Nonlinear regularized decomposition of spectral x-ray projection images

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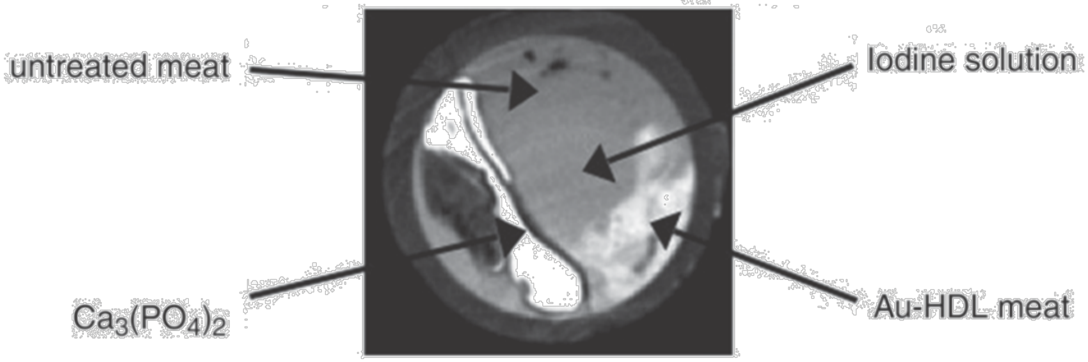
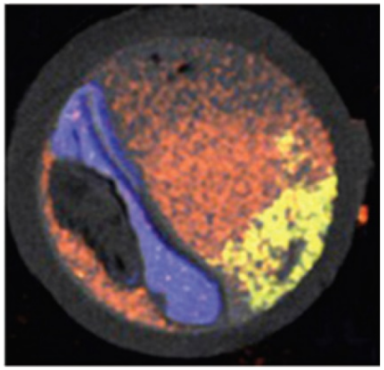
SPECTRAL CT IMAGING

- **Conventional vs. Spectral = Grey level vs Color!**

Traditional CT



Spectral CT



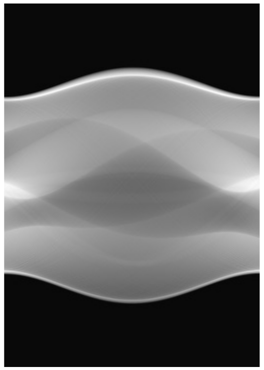
- Average attenuation
- Arbitrary units

+ Chemical components
+ Density (g.cm-3)
+ K-edge contrast imaging

[Cormode *et al.*, *Radiology*, 256 (**3**), 2010]

RECONSTRUCTION STRATEGIES

Sinogram



Tomographic Reconstruction

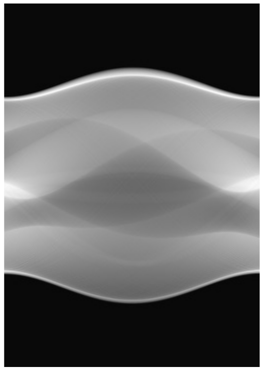
object

Linear Attenuation Coefficient

RECONSTRUCTION STRATEGIES

Sinogram

Photon Counting Detector



E

Tomographic Reconstruction

object

Linear Attenuation Coefficient

RECONSTRUCTION STRATEGIES

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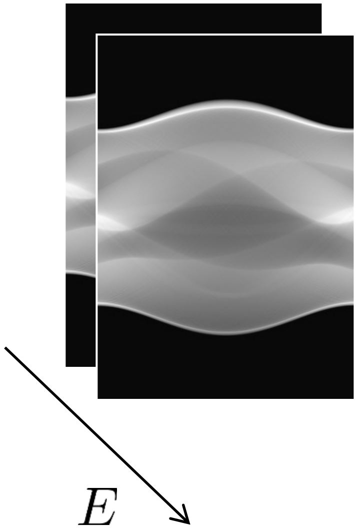

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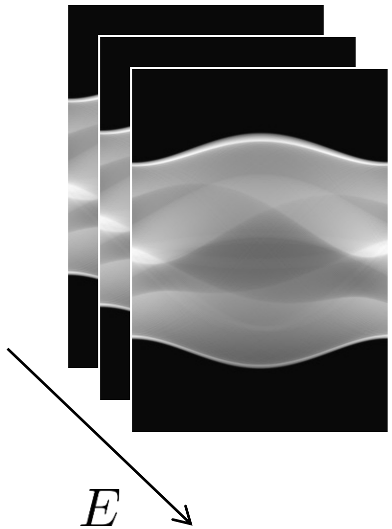
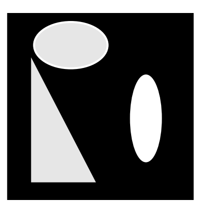


Tomographic Reconstruction

object



Linear Attenuation Coefficient

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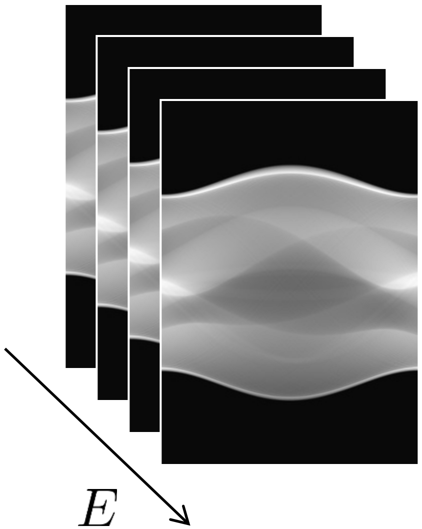

Tomographic Reconstruction

object

Linear Attenuation Coefficient

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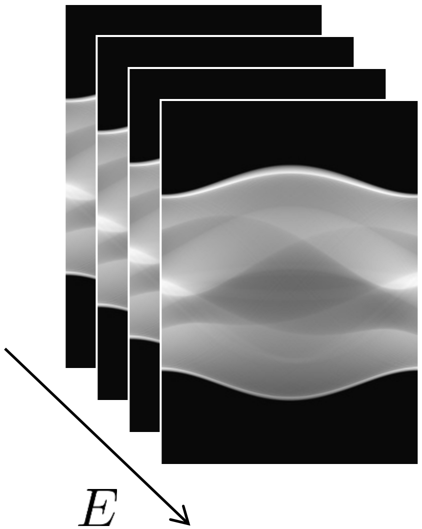
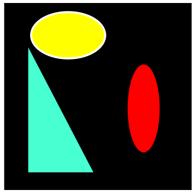


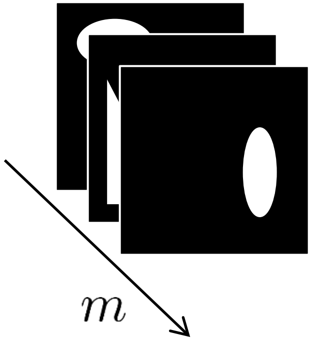
Spectral Tomographic Reconstruction

[Mendoca, et al., IEEE TMI, 2014]
[Long and Fessler, IEEE TMI, 2014]
[Zhang et al., IEEE TMI, 2014]
[Barber, Fully 3D, 2015]

object



$\Leftrightarrow$

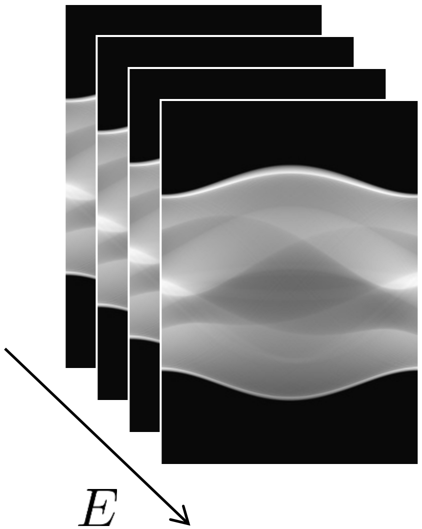


Material Mass Density

RECONSTRUCTION STRATEGIES

Sinogram

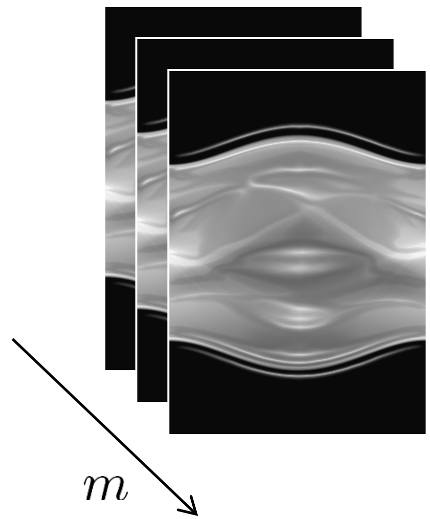
Photon Counting Detector



Decomposition

[Alvarez *et al.*, PMB, 1976]
[Brody *et al.*, Med. Phys, 1981]
[Schlomka *et al.*, PMB, 2007]
[Roessl et al., PMB, 2007]

Material sinogram



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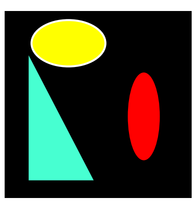

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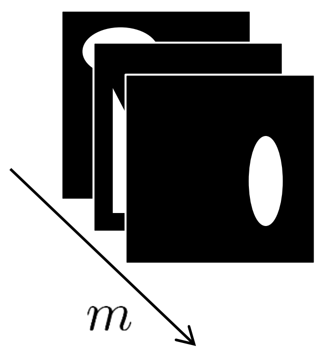

