Freeform imaging from Hadamard matrices

Nicolas Ducros[1, 2]

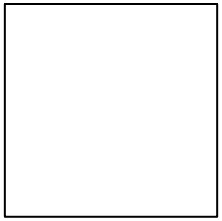
[1]CREATIS, Univ Lyon, INSA-Lyon, UCB Lyon 1,CNRS, Inserm, CREATIS UMR 5220, U1206, Lyon, France

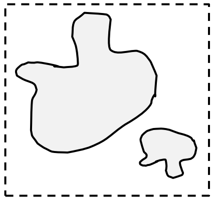
[2]IUF, Institut Universitaire de France

Joint work with J Cohen and L Mahieu-Williame

Outline

1. Introduction

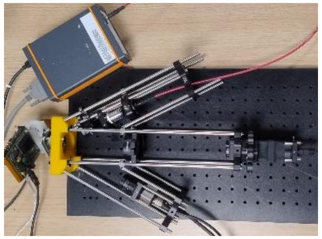




2. Freeform imaging

3. Multiplex (or Fellgett's) advantage

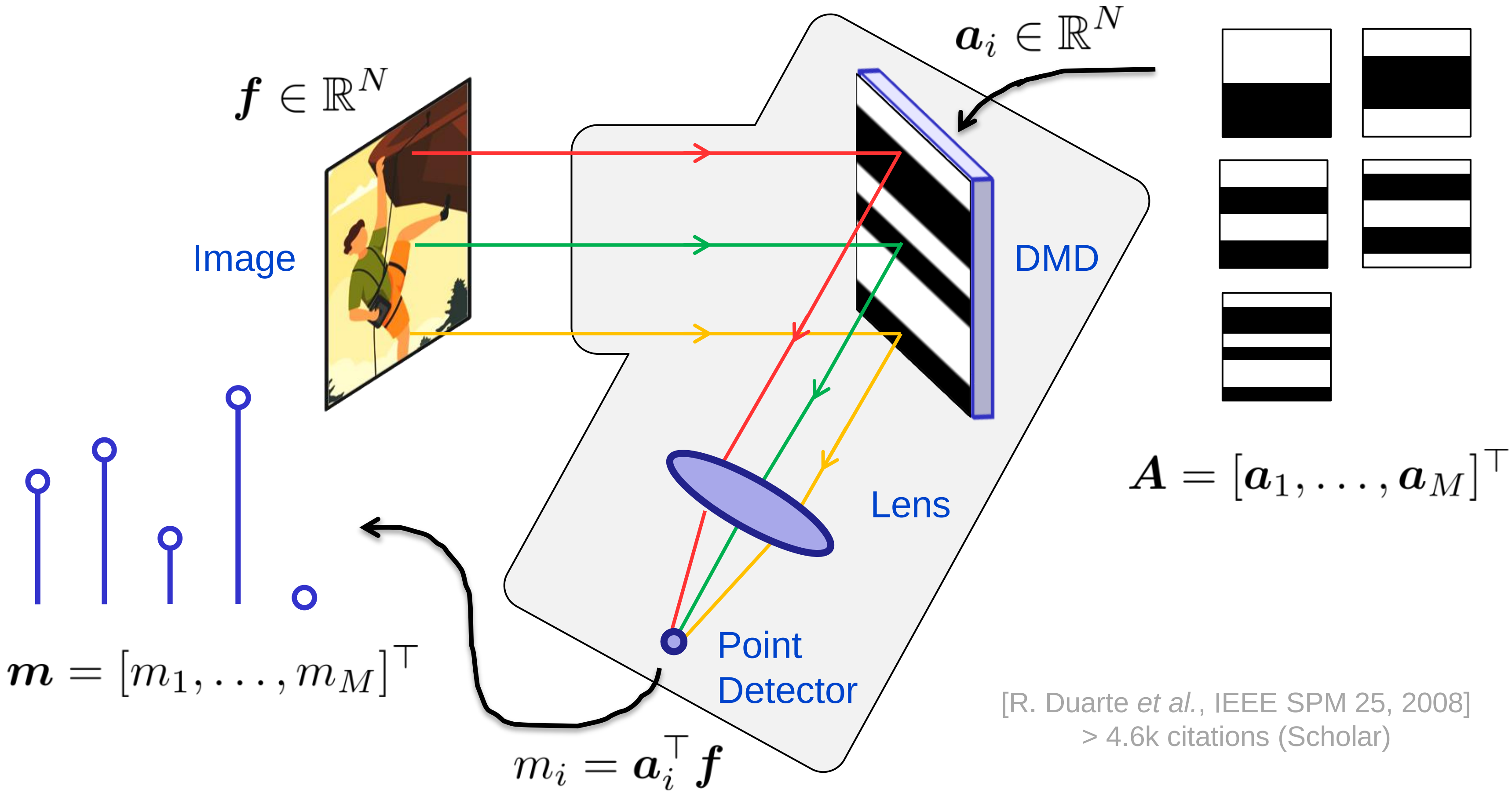
$$\mathbb{M}\mathsf{se}\left(\mathbf{f}^*\right) \approx \frac{1}{t}[2Q\bar{f} + 4M f_{\mathrm{ref}}]$$



4. Experimental results

1.
Introduction

Single-pixel imaging (Duarte *et al.*, 2008)



[R. Duarte *et al.*, IEEE SPM 25, 2008]
> 4.6k citations (Scholar)

Acquisition

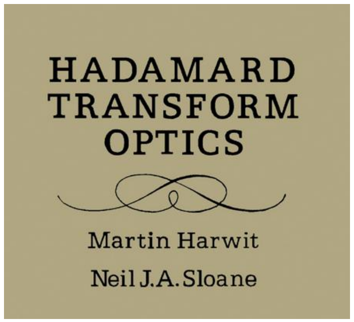
$$\boldsymbol{m} = \boldsymbol{A}\boldsymbol{f}$$

