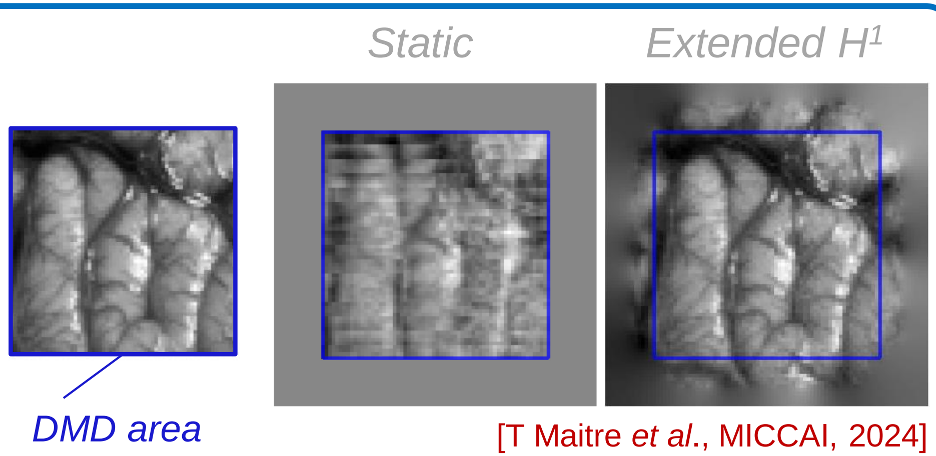
[JFJ. Abascal *et al.*, Opt. Express, 2025]

## Spectral unmixing




[S Hariga *et al.*, EUSIPCO, 2024]

## Motion-compensated imaging




[T Maitre *et al.*, MICCAI, 2024]

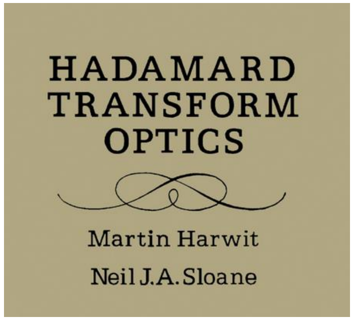
# Should I go for it?



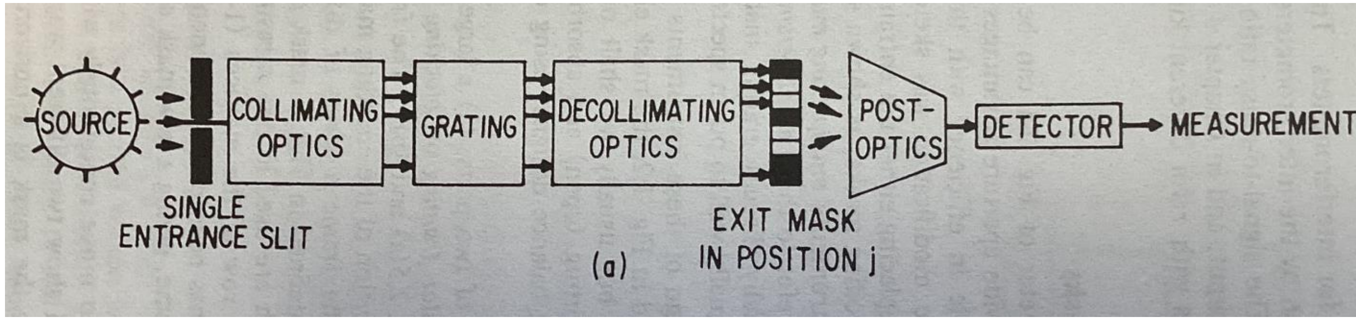
- Hardware complexity
- Need for reconstruction algorithms
- Time-consuming reconstruction

- Acquisition is fast (only few measurements)
- This is it?

# Hadamard spectroscopy (from the 70's!)



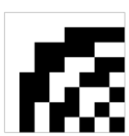

[M. Harwit and N Sloane, Academic Press, 1979]




*'(...) conventional spectrometer is modified by using a mask to encode the light at the output'*

## Hadamard

$$\boldsymbol{A} = \boldsymbol{H} \in \{-1, 1\}^{N \times N}$$



$$\boldsymbol{H}^\top \boldsymbol{H} = N \boldsymbol{I}_N$$

## L2 reconstruction

$$\boldsymbol{f}^* = \frac{1}{N} \boldsymbol{H}^\top \boldsymbol{m}$$

## Fellgett's advantage

$$\mathrm{var}\left(f_n^*\right) = \frac{1}{N}\sigma^2 < \sigma^2$$

**Improved SNR w.r.t. raster scan**

**Not observed in practice**

# Fellgett’s advantage, revisited

# Practical models



## Poisson-Gaussian noise

$$\boxed{\mathbf{m} \sim \gamma\,\mathcal{P}(\boldsymbol{A}\boldsymbol{f}\Delta t) + \mathcal{N}(\mu_\mathrm{d}, \sigma_\mathrm{d}^2),}$$