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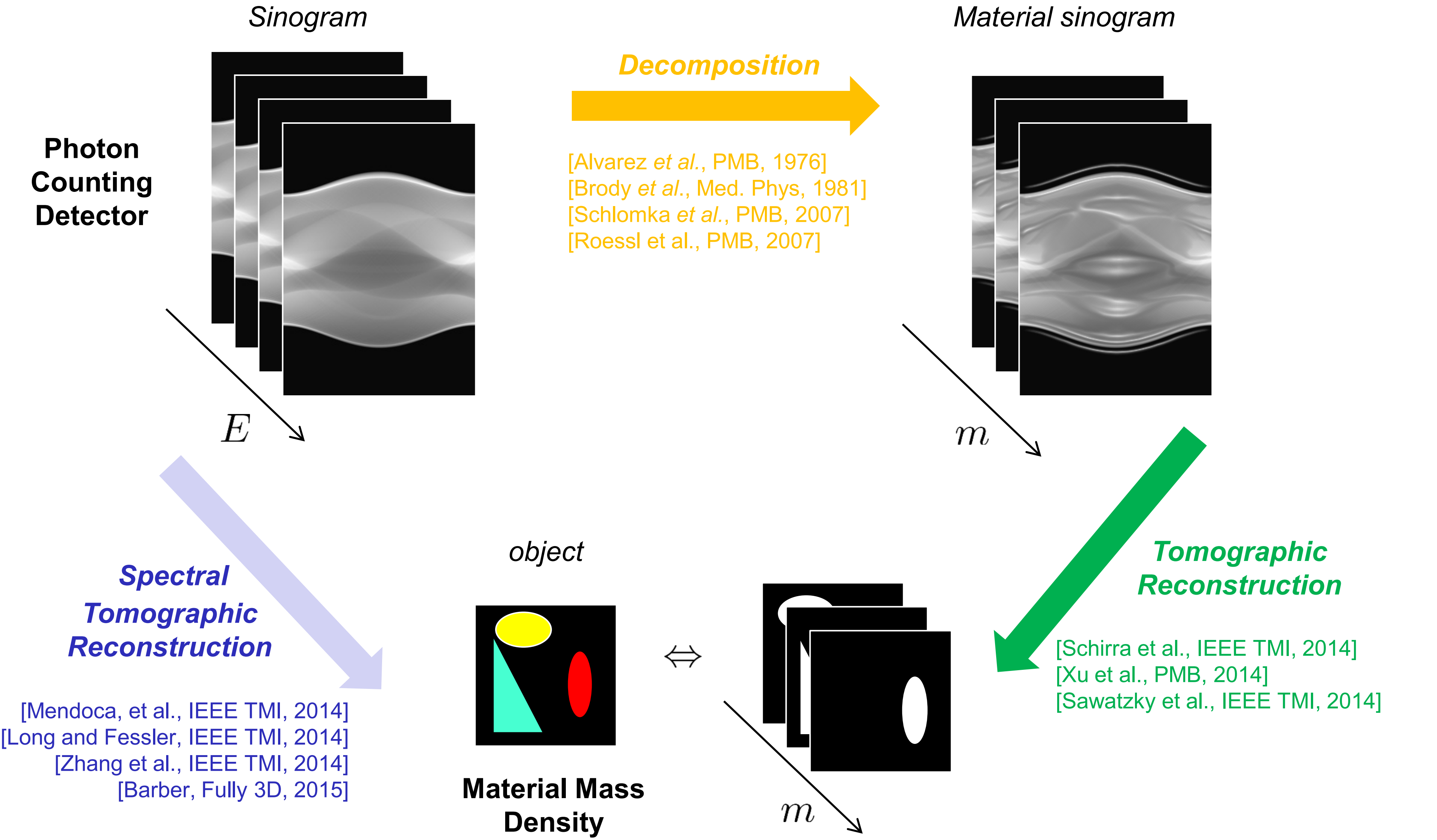
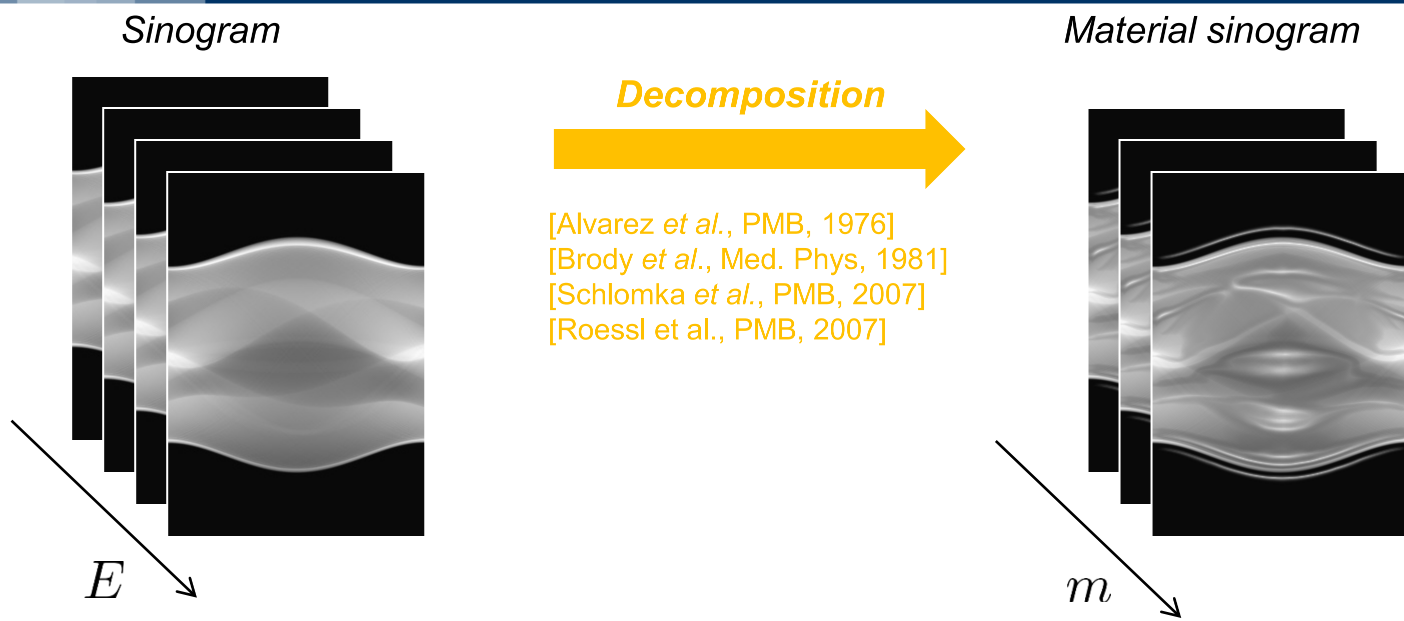


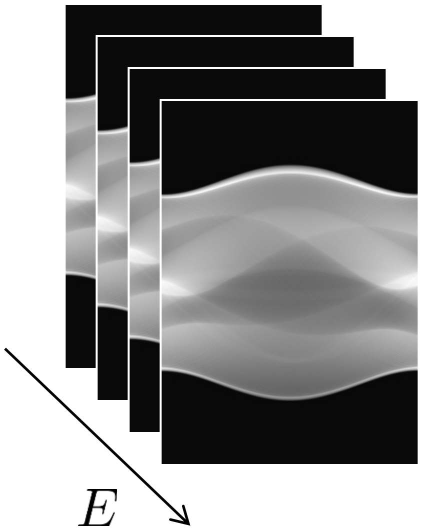
GOAL AND MOTIVATION



- **Why material decomposition of the sinogram?**
  1. Embeds all the **nonlinearities** of spectral CT
  2. Naturally **parallelizable** across the projection views
  3. Applicable to both CT and interventional radiography

GOAL AND MOTIVATION

Sinogram



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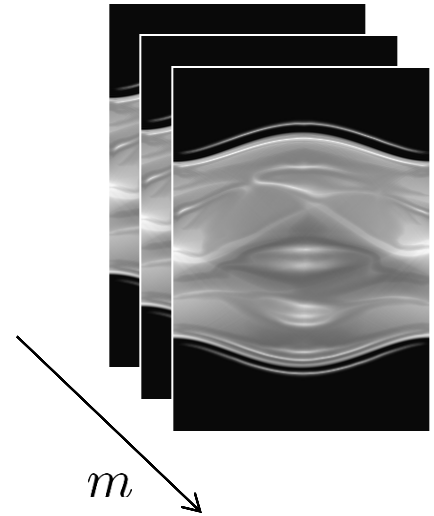
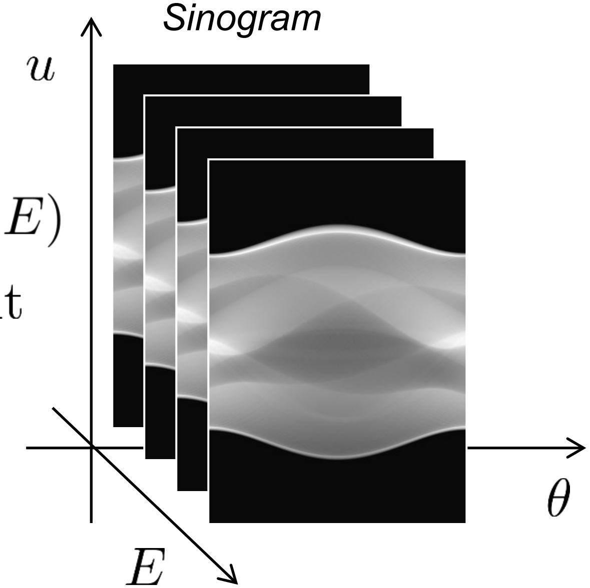


GOAL AND MOTIVATION

Sinogram

$s(u, \theta, E)$

no unit



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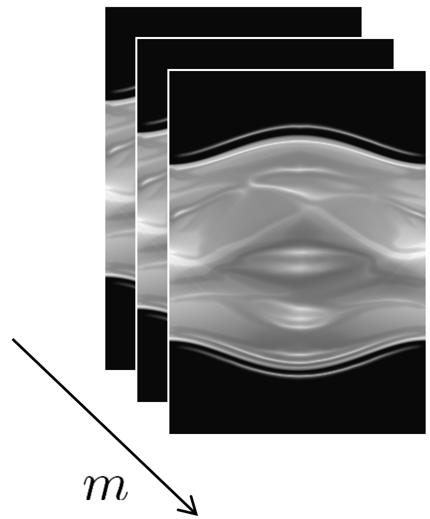

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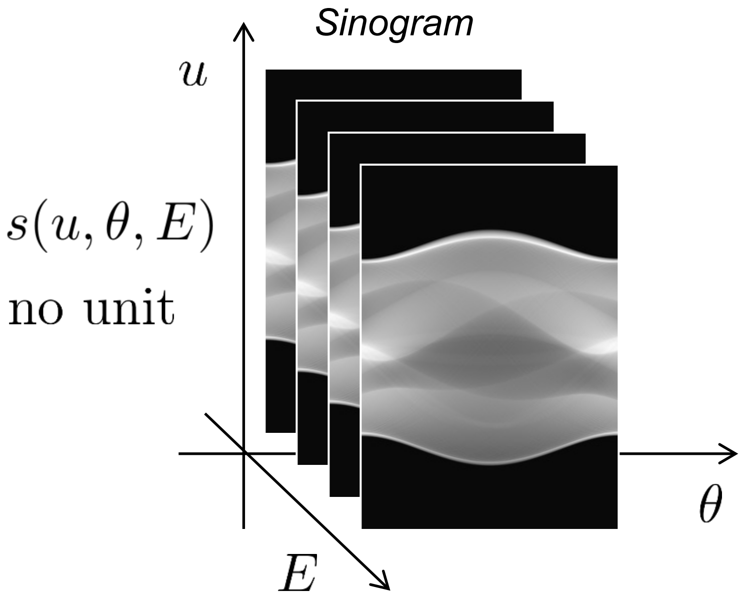

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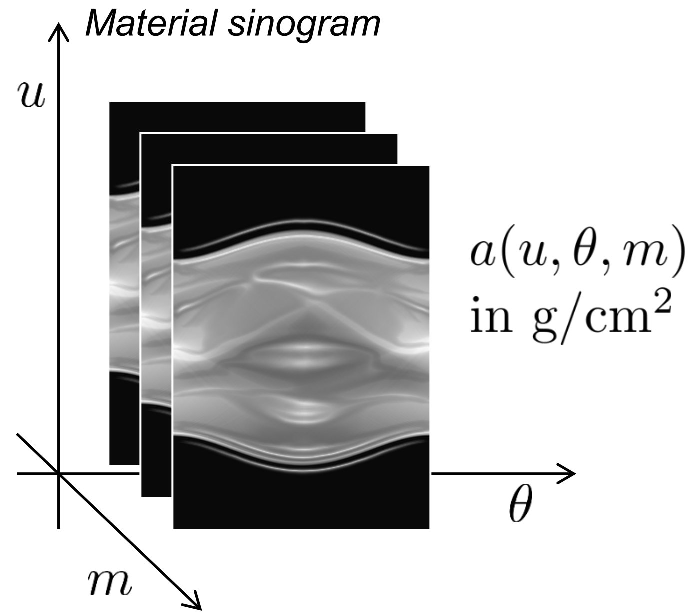