$\boldsymbol{A} \in \mathbb{R}^{M \times N}$

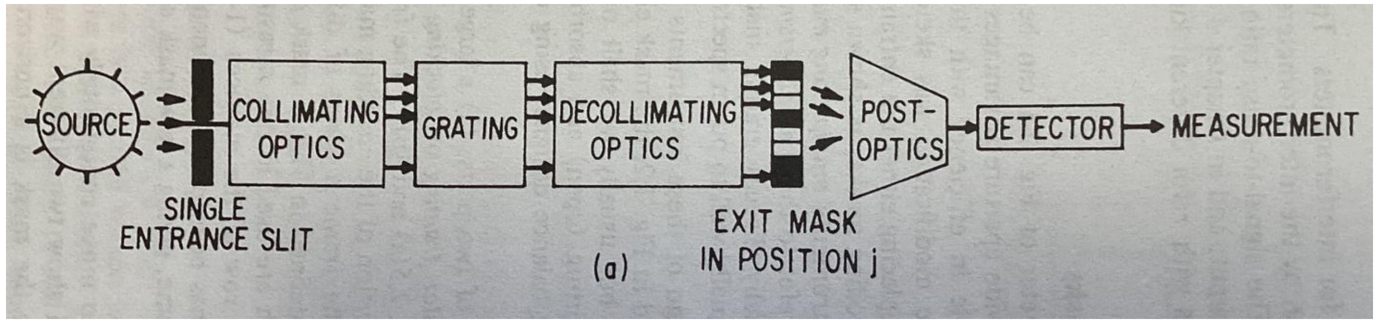
Reconstruction (*M* << *N*)

$$\boldsymbol{f}^* \leftarrow \min_{\boldsymbol{f}} \|\boldsymbol{m} - \boldsymbol{A}\boldsymbol{f}\|_2^2 + \lambda\|\boldsymbol{\Phi}\boldsymbol{f}\|_1$$

Hadamard spectroscopy (from the 70's)



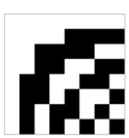
[M. Harwit and N Sloane, Academic Press, 1979]



*'(…) conventional spectrometer is modified by using a **mask to encode the light** at the output'*

Hadamard

$$\boldsymbol{A} = \boldsymbol{H} \in \{-1, 1\}^{N \times N}$$



$$\boldsymbol{H}^\top \boldsymbol{H} = N \boldsymbol{I}_N$$

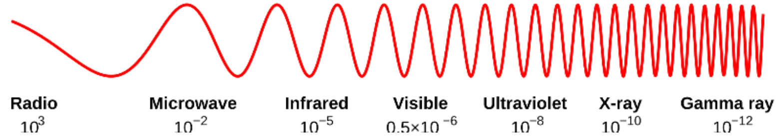
L2 reconstruction

$$\boldsymbol{f}^* = \frac{1}{N} \boldsymbol{H}^\top \boldsymbol{m}$$

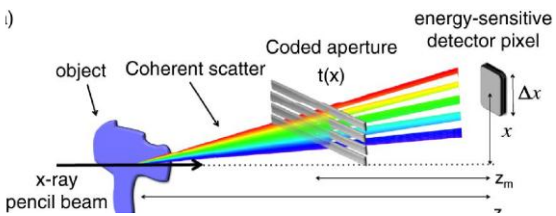
Fellgett's advantage

$$\mathrm{var}\,(f_n^*) = \frac{1}{N}\sigma^2 < \sigma^2$$

Applications

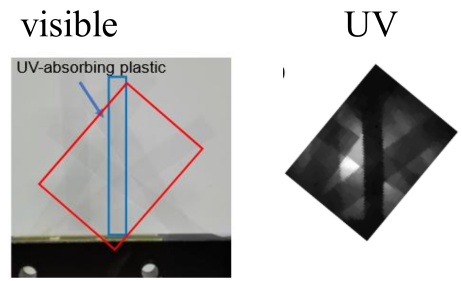


X rays



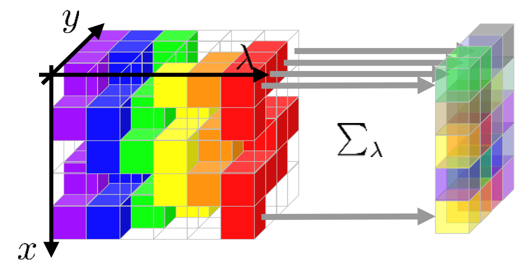
[J. Greenberg *et al.*, Optics letters 39, 2014]

Ultraviolet



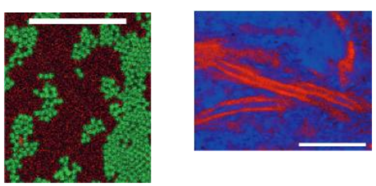
[J. T. Ye *et al.*, Appl. Phys. Lett. 123, 2023]

Hyperspectral



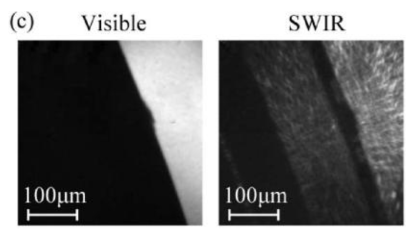
[G Beneti Martins et al., Opt. Express, 2023]
[E Hemsley *et al.*, JOSAA 37 (12), 2020]

Raman imaging



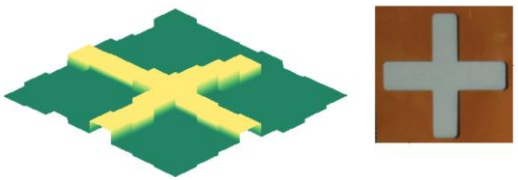
[F Soldevilla *et al.*, Optica 6(3), 2019]
[Scotte *et al.*, Jphys Photonics 5(3), 2023]

Infrared



[N. Radwell *et al.*, Optica 1(5), 2014]

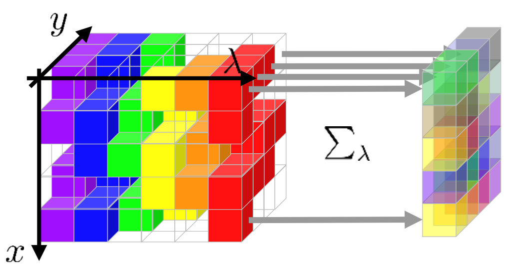
Terahertz imaging



[C. Watts *et al.*, Nature Photonics 8(8), 2014]