[EMVA standard, 2021]

gain

Integration time

dark current

dark noise

## Hadamard-based nonnegative acquisition matrix

Given

$$\boldsymbol{A} \in \{-1, 1\}^{N\times N}$$

Main options are:

$$\boldsymbol{A} = \boldsymbol{H}_+ \in \{0, 1\}^{N\times N}$$

$$\boldsymbol{A} = \boldsymbol{H}_- \in \{0, 1\}^{N\times N}$$

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{H}_+ \\ \boldsymbol{H}_- \end{bmatrix} \in \{0, 1\}^{2N\times N}$$

## L2 reconstructions

$$\boxed{\mathbf{f}^* = \tilde{\boldsymbol{A}}^\dagger \tilde{\mathbf{m}}}$$

$$\mathbb{E}(\tilde{\mathbf{m}}) = \tilde{\boldsymbol{A}}\boldsymbol{f}$$

preprocessed measurements

virtual matrix

**Affine:**
$$\begin{cases} \tilde{\mathbf{m}} = \frac{1}{\gamma\Delta t}(\mathbf{m} - \mu_\mathrm{d}) \\ \tilde{\boldsymbol{A}} = \boldsymbol{A} \end{cases}$$

**Differential:**
$$\begin{cases} \tilde{\mathbf{m}} = \frac{1}{\gamma\Delta t}(\mathbf{m}_+ - \mathbf{m}_-) \\ \tilde{\boldsymbol{A}} = \boldsymbol{H} \end{cases}$$

# Time-bound mean squared error



(only depends on the acquisition matrix)

**Reconstruction MSE**

(The lower, the better)

$$\boxed{\mathbb{M}\mathrm{se}\,(\mathbf{f}^*) \approx \frac{1}{t}[\alpha N \bar{f} + \beta N f_{\mathrm{ref}}]}$$

**reference flux**
(only depends on experimental parameters)

$$f_{\mathrm{ref}} = \frac{\sigma_{\mathrm{d}}^2}{\gamma^2 t}$$

dark noise

gain

**Fixed time budget**

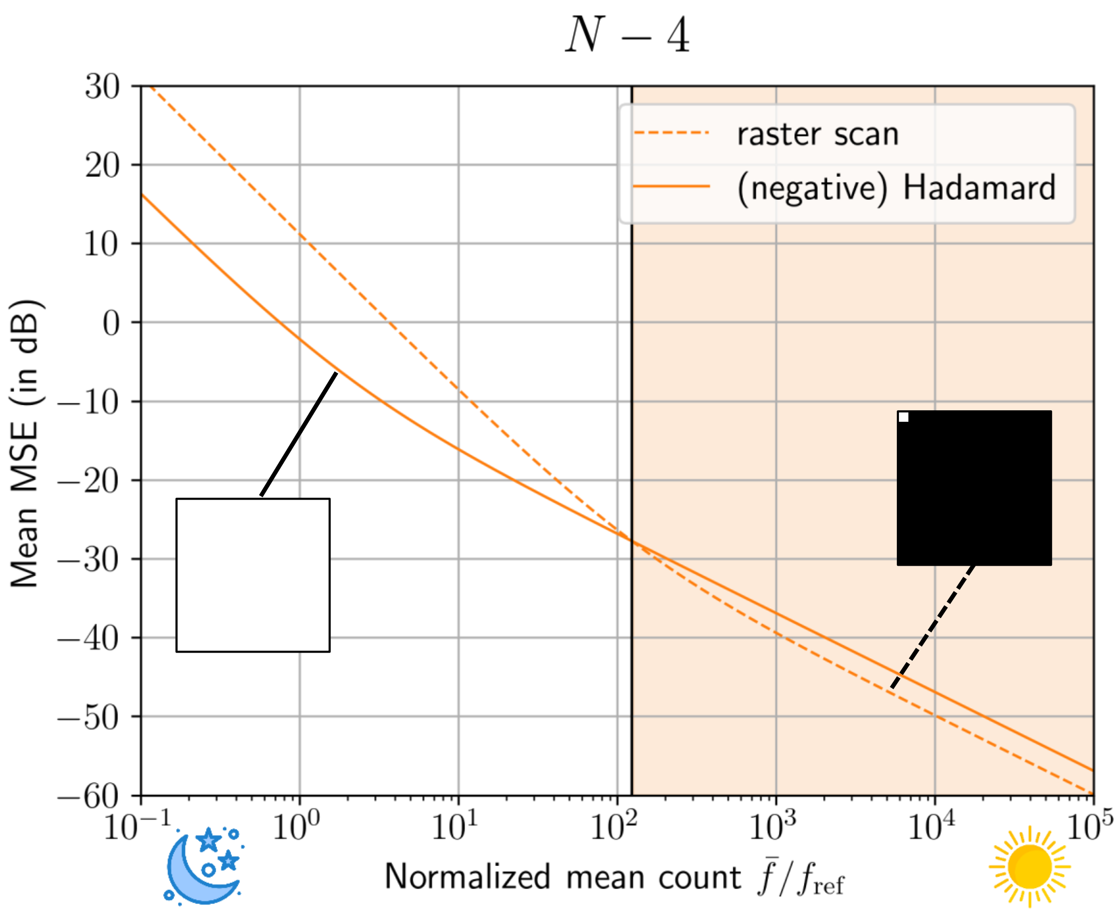
$$\boxed{t = N\Delta t}$$

**image mean flux**
(only depends on the scene)

- For **negative Hadamard** (affine preprocessing):
  $\alpha = 2 \qquad \beta = 4$
- For a **raster scan** (affine preprocessing):
  $\alpha = 1 \qquad \beta = N$

# Fellgett’s advantage depends on light flux






→ **Hadamard** multiplexing more effective in **low-light** conditions.