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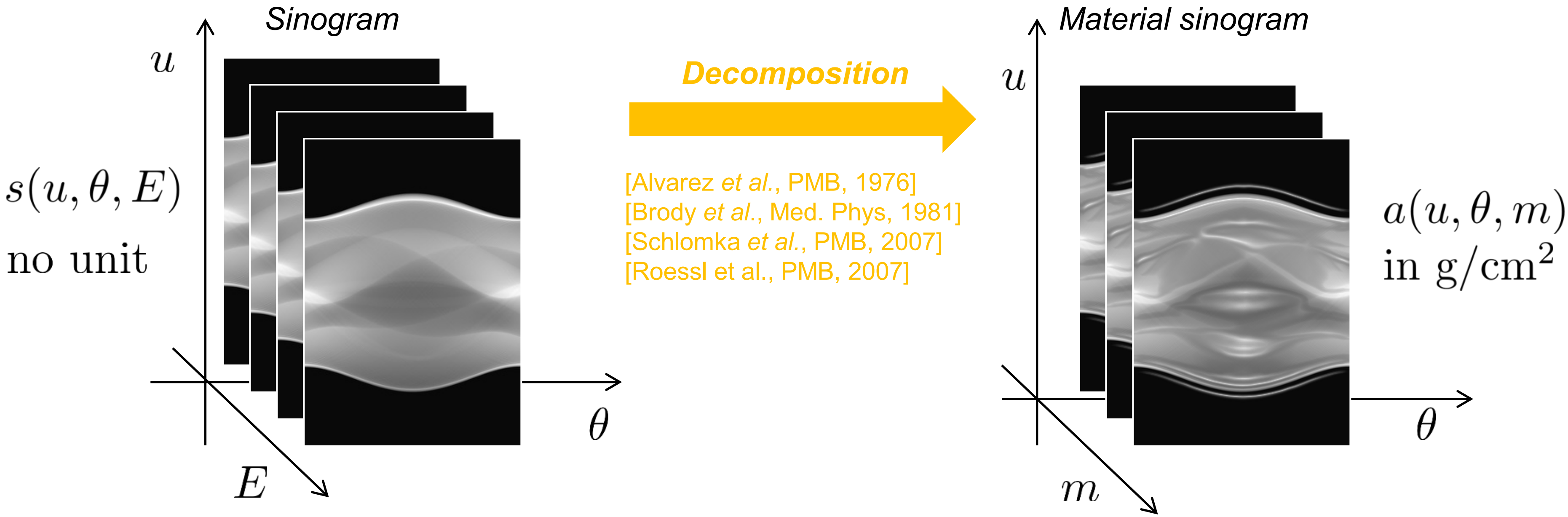
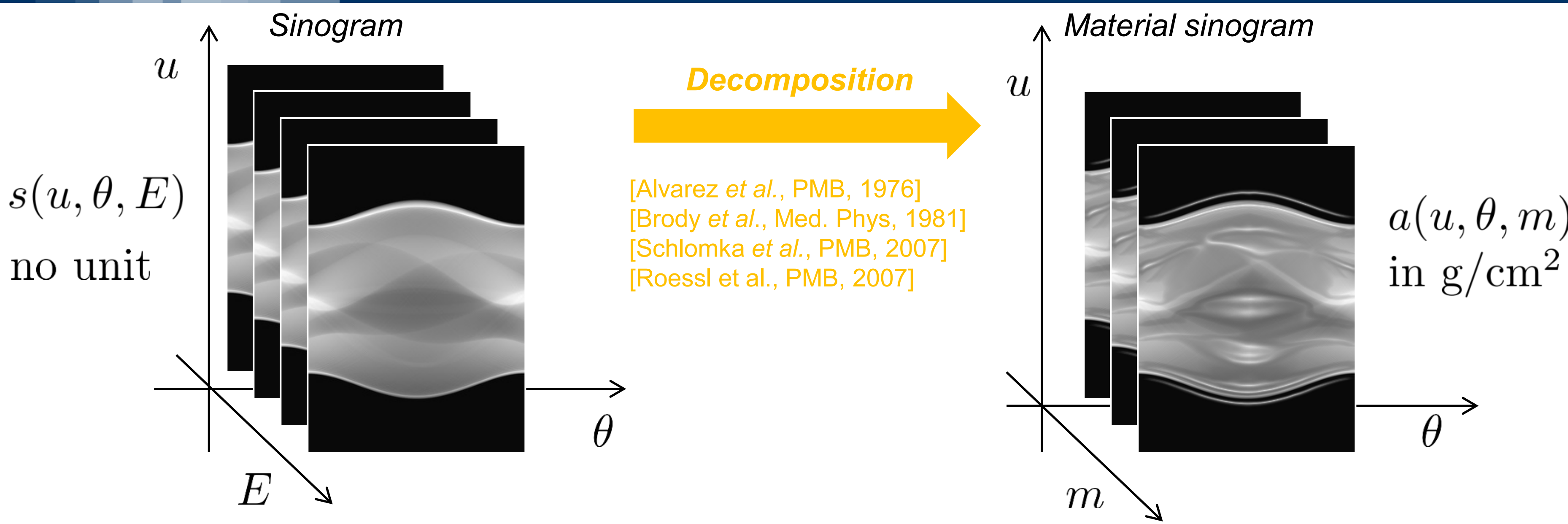


- **2D projection images at a fixed view**

GOAL AND MOTIVATION



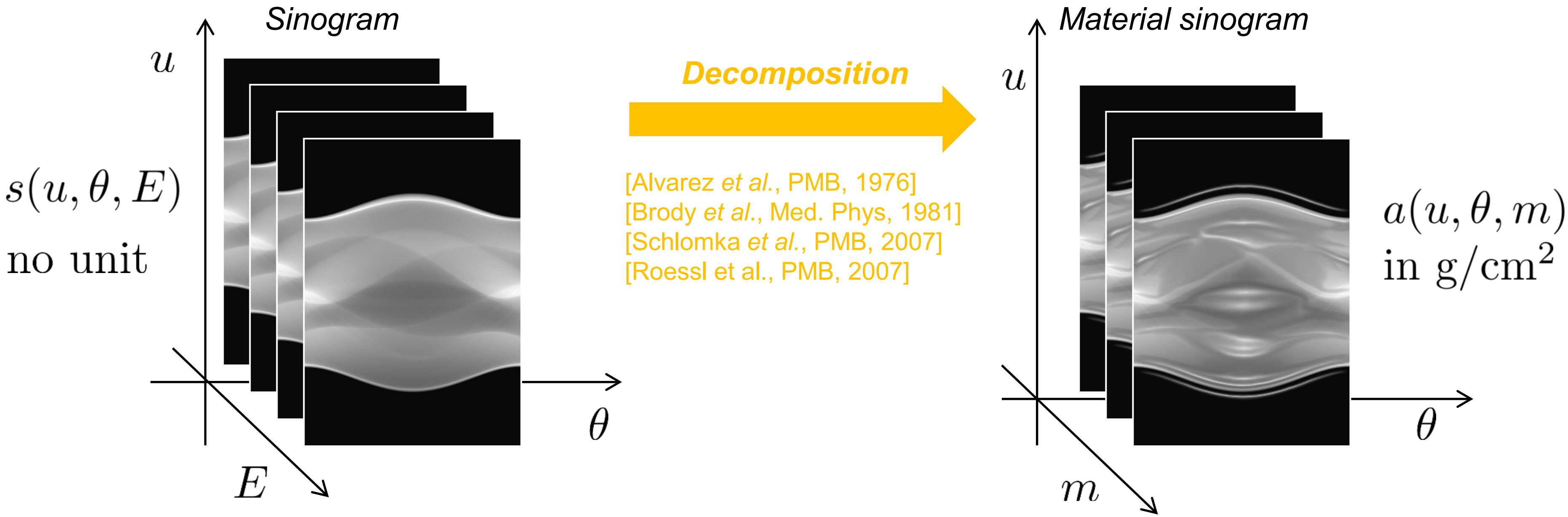
- **2D projection images at a fixed view**

- **Notations**

  Photon numbers (no units) $\mathbf{s} =$

  Projected mass (in g.cm^{-2}) $\mathbf{a} =$

GOAL AND MOTIVATION



- **2D projection images at a fixed view**

- **Notations**

  Photon numbers (no units) $\mathbf{s} =$ *Projected mass (in g.cm^{-2})* $\mathbf{a} =$

$$\text{Find } \mathbf{a} \text{ given } \mathbf{s} \text{ and } \mathcal{F}(\mathbf{a}) \qquad \text{(P)}$$

CONTRIBUTION

- **Variational framework**
  - Nonlinear decomposition
  - Regularized decomposition

$$\mathcal{C}(\mathbf{a}) = \mathcal{D}(\mathbf{s}, \boldsymbol{\mathcal{F}}(\mathbf{a})) + \alpha\mathcal{R}(\mathbf{a})$$

- **Gauss-Newton algorithm**

$$\mathbf{a}^{(k+1)} = \mathbf{a}^{(k)} + \lambda^{(k)}\Delta\mathbf{a}^{(k)}$$

1. Descent direction:

$$(2{\mathbf{J}^{(k)}}^\top \mathbf{J}^{(k)} + \alpha\mathbf{H}^{(k)})\Delta\mathbf{a}^{(k)} = -\mathbf{g}^{(k)}$$

2. Line Search:

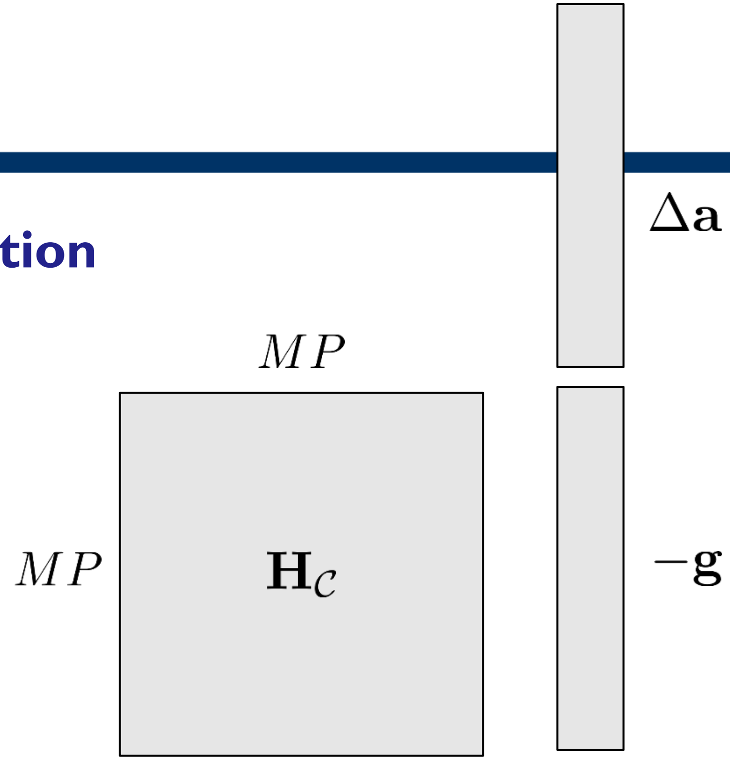
$$\lambda^{(k)} = \arg\min\ \mathcal{C}(\mathbf{a}^{(k)} + \lambda\Delta\mathbf{a}^{(k)})$$