Current projects in Lyon

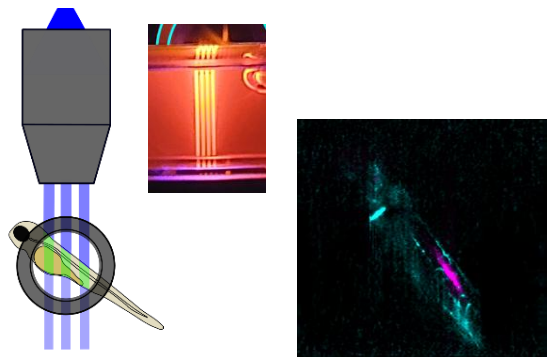
Hyperspectral



200+ hypercubes in open access
https://pilot-warehouse.creatis.insa-lyon.fr/

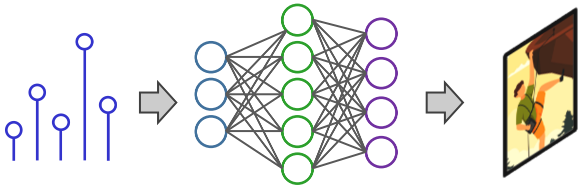
[G Beneti Martins *et al.*, Opt. Express, 2023]

Fluorescence light sheet



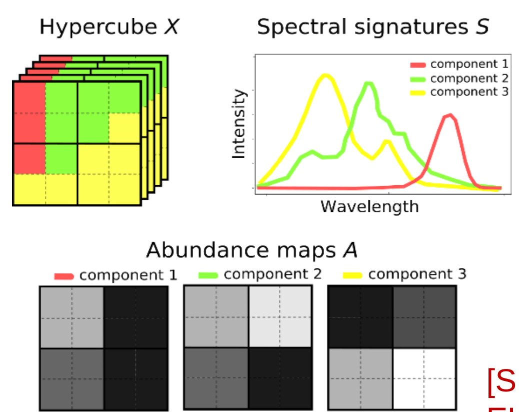
[S Crombez *et al.*, Opt. Express, 2025]

DL-based reconstruction



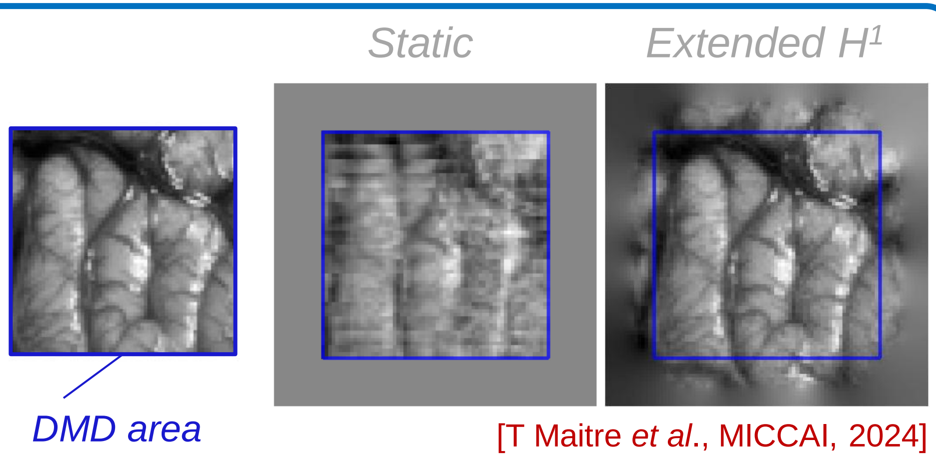
[JFJ. Abascal *et al.*, Opt. Express, 2025]

Spectral unmixing



[S Hariga *et al.*, EUSIPCO, 2024]

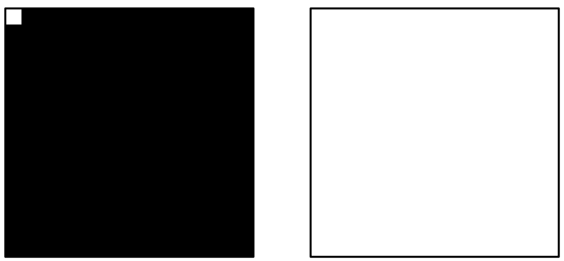
Motion-compensated imaging



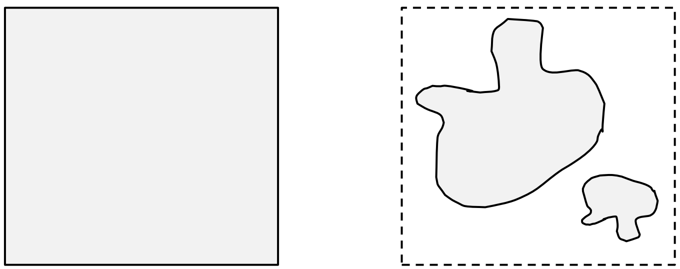
[T Maitre *et al.*, MICCAI, 2024]

Two simple ideas

Idea#1: Hadamard modulation to reduce noise



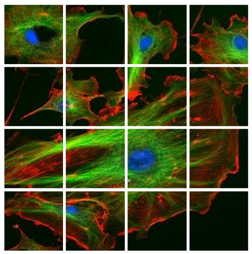
Idea#2: Freeform to capture only relevant information



Easily combined to leverage both advantages

2.
Freeform Imaging

Motivation and definition

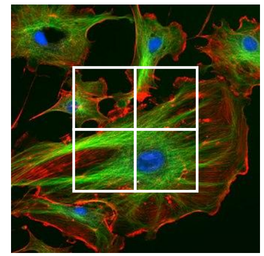


4x4 image

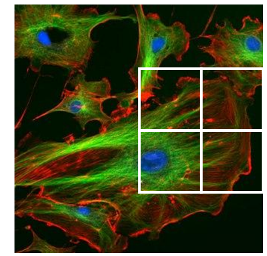
Given a time budget

→

Fewer pixels = reduced noise

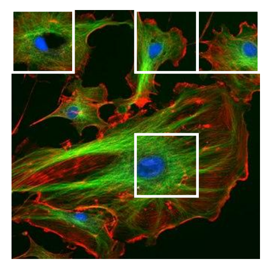


2x2 image



2x2 image

Why not?



4-pixel image

→ **Freeform imaging**

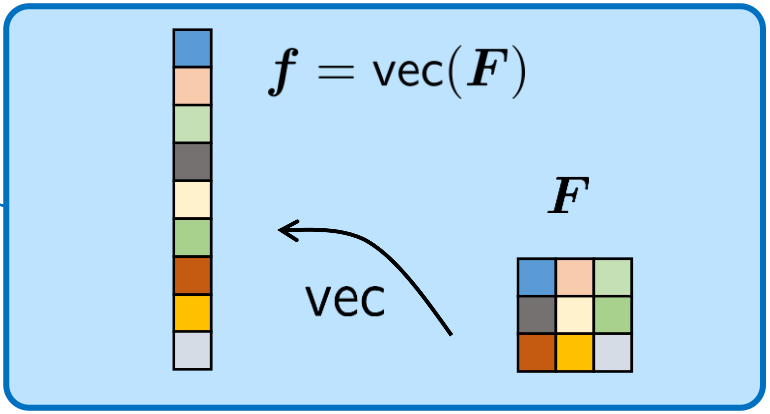
= Capturing an arbitrary pixel subset within the FOV.

https://commons.wikimedia.org/wiki/File:FluorescentCells.jpg

How can freeform imaging be achieved?