# Freeform Imaging

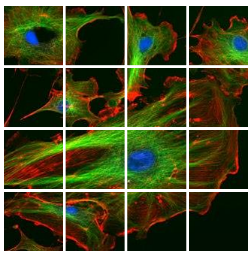
# Motivation and definition



## Motivation

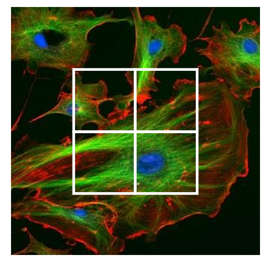
$$\boxed{\mathbb{M}\mathrm{se}\,(\mathbf{f}^*) \searrow \text{ when } N \searrow}$$

<u>Recall:</u> $\mathbb{M}\mathrm{se}\,(\mathbf{f}^*) \approx \frac{1}{t}\left[2N\bar{f} + 4N f_{\mathrm{ref}}\right]$

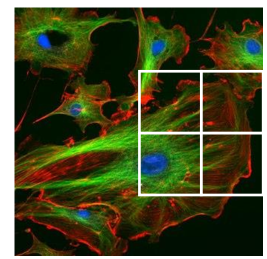


4x4 image

**Lowering *N* improves the MSE**

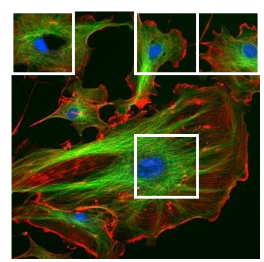


2x2 image



2x2 image

**Why not?**



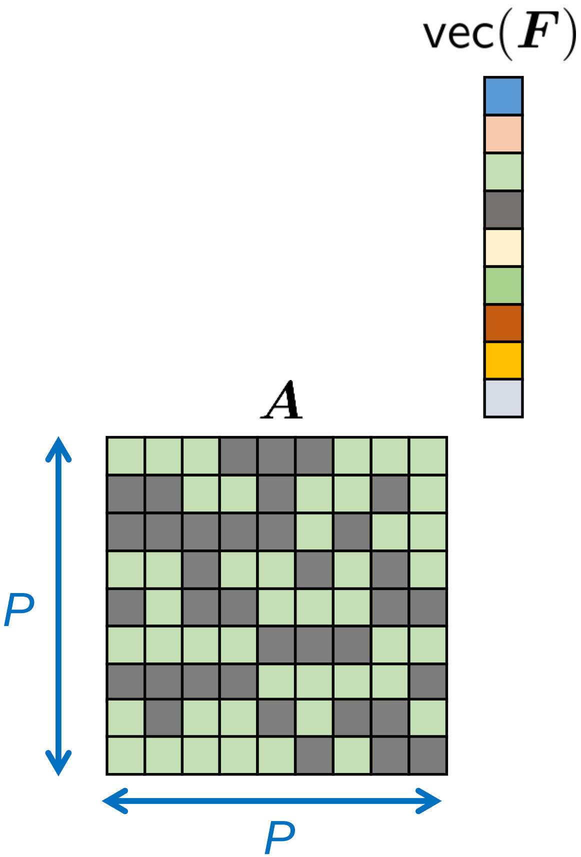
4-pixel image

**→ Freeform imaging!**

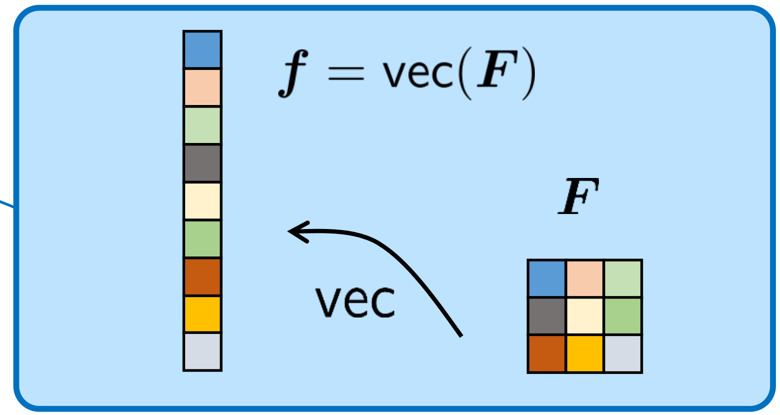
https://commons.wikimedia.org/wiki/File:FluorescentCells.jpg