  - Gold standard
  - Superlinear convergence rates

CONTRIBUTION

- **Descent direction computation**

$$\underbrace{(2\mathbf{J}^\top\mathbf{J} + \alpha\mathbf{H})}_{\mathbf{H}_\mathcal{C}}\Delta\mathbf{a} = -\mathbf{g}$$



CONTRIBUTION

➢ Descent direction computation

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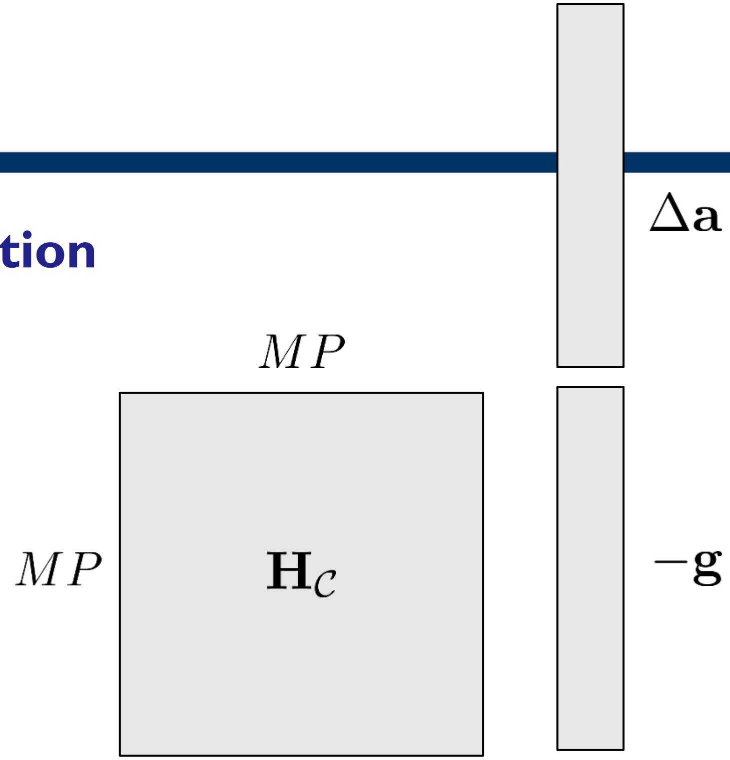

- Study case
  - M = 3
  - P = 256 x 256

	$-\mathbf{g}$	$\Delta \mathbf{a}$	$\mathbf{H}_\mathcal{C}$	
Size	1.5 MiB	1.5 MiB	288 GiB	

CONTRIBUTION

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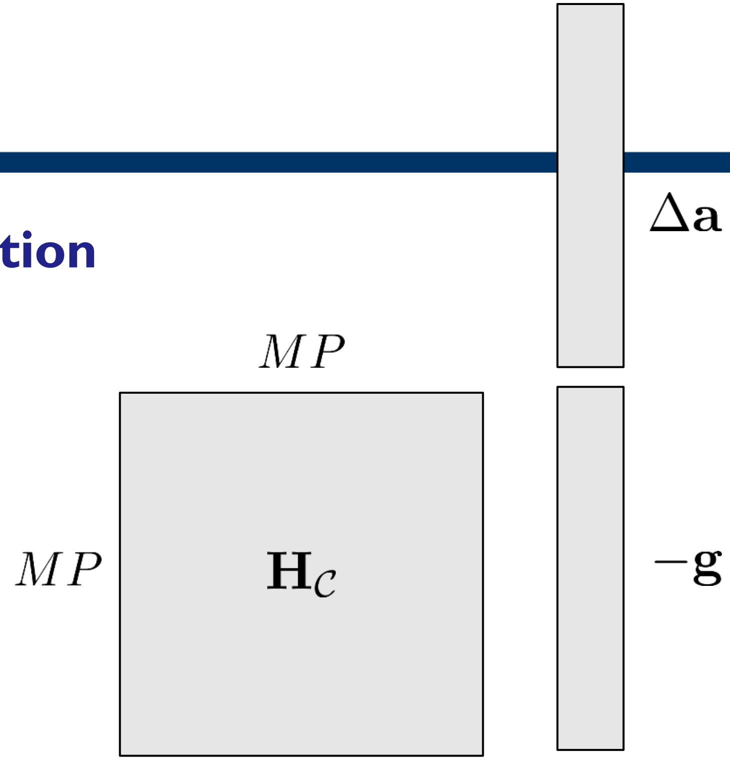

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- Storage of the full Hessian is intractable!

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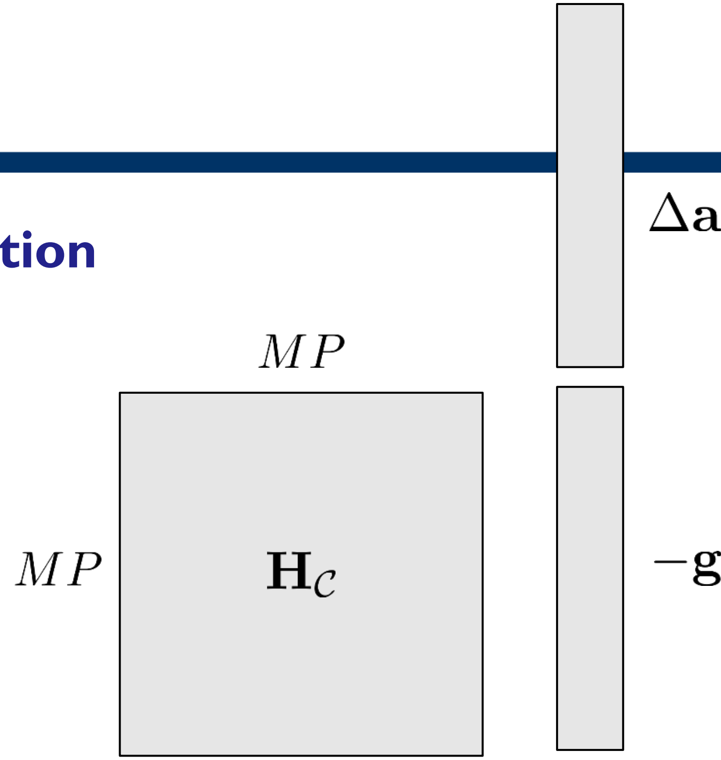


- Study case
  - M = 3
  - P = 256 x 256

	$-\mathbf{g}$	$\Delta\mathbf{a}$	$\mathbf{H}_\mathcal{C}$	$\mathbf{H}_\mathcal{C}^{\text{sparse}}$
Size	1.5 MiB	1.5 MiB	288 GiB	4.5 MiB

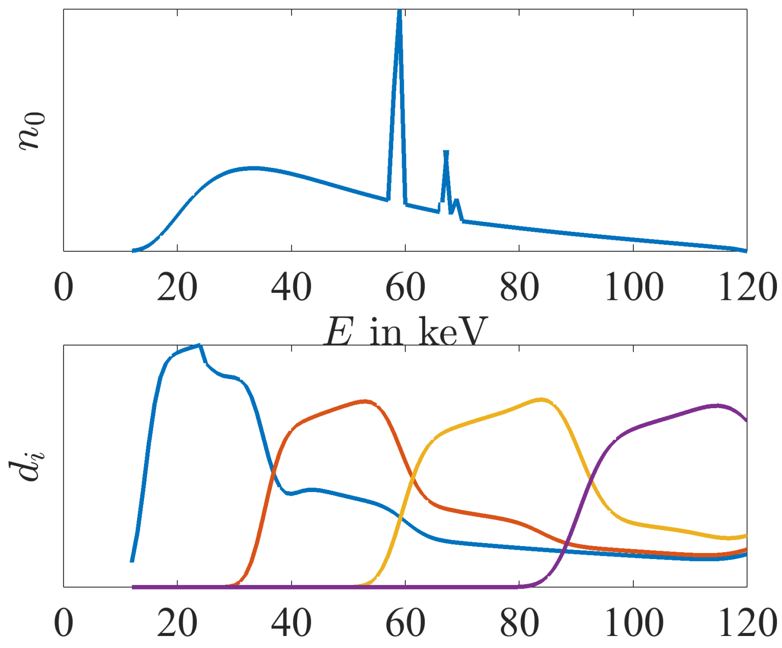
- Storage of the full Hessian is intractable!
- Hessian is block diagonal with only PM^2 non zero entries

NUMERICAL SIMULATIONS

Forward model

$$s_i(\mathbf{u}) = \int_{\mathbb{R}} n_0(E) d_i(E) \exp\left(-\sum_{m=1}^{M} a_m(\mathbf{u}) \tau_m(E)\right) \mathrm{d}E$$

[Poludniowski *et al.*, Med Phys, 2007]



[Schlomka *et al.*, PMB, 2008]

4 bins

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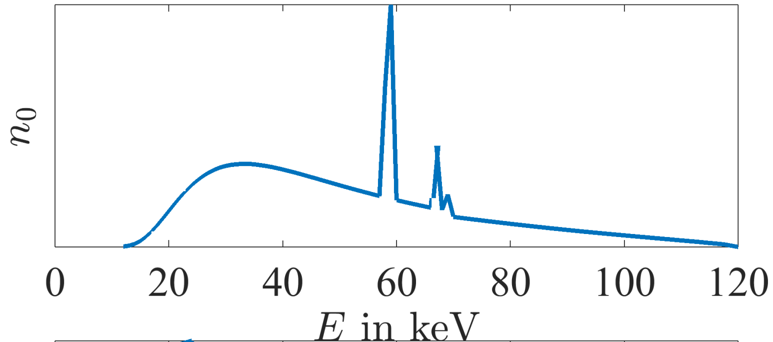
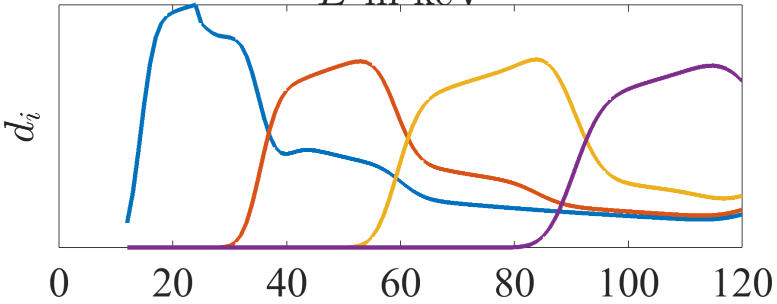
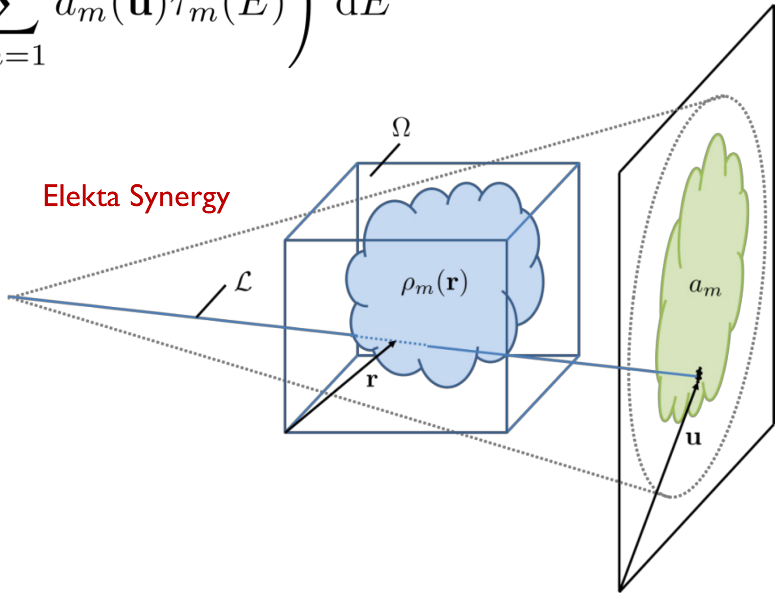