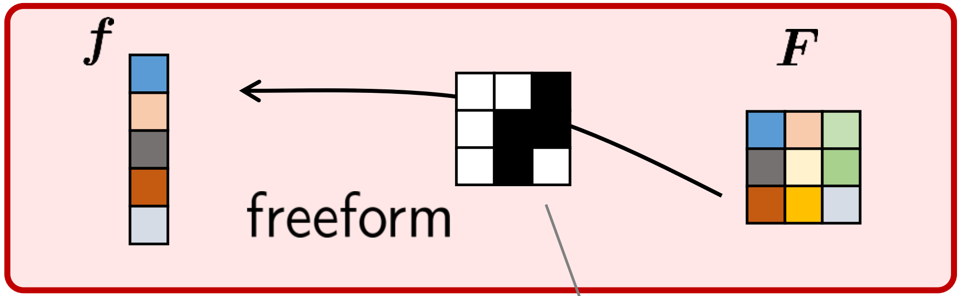
$$\boldsymbol{m} = \boldsymbol{A}\boldsymbol{f}$$

Full (*P* pixels) Here: *P* = 9 pixels



P: Total number of pixel within the full FOV

Freeform (*N* pixels) Here: *N* = 5 pixels

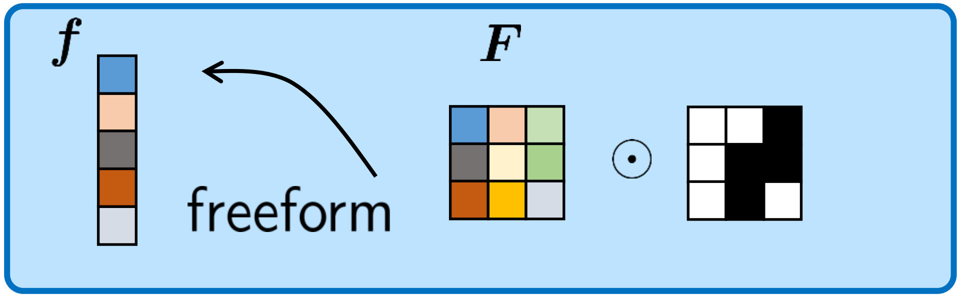


N: Number of pixels within the freeform region

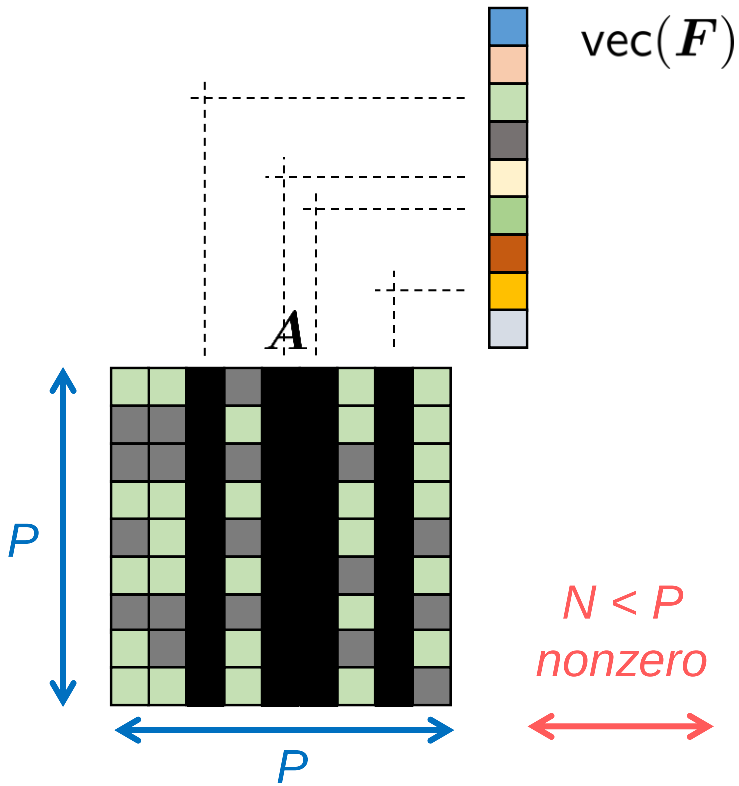
Mask indicating the pixels within the freeform region (white = in; black = out)

How can freeform imaging be achieved?

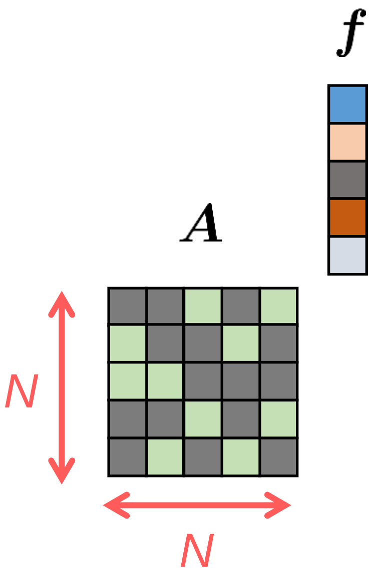
$$\boldsymbol{m} = \boldsymbol{A}\boldsymbol{f}$$



1. Masked matrices



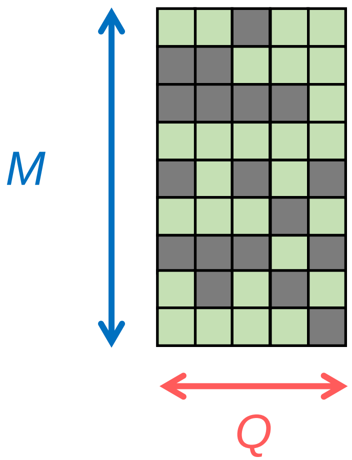
2. Non imaging matrices



3.
Fellgett's advantage in freeform imaging

Column-decimated Hadamard imaging

Acquisition matrix $M \geq Q$



M measurements
for *Q* pixels

$$\boldsymbol{A}^\top \boldsymbol{A} = K \boldsymbol{I}_Q$$

Mean Squared Error* (MSE)

$$\mathbb{M}\mathrm{se}\,(\mathbf{f}^*) \approx \frac{1}{t}\left[2Q\bar{f} + 4M\, f_{\mathrm{ref}}\right]$$