# Full acquisition matrices ("Full 2D")

$$\boxed{\boldsymbol{m} = \boldsymbol{A}\boldsymbol{f}}$$




**Full ($P$ pixels)**




Here: $P$ = 9 pixels

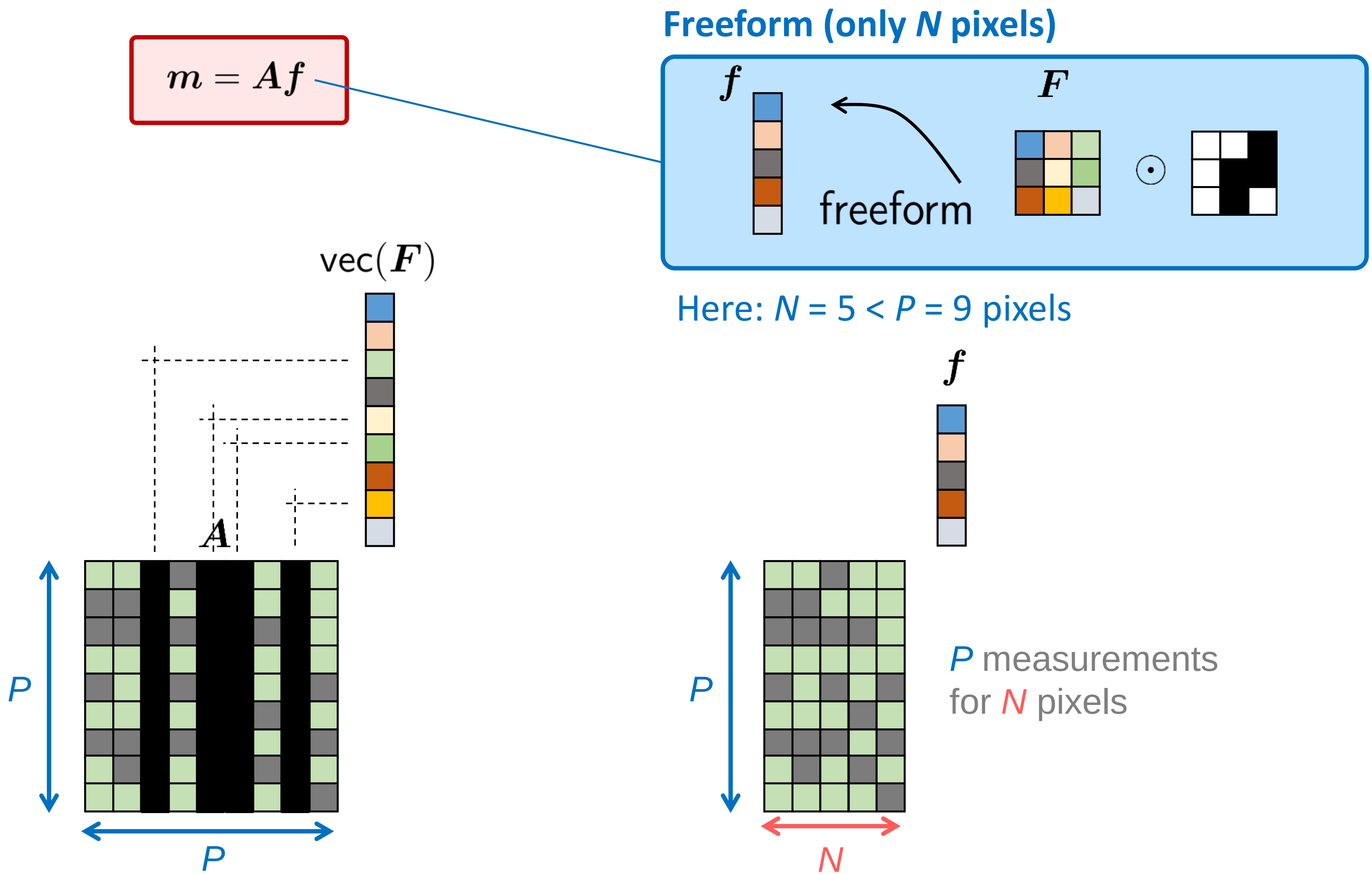
**How can freeform imaging be achieved?**

→ **Masked 2D acquisition matrices**

→ **Non imaging 1D matrices**

# Masked acquisition matrices (“masked 2D”)

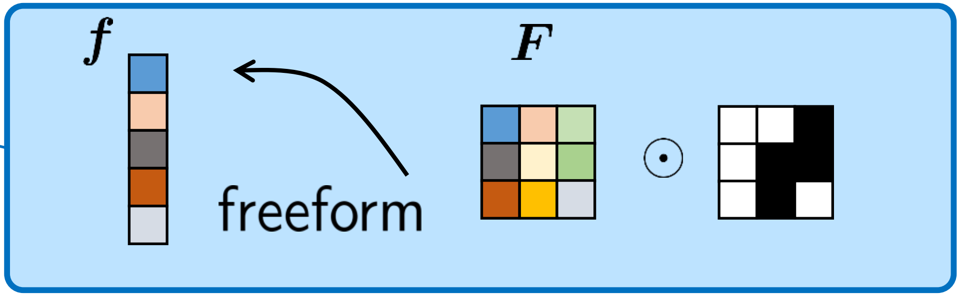




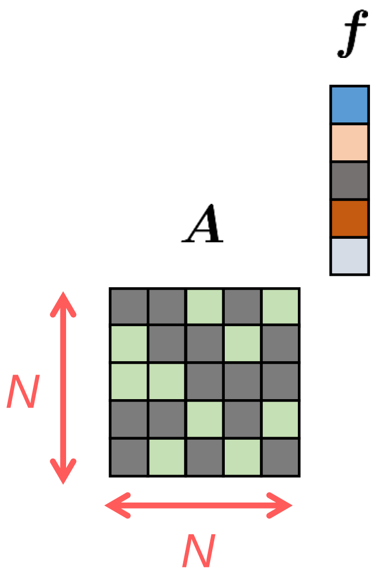
# Non-imaging acquisition matrices (“1D”)