[Schlomka *et al.*, PMB, 2008]

4 bins

Elekta Synergy



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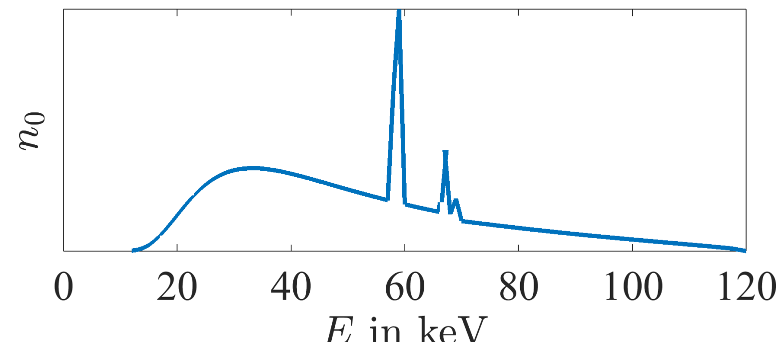

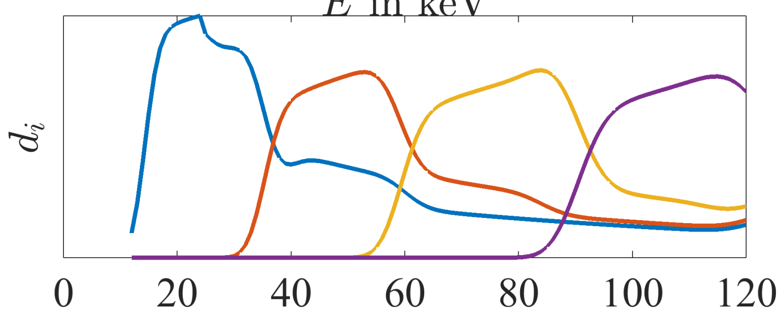

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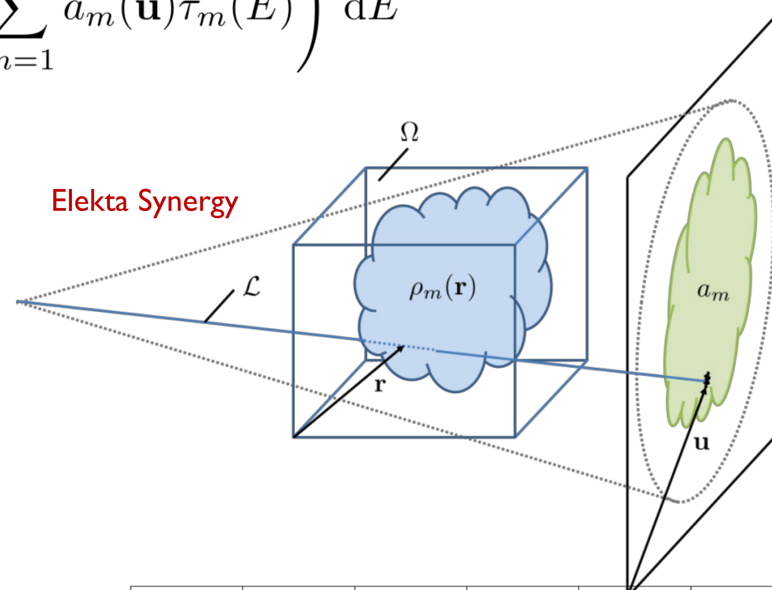
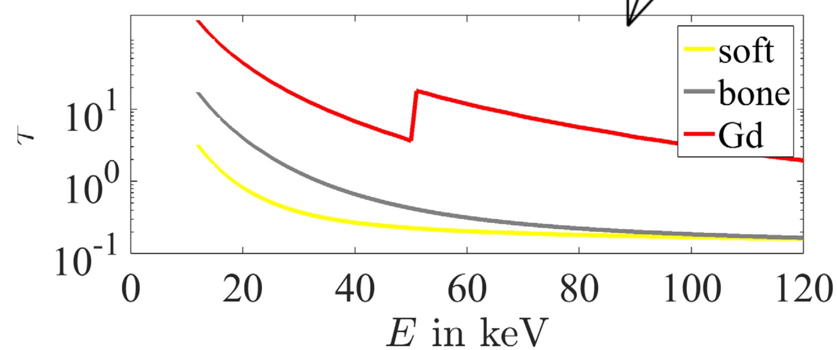


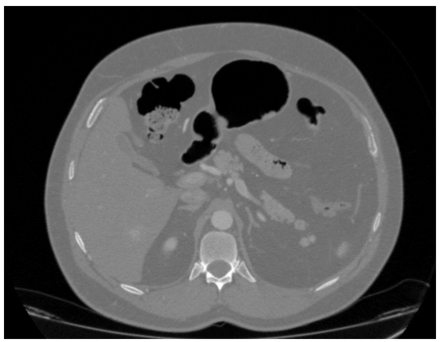


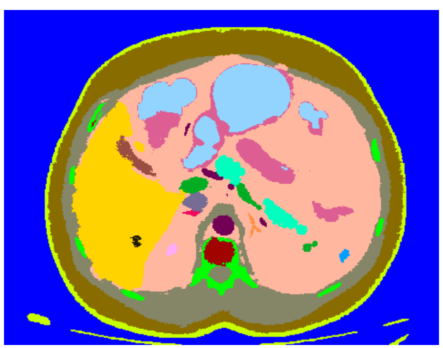
NUMERICAL SIMULATIONS

➢ Numerical Phantom

❖ 3-material Thorax Phantom

3D CT scan (a.u)
[3D-IRCADb data set]



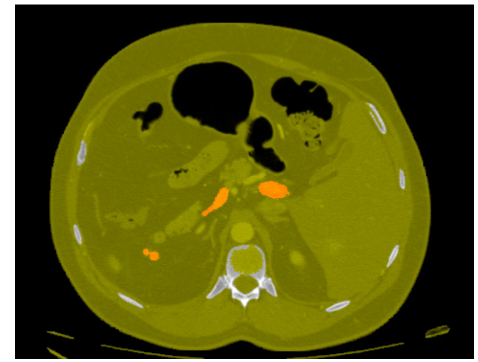


3D atlas (labels)
[Kéchichian *et al.*, IEEE TIP, 2013]

Materials

- Soft tissue
- Bone
- Gadolinium (Portal vein)

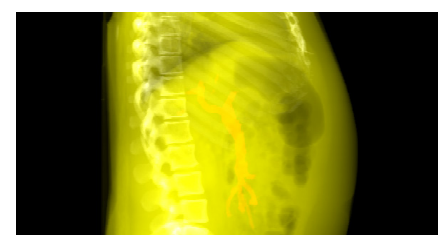
3D volumes (g.cm^{-3})



Projection (RTK)
http://www.openrtk.org/

2D image (g.cm^{-2})

frontal view



lateral view

A typical decomposition

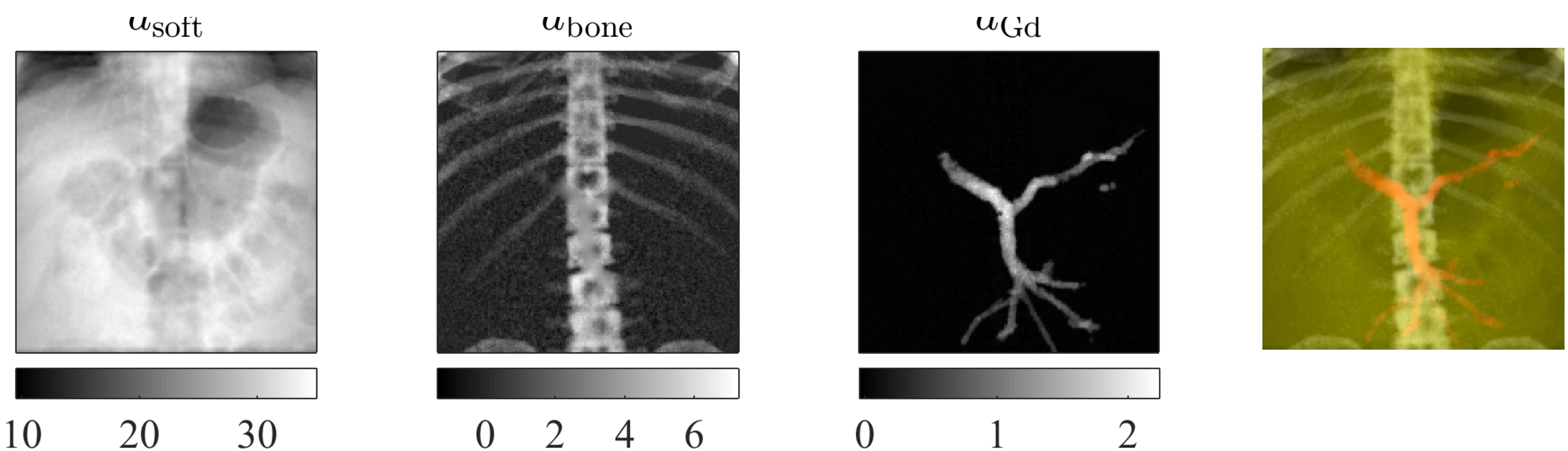
$$\mathcal{C}(\mathbf{a}) = \mathcal{D}(\mathbf{s}, \boldsymbol{\mathcal{F}}(\mathbf{a})) + \alpha \mathcal{R}(\mathbf{a})$$

- Weighted least squares

$$\mathcal{D}(\mathbf{s}, \boldsymbol{\mathcal{F}}(\mathbf{a})) = \|\mathbf{s} - \boldsymbol{\mathcal{F}}(\mathbf{a})\|_{\mathbf{W}}^2$$

- Material-dependent regularization

$$\mathcal{R}(\mathbf{a}) = \|\Delta \mathbf{a}_{\text{soft}}\|_2^2 + \|\nabla \mathbf{a}_{\text{bone}}\|_2^2 + \|\nabla \mathbf{a}_{\text{Gd}}\|_1$$