M* ≥ *Q

Reference flux
(depends only on
experimental parameters)

Number of pixels

Number of
measurements

Image mean flux
(only depends on the scene)

*Assumes:

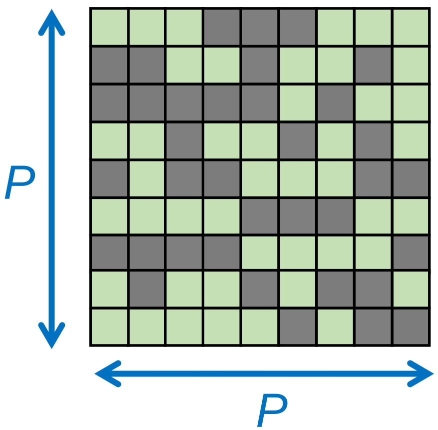
- Poisson-Gaussian noise [EMVA standard, 2021]
- Hadamard = negative part/S matrix
- Pseudoinverse reconstruction

Freeform MSE

#px in freeform region — #measurements

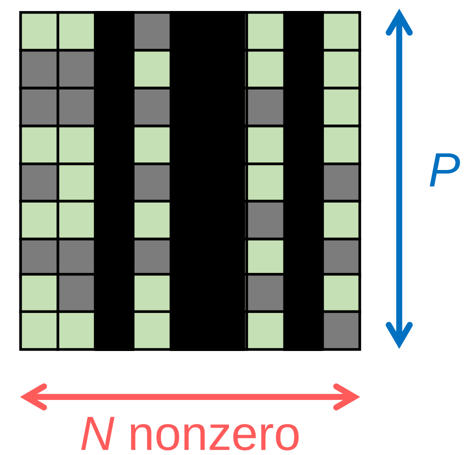
$$\mathbb{M}\text{se}\,(\mathbf{f}^*) \approx \frac{1}{t}[2Q\bar{f} + 4M f_{\text{ref}}], \quad M \geq Q$$

Full



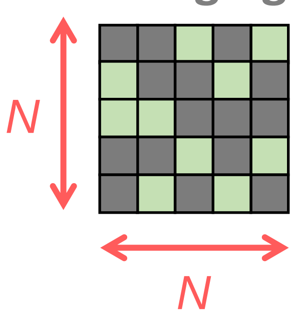
$$\mathbb{M}\text{se}\,(\mathbf{f}^*) \approx \frac{1}{t}[2P\bar{f} + 4P f_{\text{ref}}],$$

Masked



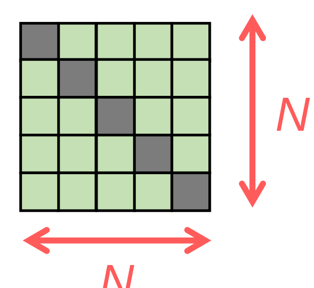
$$\mathbb{M}\text{se}\,(\mathbf{f}^*) \approx \frac{1}{t}[2N\bar{f} + 4P f_{\text{ref}}],$$

Non imaging



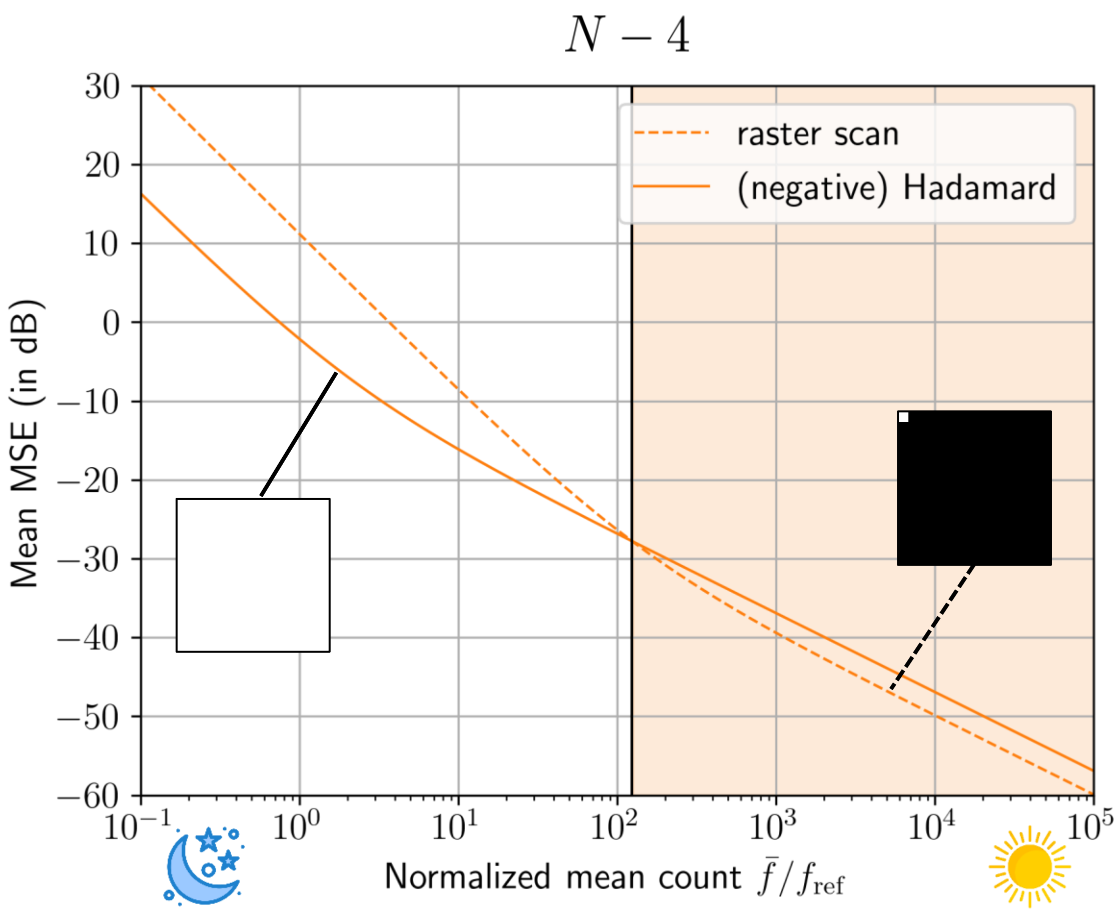
$$\mathbb{M}\text{se}\,(\mathbf{f}^*) \approx \frac{1}{t}[2N\bar{f} + 4N f_{\text{ref}}],$$

Raster Scan



$$\mathbb{M}\text{se}\,(\mathbf{f}^*) \approx \frac{1}{t}[N\bar{f} + N^2 f_{\text{ref}}],$$

Low flux increases Fellgett's advantage



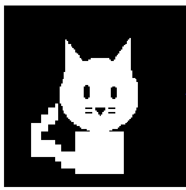
→ **Hadamard** multiplexing is more effective in **low-light** conditions.