$$\boxed{\boldsymbol{m} = \boldsymbol{A}\boldsymbol{f}}$$

## Freeform (only *N* pixels)




Here: $N = 5 < P = 9$ pixels



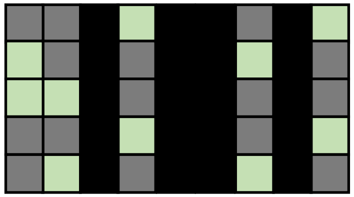

$$\boldsymbol{A} = \boldsymbol{H}_+ \in \mathbb{R}^{N\times N}$$

NB: No underlying 2D transform**

**Only** ***N*** measurements for *N* pixels

→ Fewer measurements than with masking (as in previous slide)

**The actual matrix loaded onto the DMD is



# Theory, updated

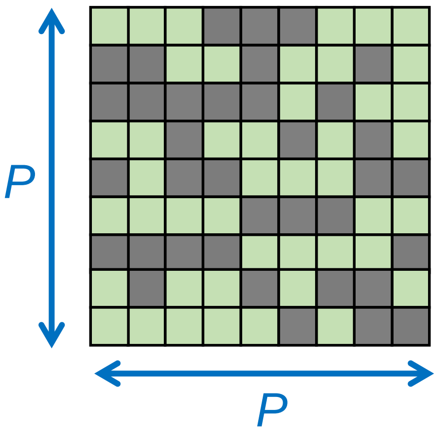


#px in the freeform region

#measurements

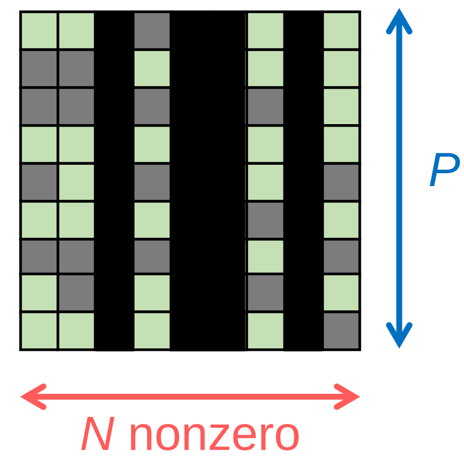
$$\boxed{\mathbb{M}\mathsf{se}\left(\mathbf{f}^*\right) \approx \frac{1}{t}[2Q\bar{f} + 4K f_{\mathrm{ref}}], \quad K \geq Q}$$

## Full 2D



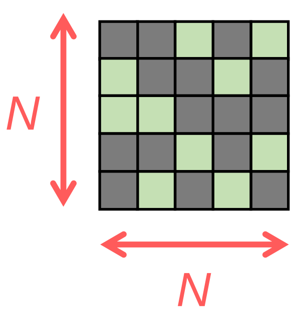

$$\mathbb{M}\mathsf{se}\left(\mathbf{f}^*\right) \approx \frac{1}{t}[2P\bar{f} + 4P f_{\mathrm{ref}}],$$

## Masked 2D



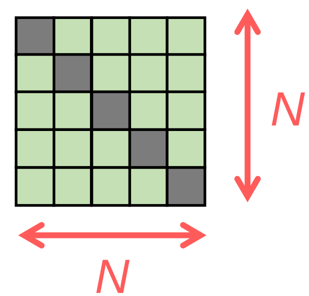

$$\mathbb{M}\mathsf{se}\left(\mathbf{f}^*\right) \approx \frac{1}{t}[2N\bar{f} + 4P f_{\mathrm{ref}}],$$

## 1D




$$\mathbb{M}\mathsf{se}\left(\mathbf{f}^*\right) \approx \frac{1}{t}[2N\bar{f} + 4N f_{\mathrm{ref}}],$$