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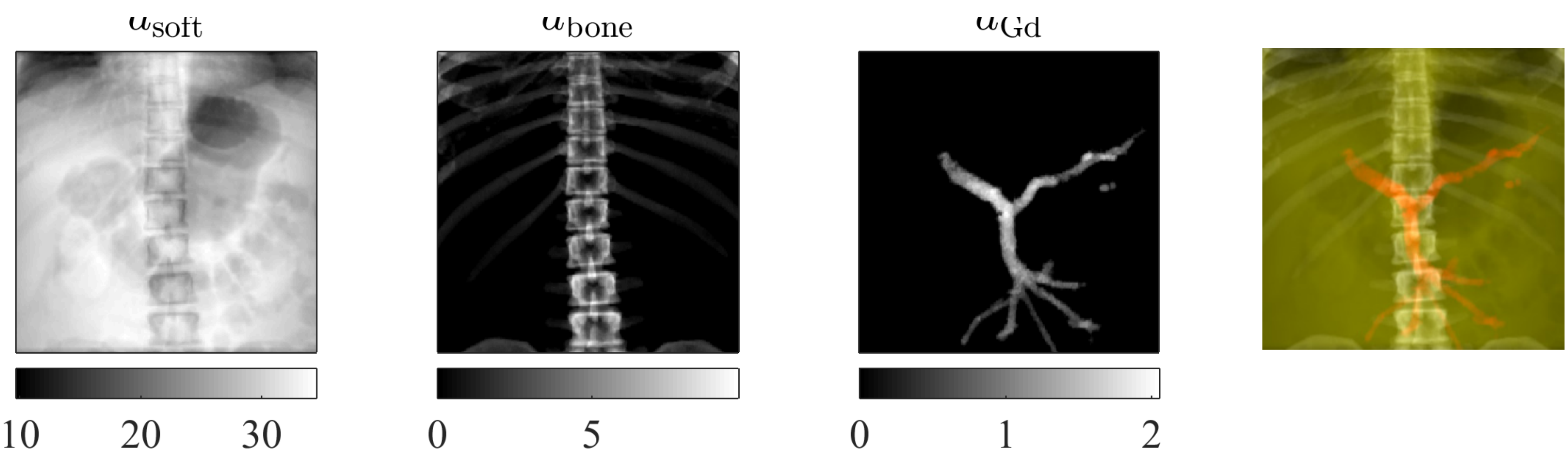
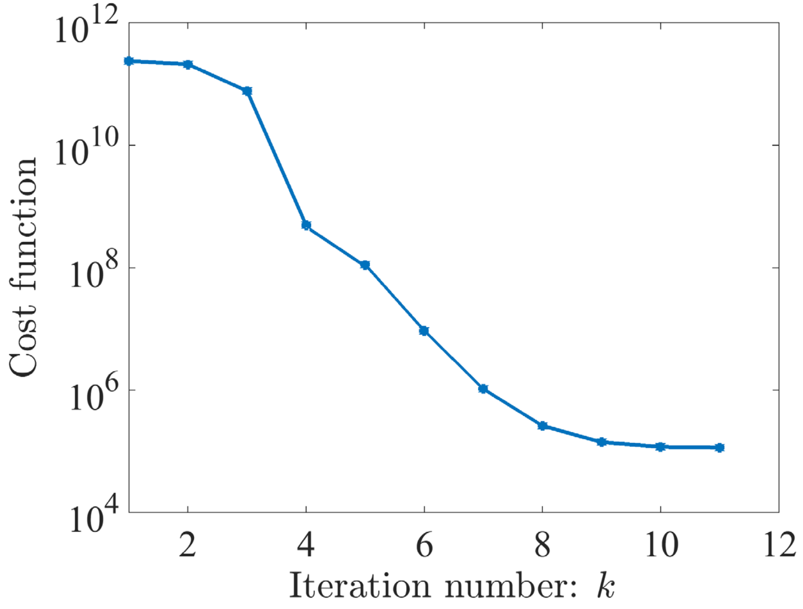


RESULTS

➢ **Convergence of the algorithm**



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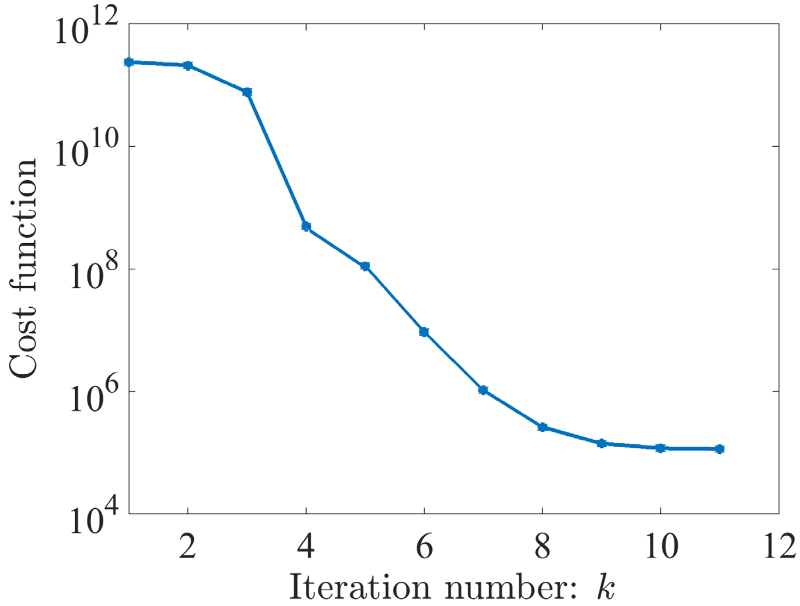
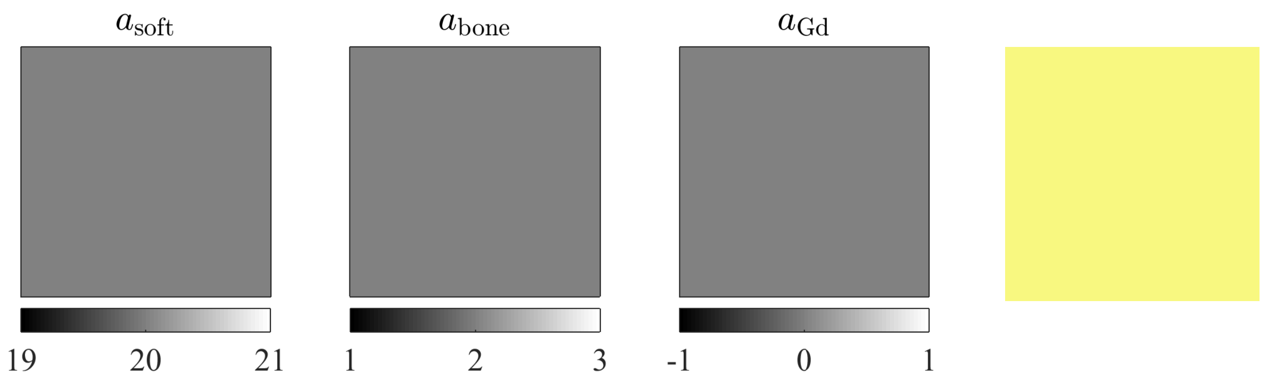


Init:

$$a^0_{\text{soft}} = 20 \text{ g.cm}^{-2}$$
$$a^0_{\text{bone}} = 2 \text{ g.cm}^{-2}$$
$$a^0_{\text{Gd}} = 0 \text{ g.cm}^{-2}$$

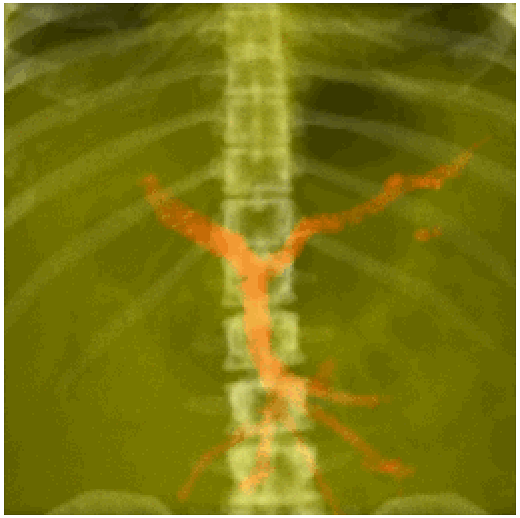


RESULTS

➢ Influence of noise

❖ Poisson noise

$$\tilde{s}_i = \mathcal{P}(\mu = s_i)$$



Decreasing s_i i.e. decreasing SNR

RESULTS

Influence of noise

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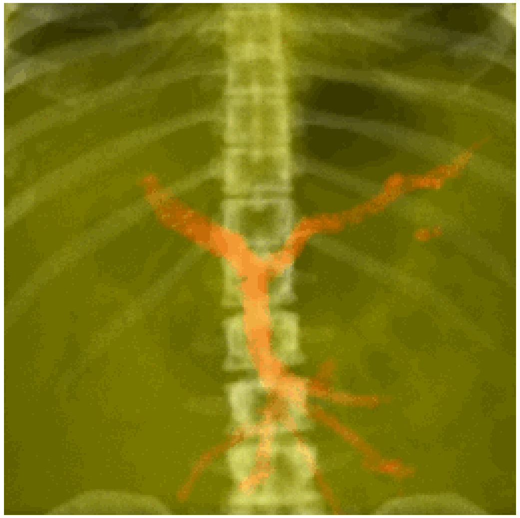
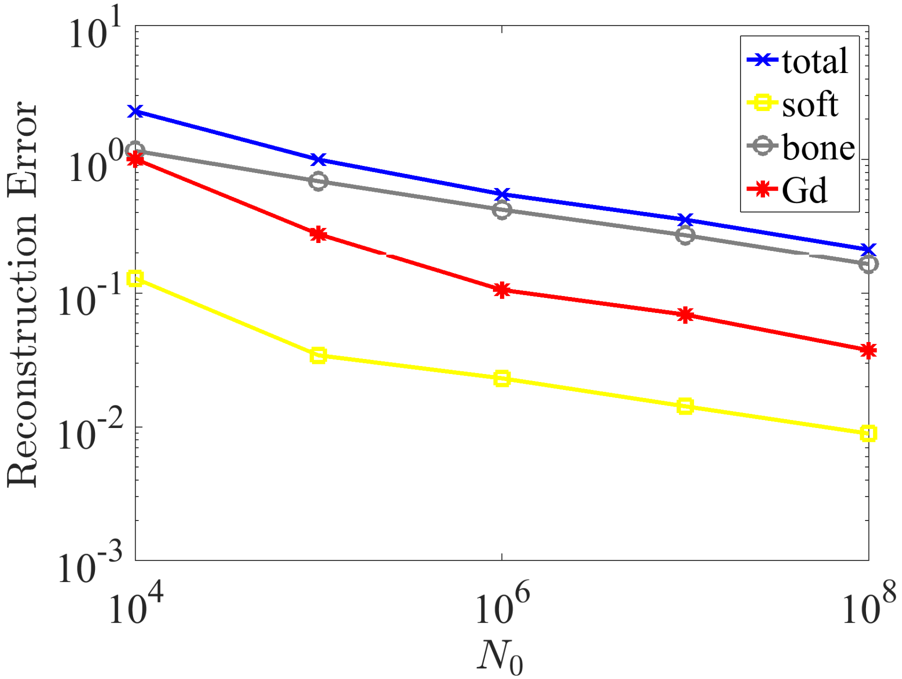