4.

Experimental Results

Acquisition device

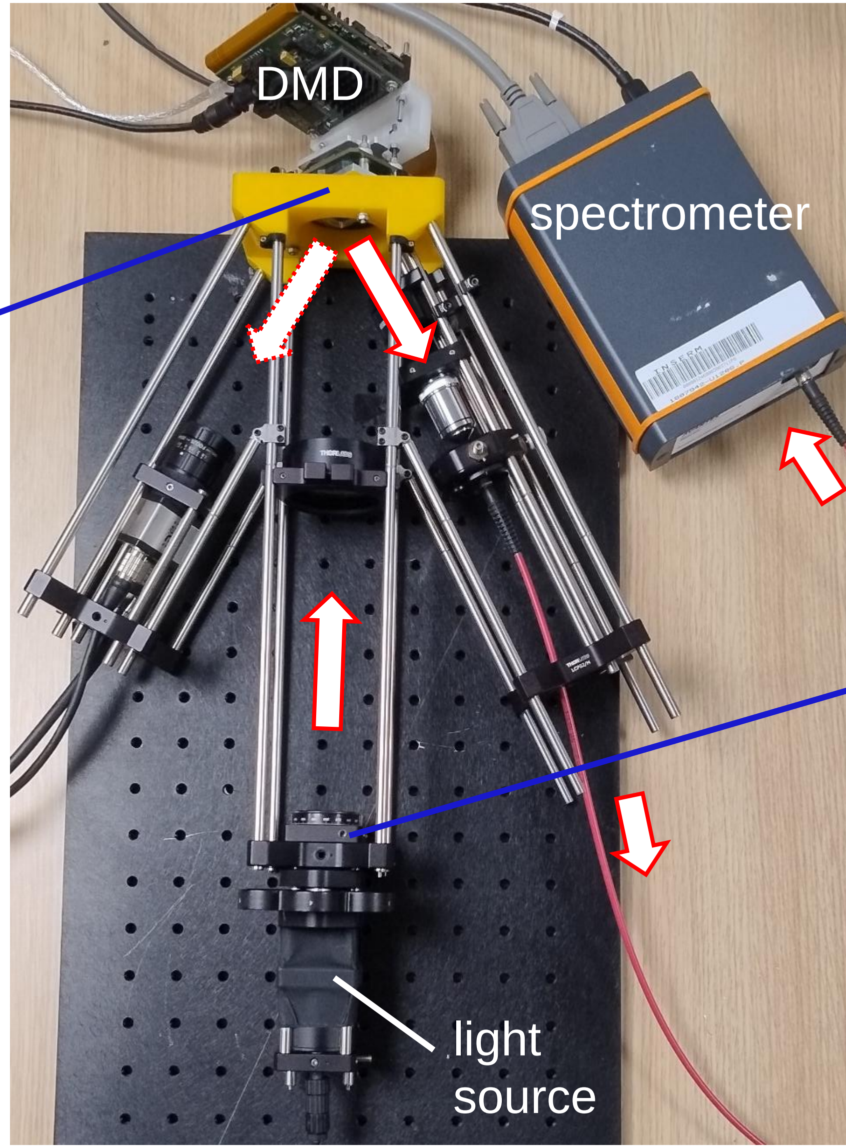
Freeform mask



total #px

$$\frac{P}{N} = 4$$

#px in the freeform region

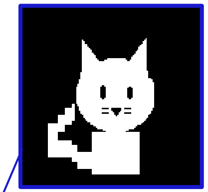


Siemens star resolution target

Freeform image at high flux

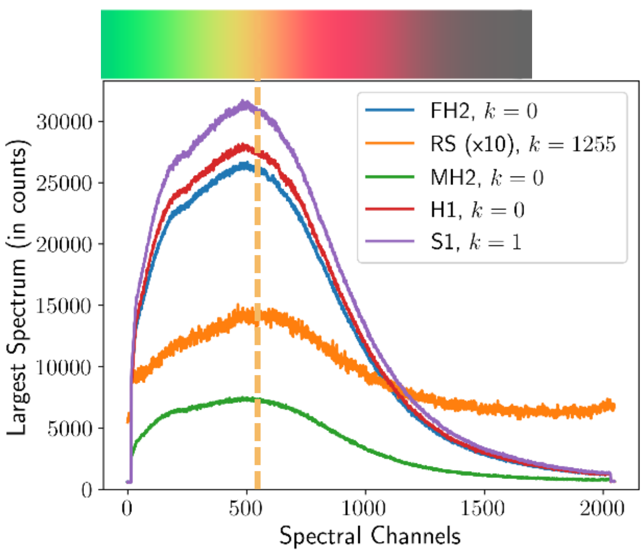
Siemens star resolution target

Freeform region

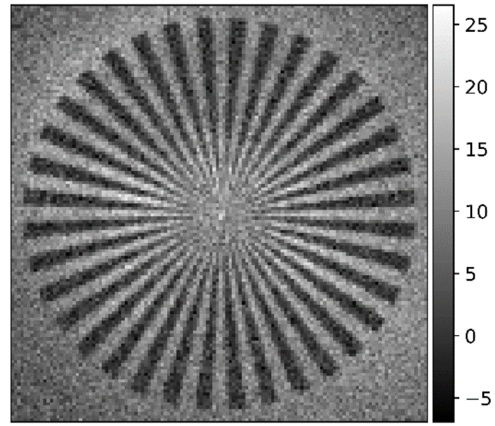


DMD area

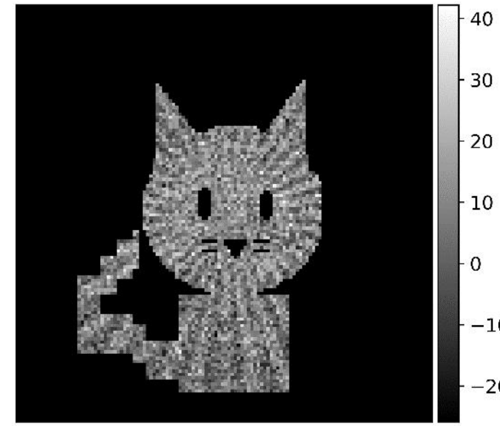
581 nm



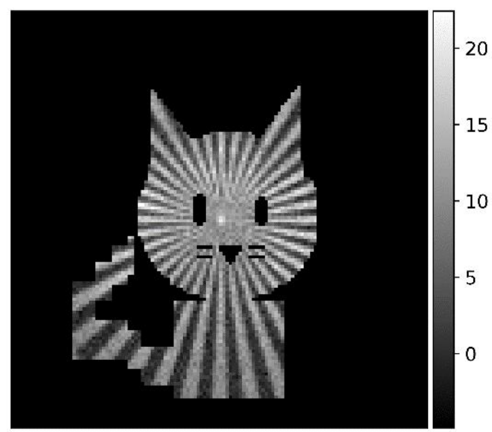
Full



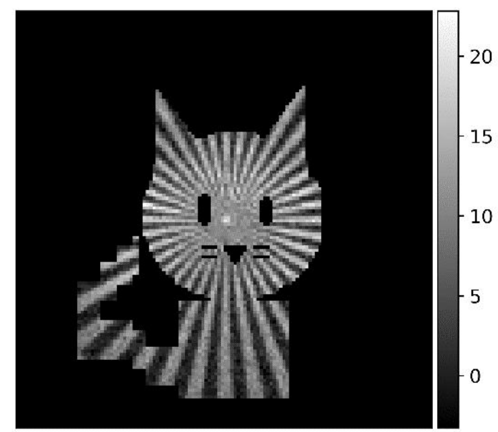