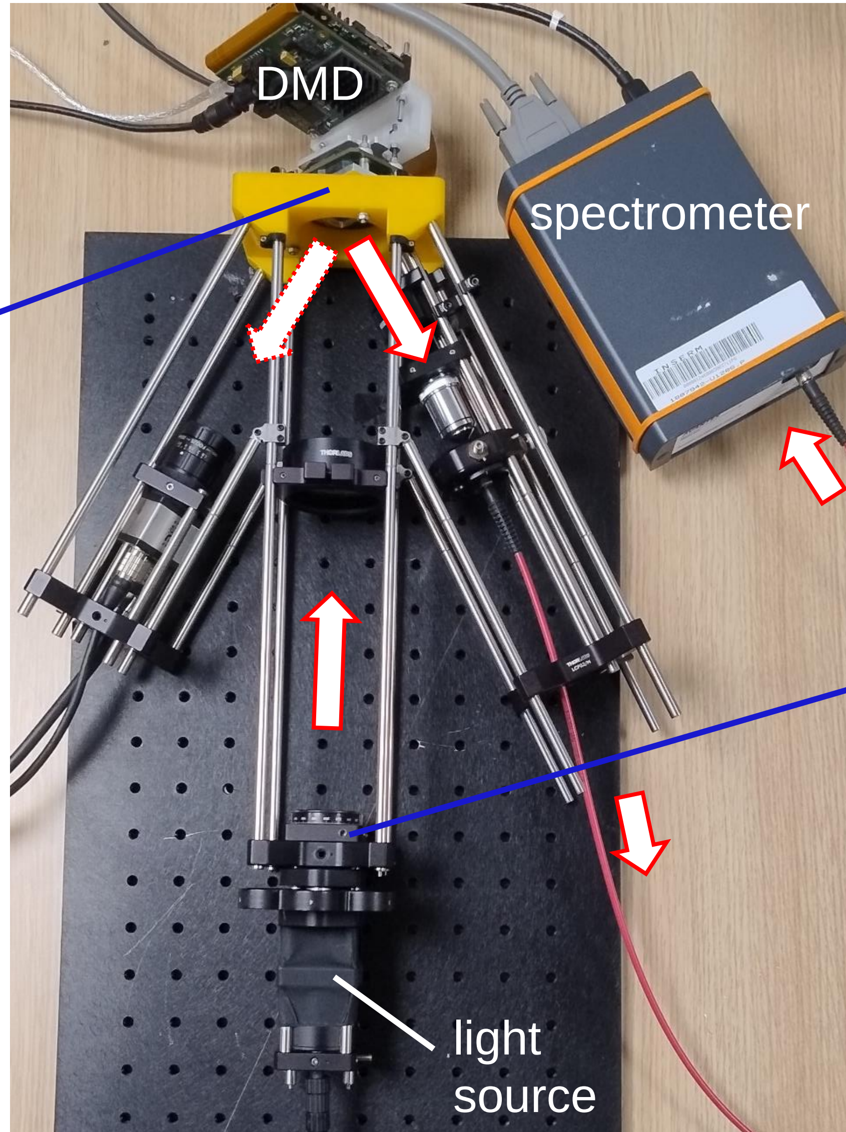
## Raster Scan



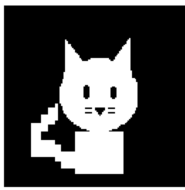

$$\mathbb{M}\mathsf{se}\left(\mathbf{f}^*\right) \approx \frac{1}{t}[N\bar{f} + N^2 f_{\mathrm{ref}}],$$