Decreasing s_i i.e. decreasing SNR

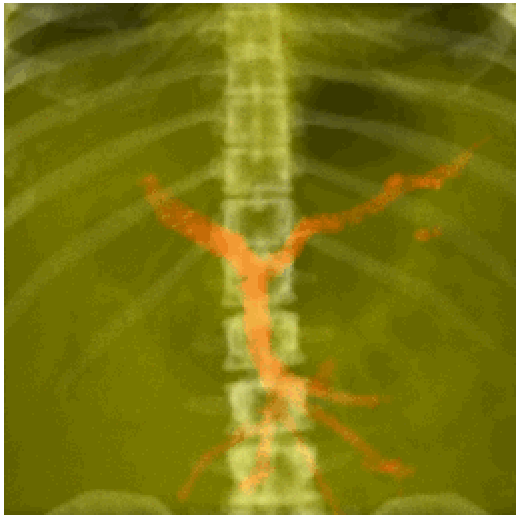
- Error vs noise



Influence of noise

Poisson noise

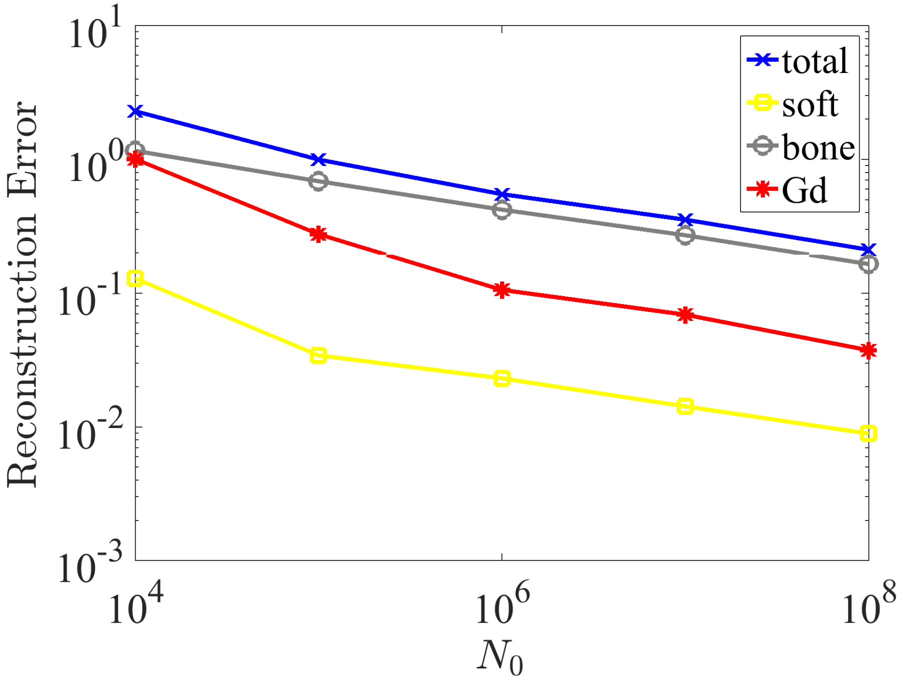
$$\tilde{s}_i = \mathcal{P}(\mu = s_i)$$



Decreasing s_i i.e. decreasing SNR

Error vs noise

$$\text{error :} \quad \frac{\|\hat{\mathbf{a}}_m - \mathbf{a}_m^{\text{true}}\|_2}{\|\mathbf{a}_m^{\text{true}}\|_2}$$

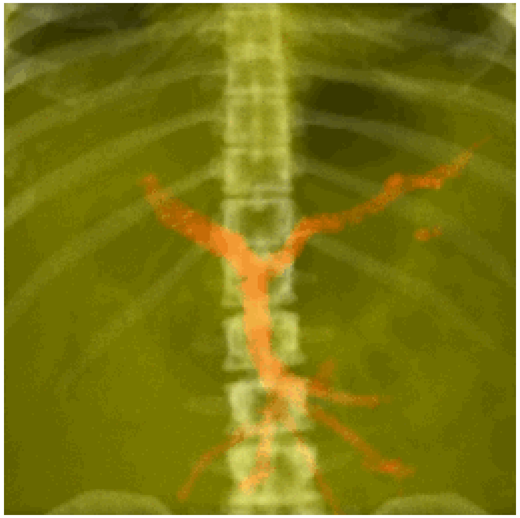


RESULTS

Influence of noise

Poisson noise

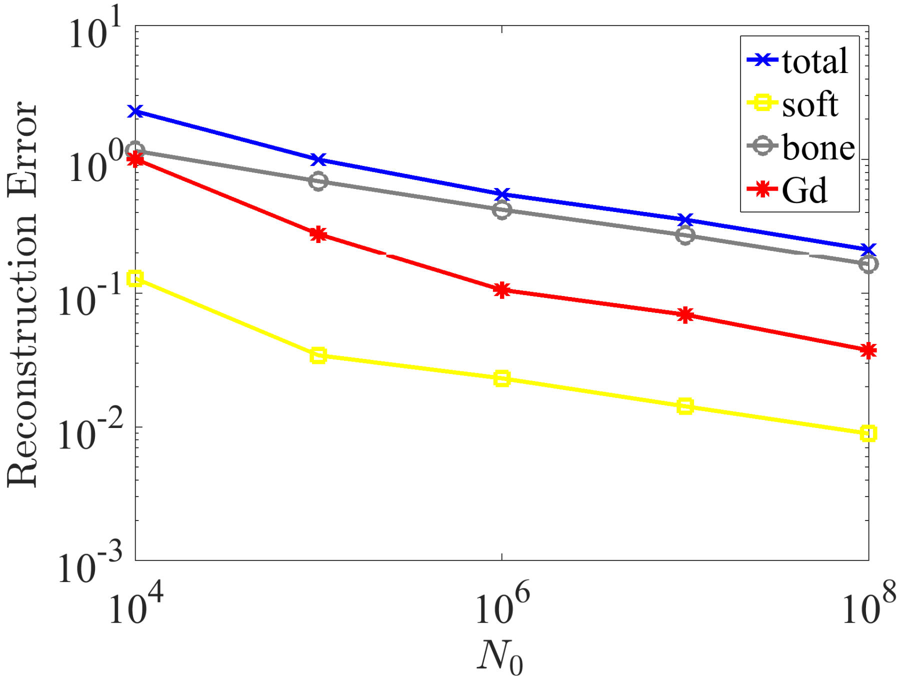
$$\tilde{s}_i = \mathcal{P}(\mu = s_i)$$



Decreasing s_i i.e. decreasing SNR

Error vs noise

$$\text{error} : \quad \frac{\|\hat{\mathbf{a}}_m - \mathbf{a}_m^{\text{true}}\|_2}{\|\mathbf{a}_m^{\text{true}}\|_2}$$



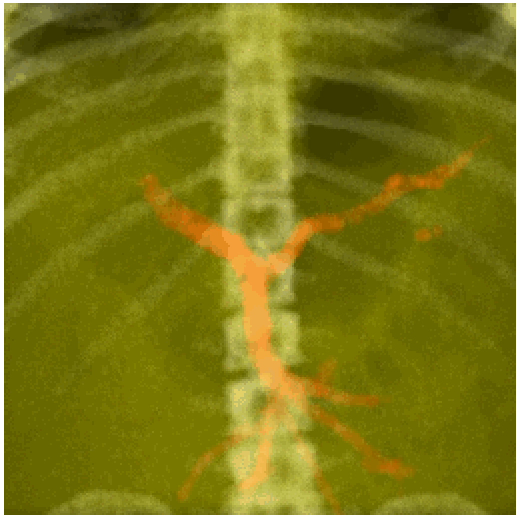
Number of incident photons (per pixel)

$$N_0 = \int_{\mathbb{R}} n_0(E)\,\mathrm{d}E \text{ in photons}$$

RESULTS

- **Influence of the marker concentration**

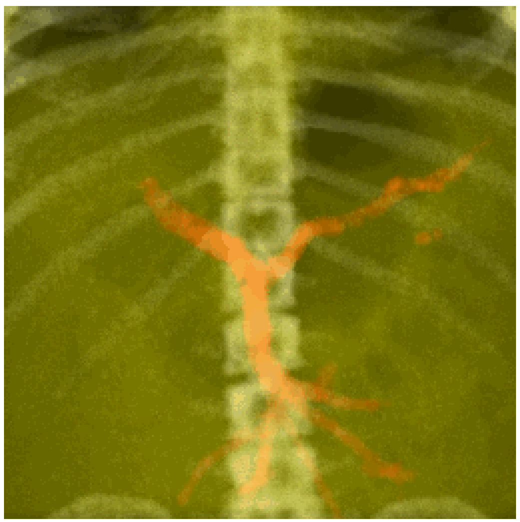
  - Concentration range: 0.01 to 1 $g.cm^{-3}$