Raster Scan



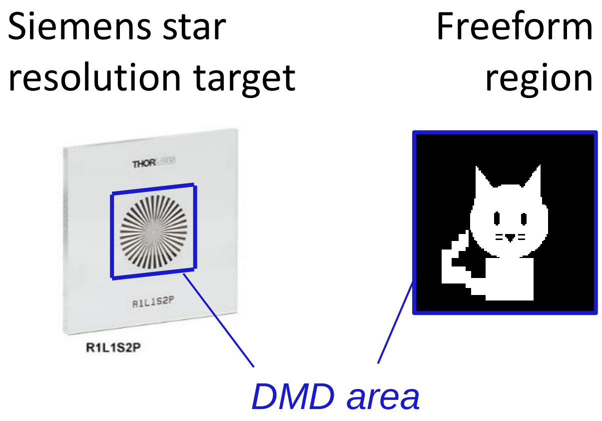
Masked

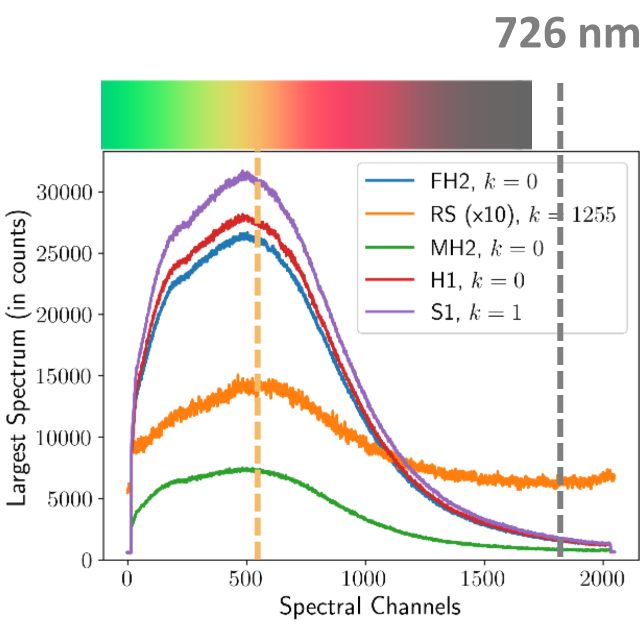


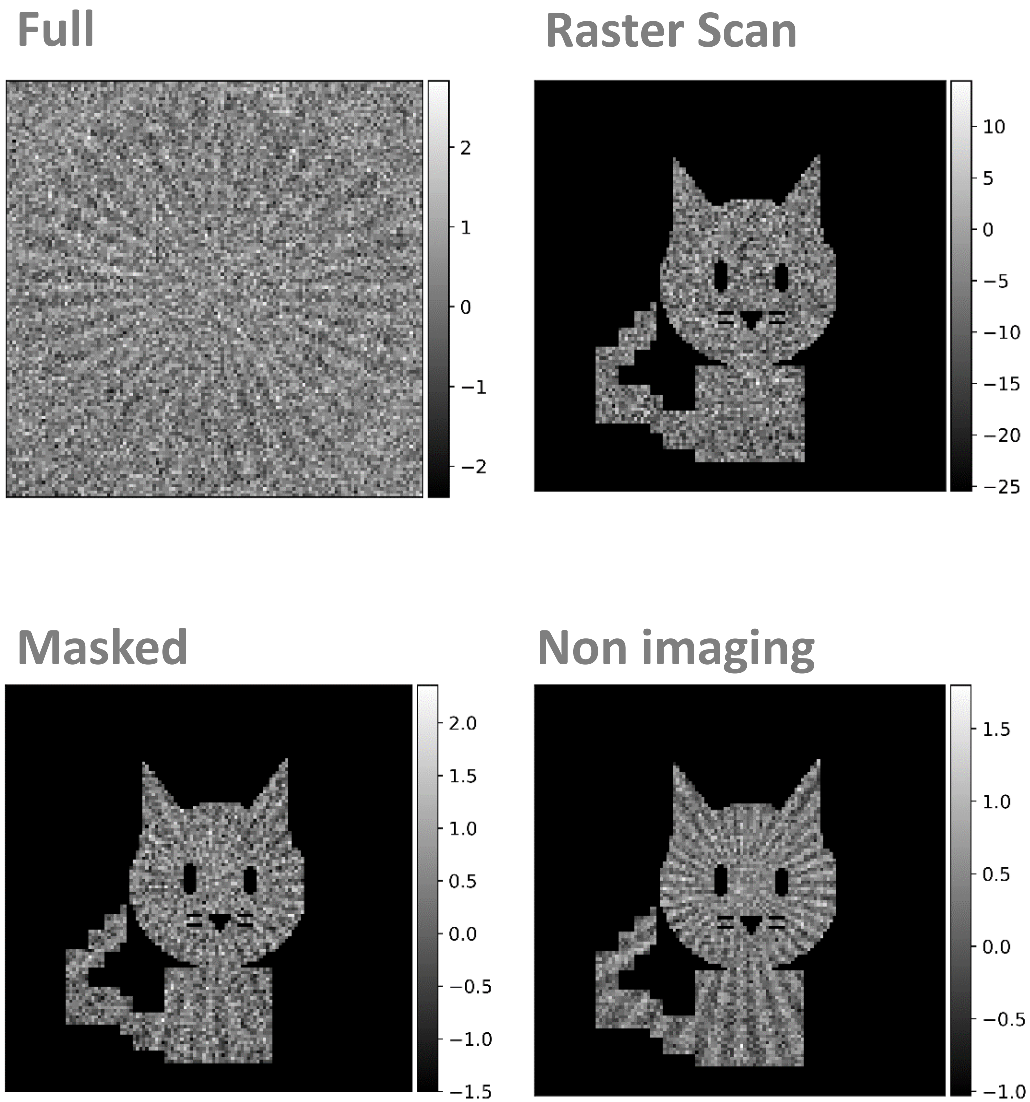
Non imaging



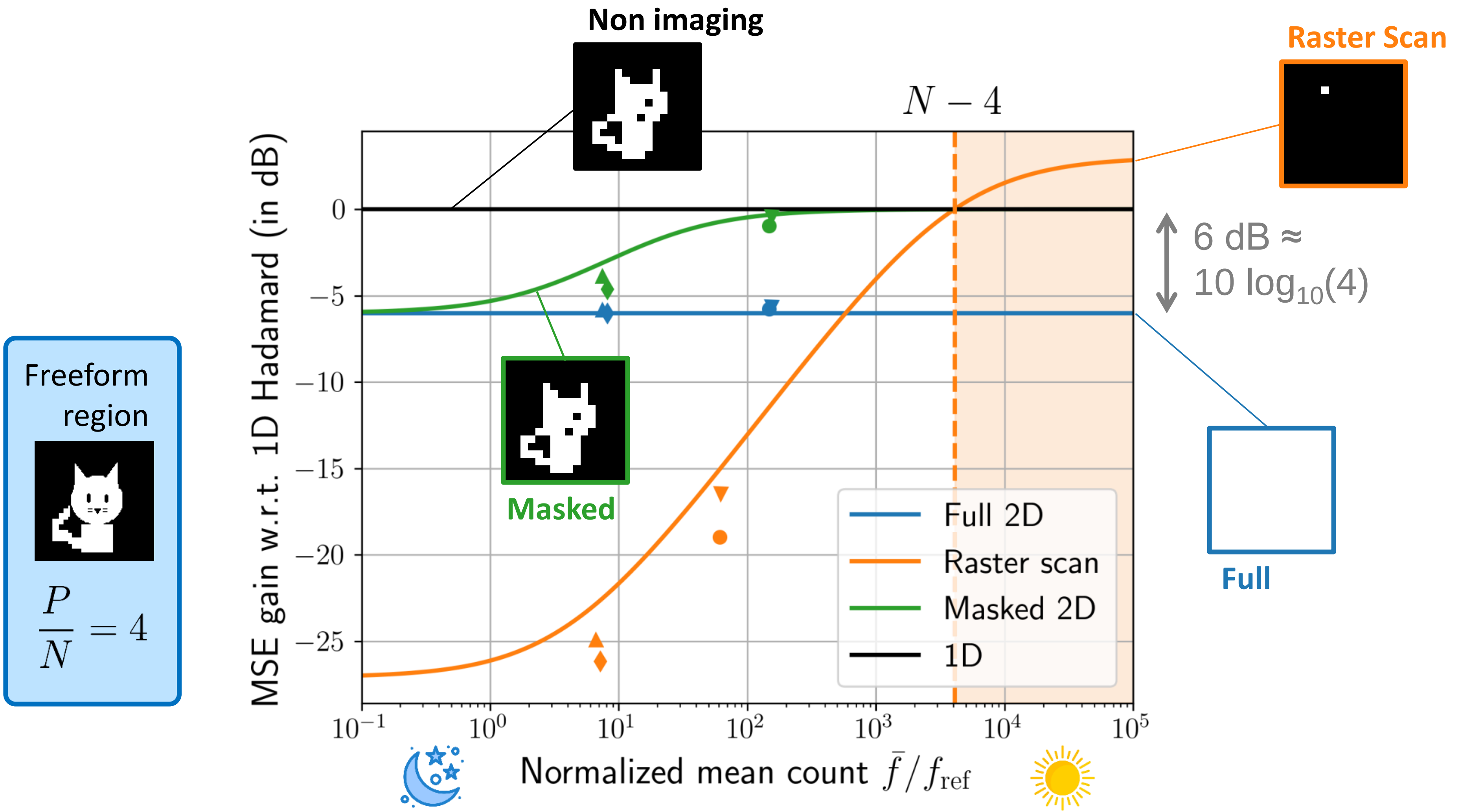
Freeform image at low flux







MSE: theory vs experiment



Note: The figure above was obtained using a split Hadamard matrix, not a negative Hadamard matrix, as the acquisition matrix.

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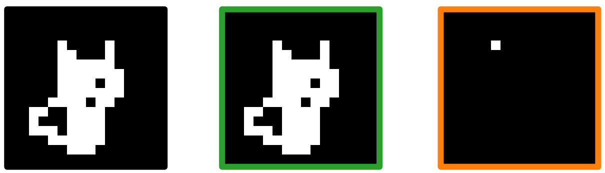
CREATIS

1. Freeform imaging



Non Imaging | Masked | Raster Scan

→ The 'non imaging' Hadamard matrix should be preferred to the masked one

2. We have characterize the MSE of Hadamard freeform imaging

→ **Freeform** imaging **outperforms** raster scan in **low/intermediate light** conditions

3. We have validated our results **experimentally** in hyperspectral imaging in the visible range

HAL archives-ouvertes.fr

"Freeform Hadamard imaging: Back to the roots of computational optics"
https://hal.science/hal-05337760v1

https://github.com/openspyrit/spyrit-examples/tree/design/2025_freeform

GitHub