# Experimental demonstration (1/4)






Freeform mask



total #px

$$\frac{P}{N} = 4$$

#px in the freeform region

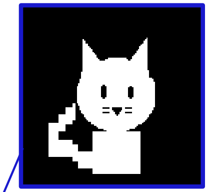
Siemens star resolution target

# Experimental demonstration (2/4)



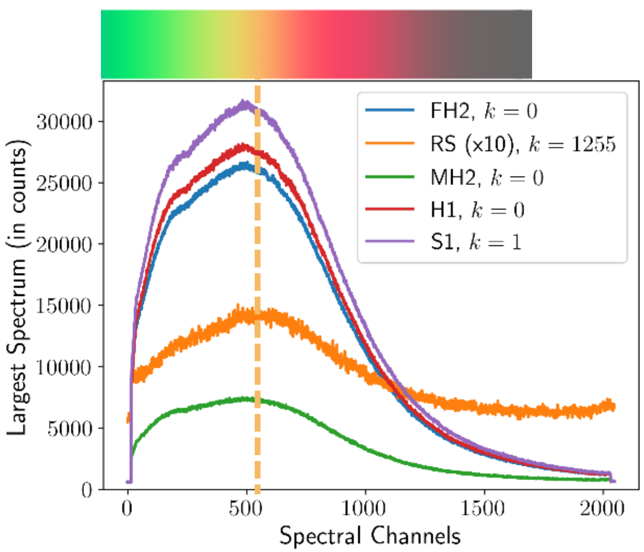
Siemens star resolution target


Freeform region

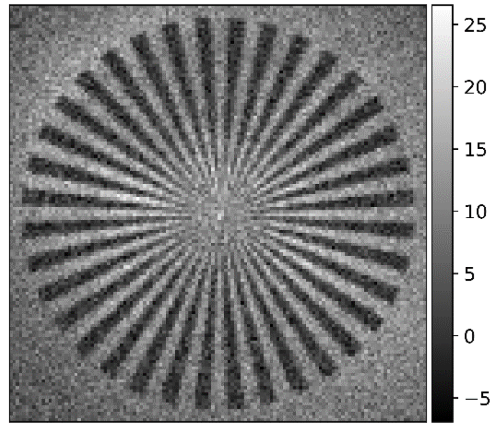


*DMD area*

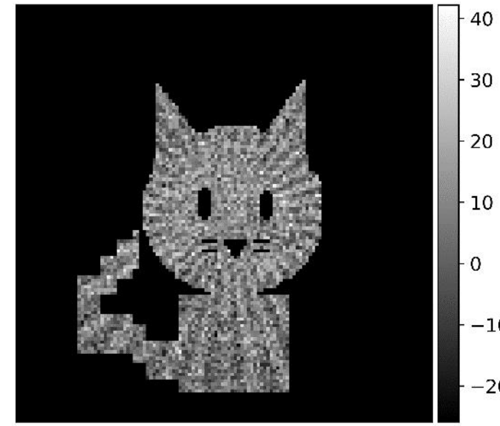
**581 nm**



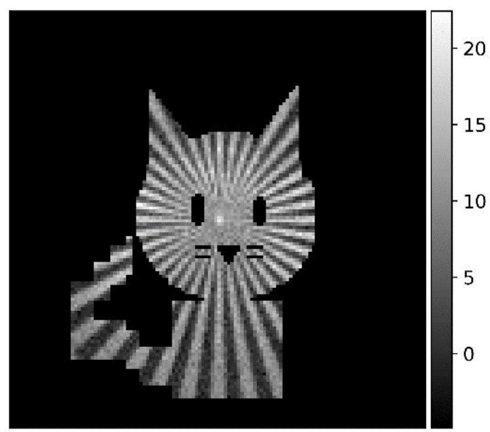

**Full 2D**



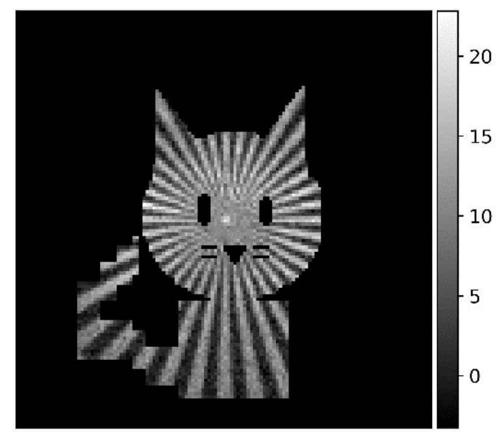

**Raster Scan**




**Masked 2D**




**1D**

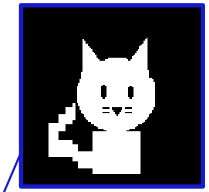


# Experimental demonstration (3/4)



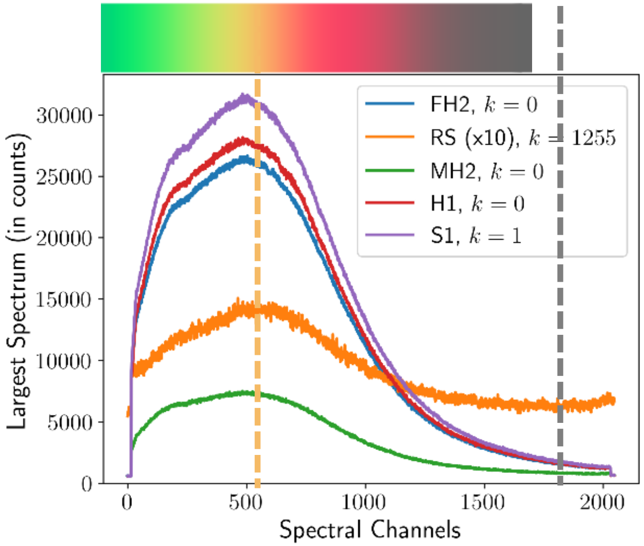
Siemens star resolution target


Freeform region

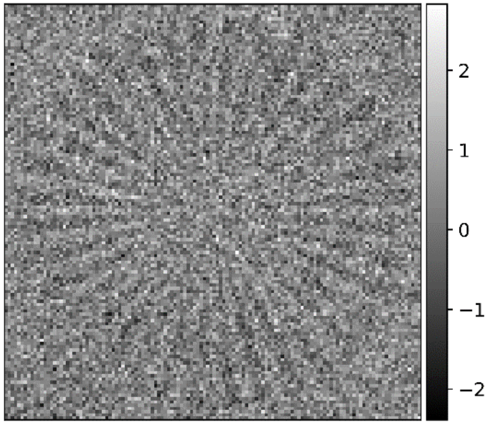


*DMD area*

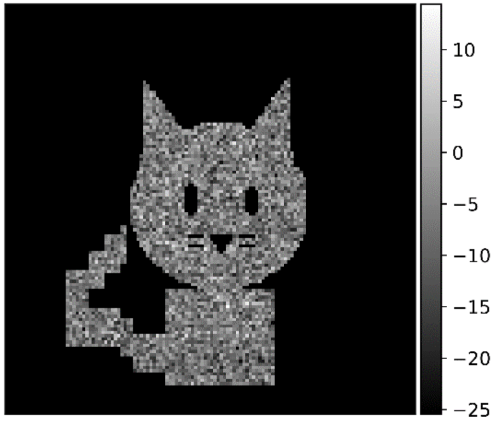