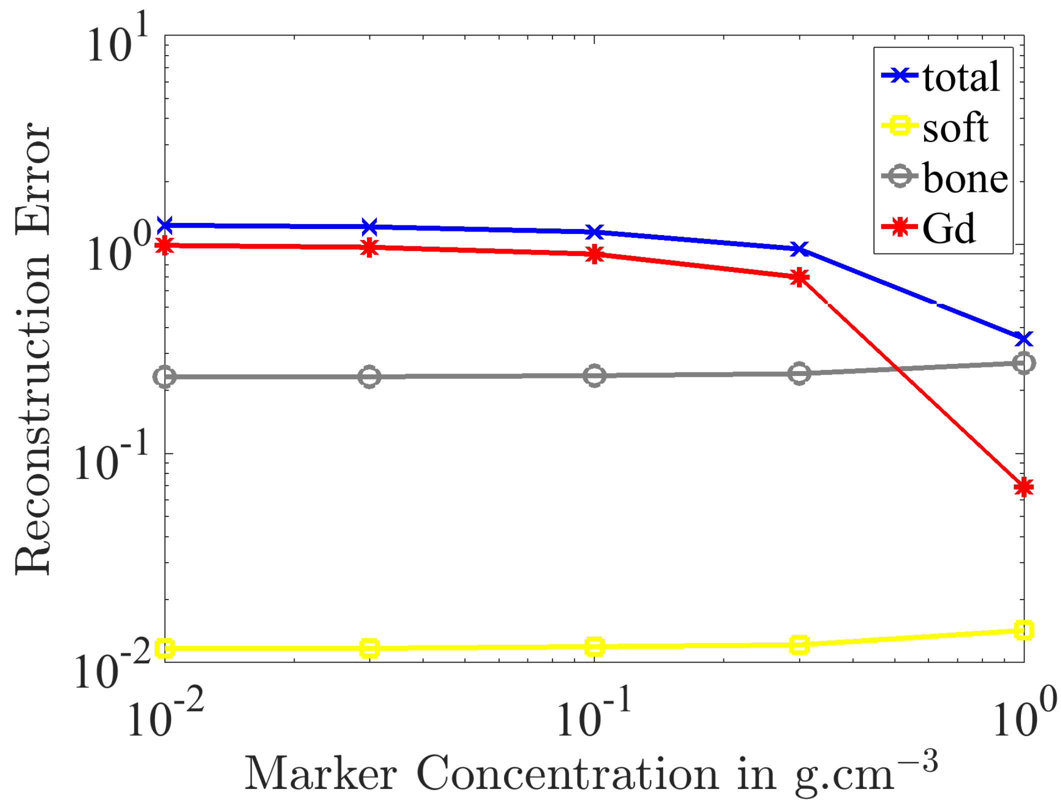


RESULTS

➢ Influence of the marker concentration

❖ Concentration range: 0.01 to 1 $g.cm^{-3}$

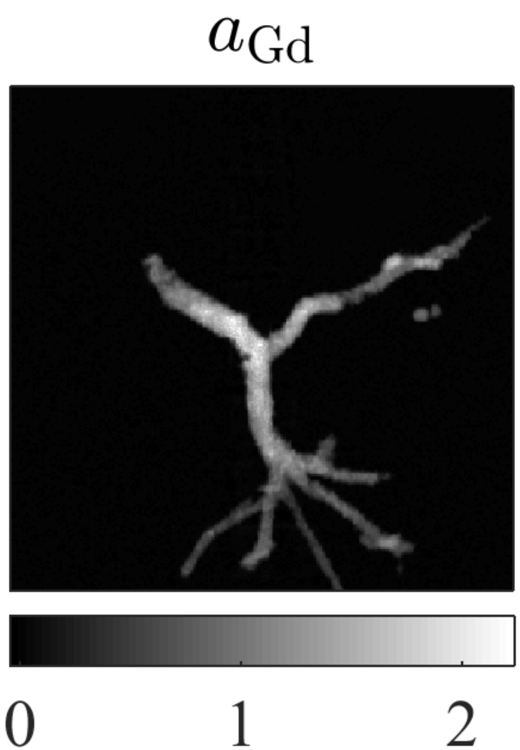


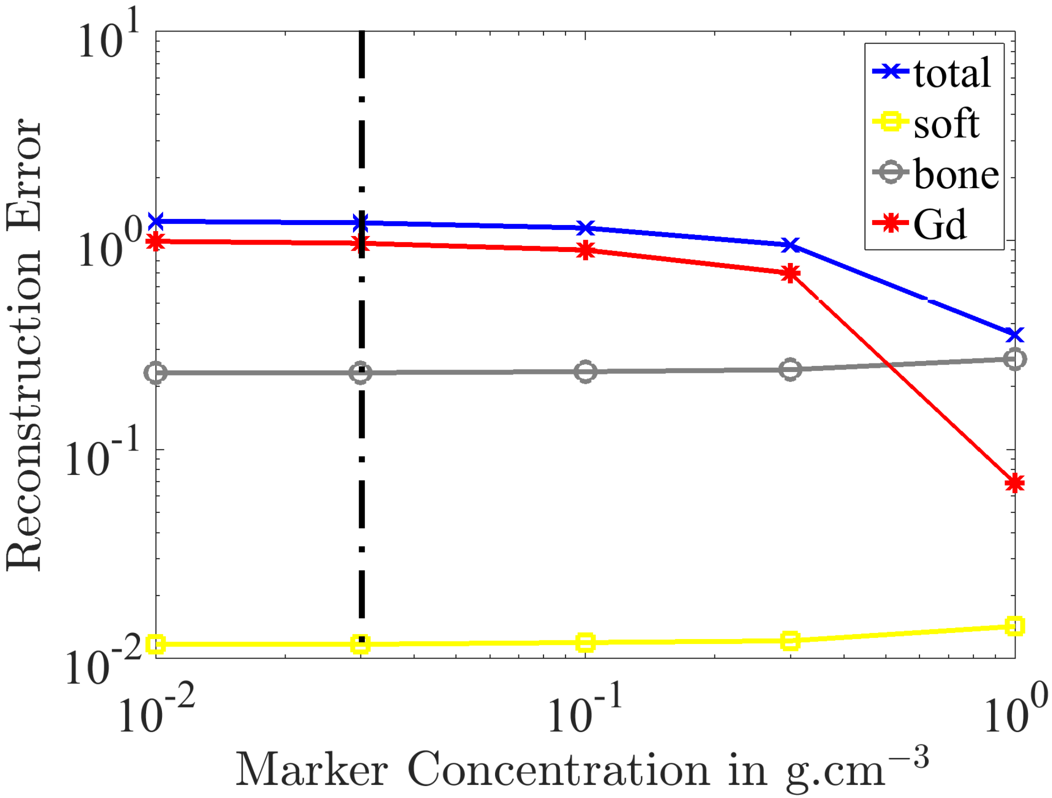


RESULTS

➢ **Influence of the marker concentration**

❖ Concentration range: 0.01 to 1 g.cm^{-3}





CONCLUSION

- **Material decomposition**

- **We proposed a GN algorithm**
  - Weighted least squares
  - L_2/L_1 material dependent regularization

- **Thorax phantom with portal vein marked with gadolinium**
  - Different number of counts
  - Different marker concentrations

- **Encouraging results**
  - Fast convergence
  - Good reconstruction quality

- **Thanks to the spectral CT team**
  - Juan FPJ ABASCAL
  - Tom HOHWEILLER
  - Jean-Michel LÉTANG
  - Cyril MORY
  - Françoise PEYRIN
  - Odran PIVOT
  - Simon RIT
  - Bruno SIXOU
  - Gloria VILCHES FREIXAS

THANK YOU

FOR

YOUR ATTENTION

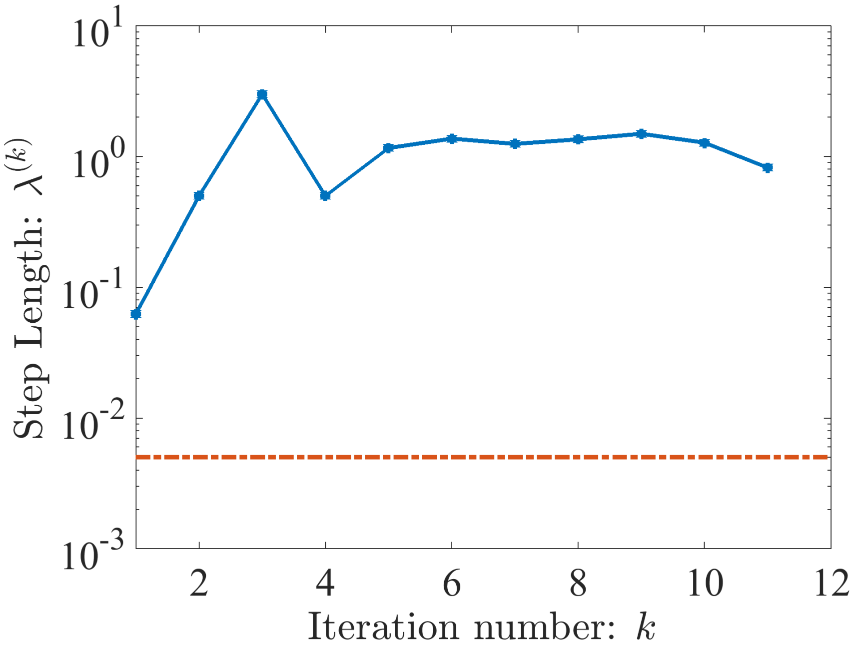
RESULTS

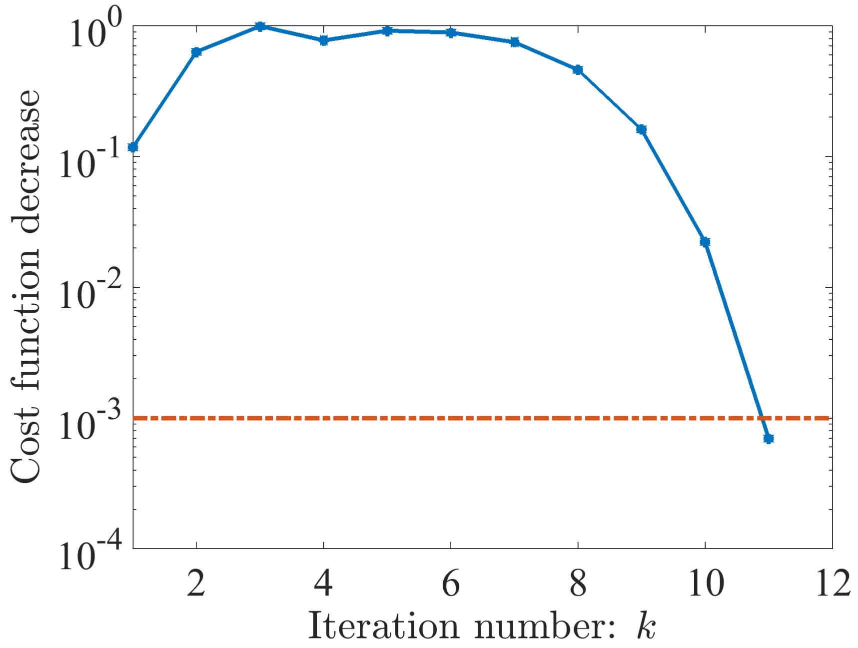
- **Convergence**
  - Stopping criteria
    - Step Length

$$\lambda^{(k)} = \arg\min \mathcal{C}(\mathbf{a}^{(k)} + \lambda \Delta \mathbf{a}^{(k)})$$

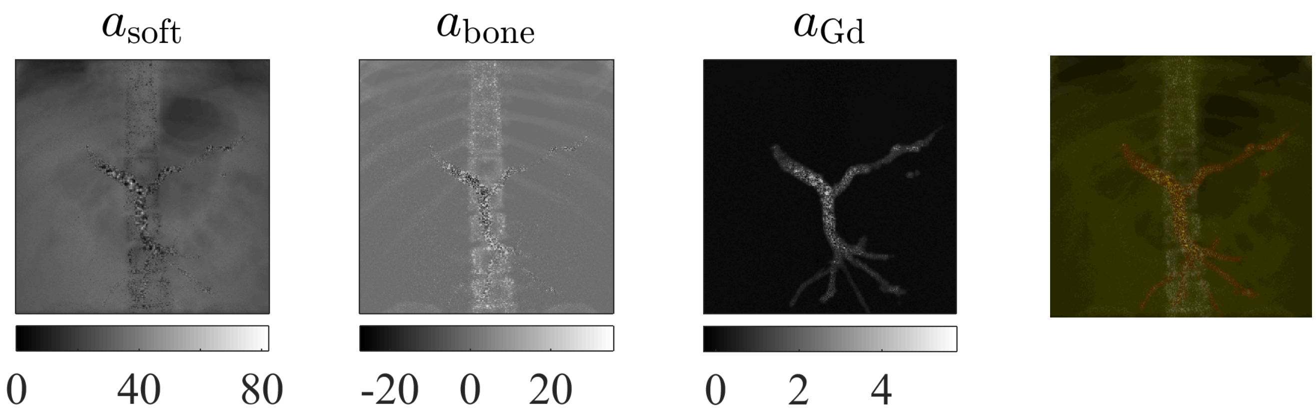
    - Cost function decrease

$$\frac{\mathcal{C}^{(k-1)} - \mathcal{C}^{(k)}}{\mathcal{C}^{(k-1)}}$$





➢ Choice of the regularization parameter



error : $$\sum_m \frac{\|a_m - a_m^{\text{true}}\|_2}{\|a_m^{\text{true}}\|_2}$$