726 nm



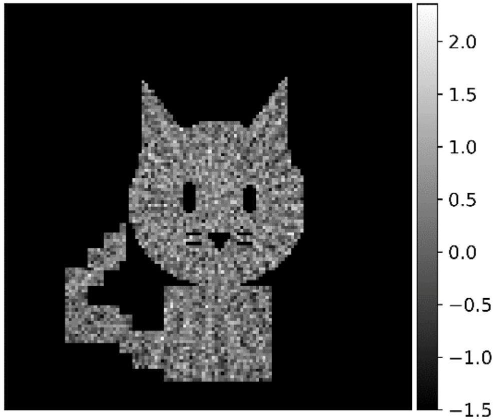

**Full 2D**



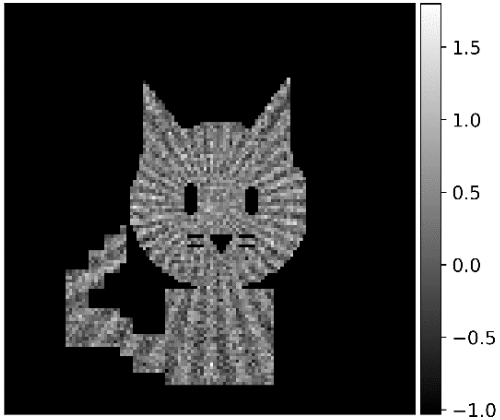

**Raster Scan**



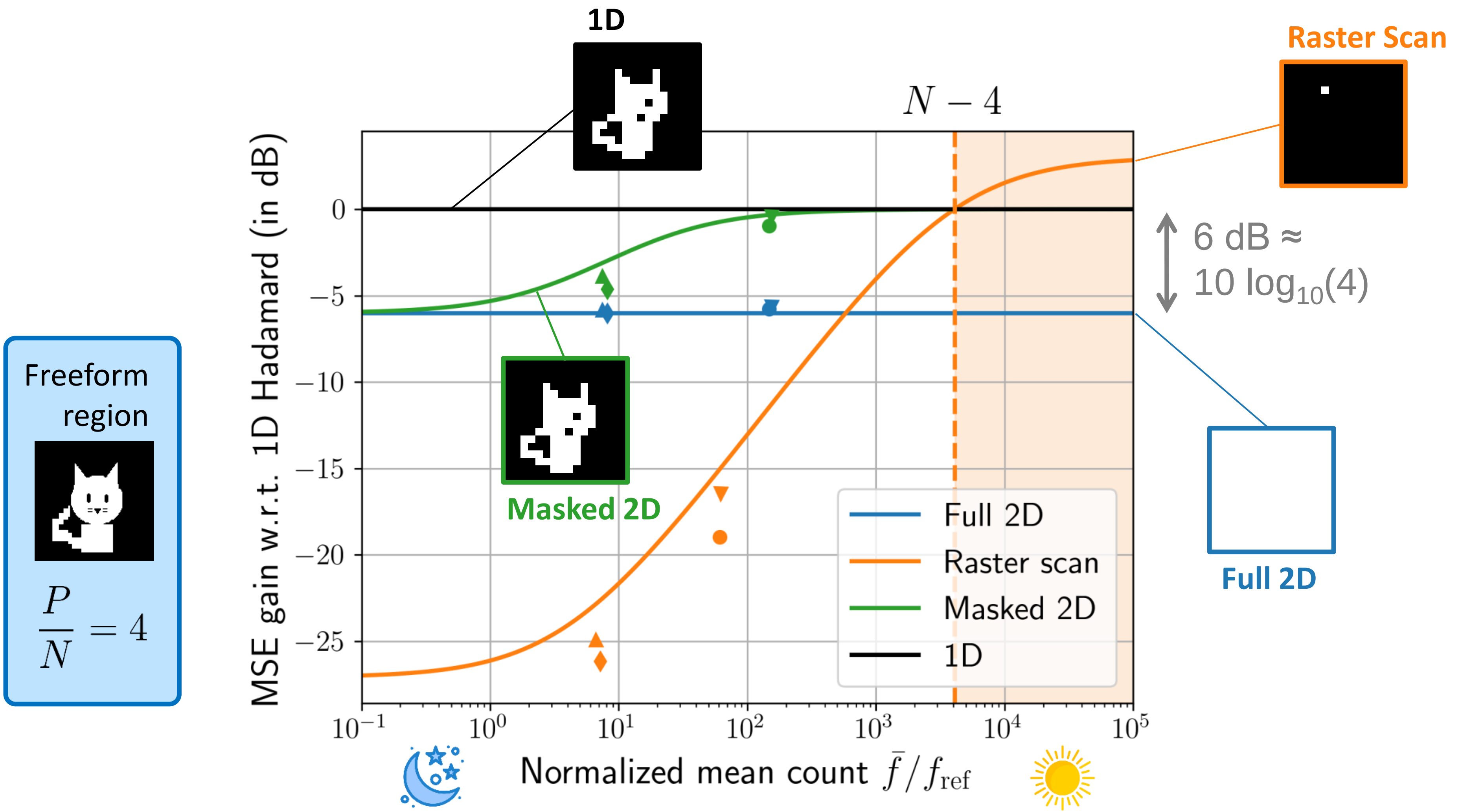

**Masked 2D**




**1D**



# Experimental demonstration (4/4)




*Note: The figure above was obtained using a split Hadamard matrix, not a negative Hadamard matrix, as the acquisition matrix.*

# Conclusions



**Fellgett's advantage**

→ Confirmed in intermediate light conditions

**Freeform imaging**

- Masking improves MSE
- Non-imaging matrices improves over masking
- Experiment and theory in good agreement
- Discrepancy for RS

→ DMD implementation is flexible

→ Could improve low light imaging in wide range of applications

→ Choice of the freeform region?

https://www.creatis.insa-lyon.fr/~ducros
(see the 'single-pixel imaging' page)