## Diffusion models

#### Olivier Bernard







#### What is the purpose of diffusion models?

- Best current methods for synthetic image generation
- Allows generating images in a conditioned form
- Many software solutions, such as Midjourney, DALL-E

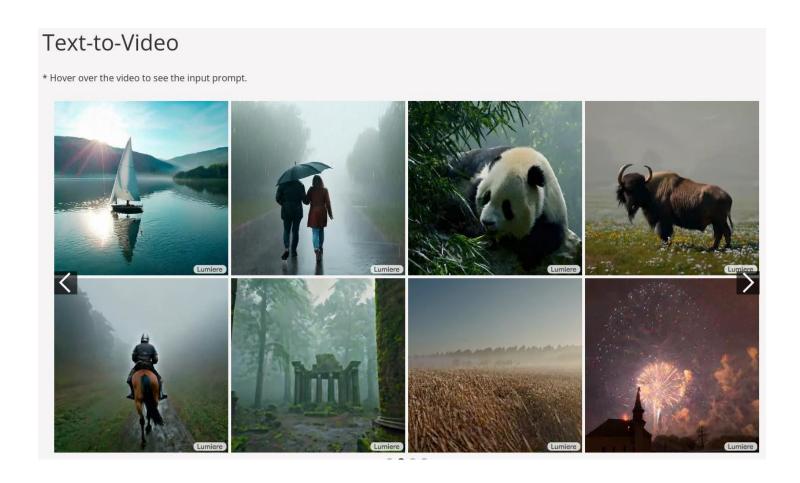
An Asian girl in ancient coarse linen clothes rides a giant panda and carries a wooden cage. A chubby little girl with two buns walks on the snow. High-precision clothing texture, real tactile skin, foggy white tone, low saturation, retro film texture, tranquil atmosphere, minimalism, long-range view, telephoto lens



#### What is the purpose of diffusion models?

Recent extensions for video synthesis

https://lumiere-video.github.io/#section\_image\_to\_video



#### What is the purpose of diffusion models?

► Family of diffusion networks

Denoising Diffusion Probabilistic models

Score-based methods

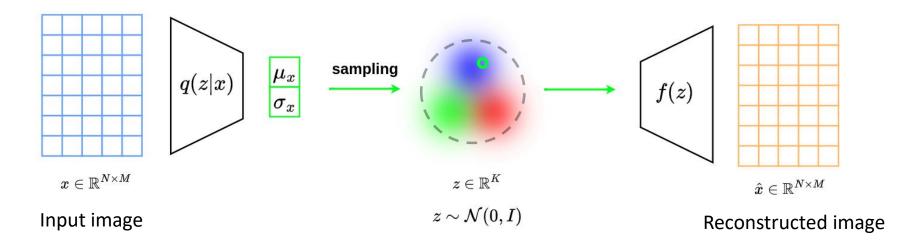
Normalizing flow methods

## Intuition behind diffusion models

Interpretation of the loss function

$$ext{loss} = D_{\mathit{KL}}\left(\mathcal{N}\left(g(x), diag\left(h(x)
ight)
ight), \mathcal{N}\left(0, I
ight)
ight) \, + \, lpha \|x - f(z)\|^2$$

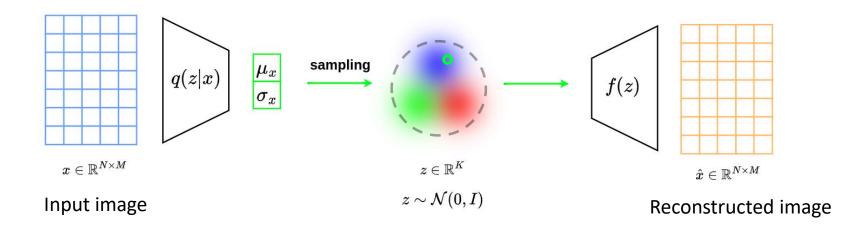
- $\rightarrow \mathcal{N}(g(x), diag(h(x)))$  imposes a local *continuity* constraint
- $\rightarrow$   $D_{KL}(\cdot, \mathcal{N}(0, I))$  imposes a global *completeness* constraint



Completeness is expressed as a soft constraint!

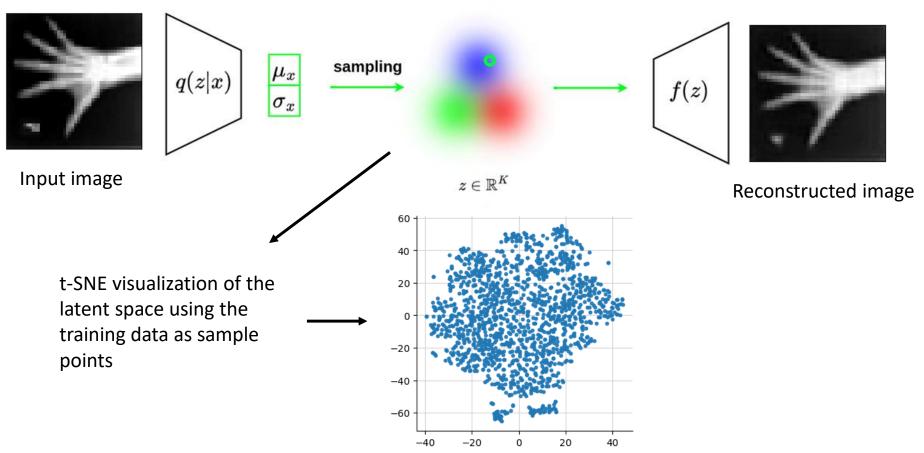
$$ext{loss} = D_{KL}\left(\mathcal{N}\left(g(x), diag\left(h(x)
ight)
ight), \mathcal{N}\left(0, I
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ight) \,+\, lpha \|x - f(z)\|^2$$

 $ightarrow \mathcal{N}\left(g(x),diag(h(x))
ight)$  and  $\mathcal{N}\left(0,I\right)$  should remain close in terms of distributional distance

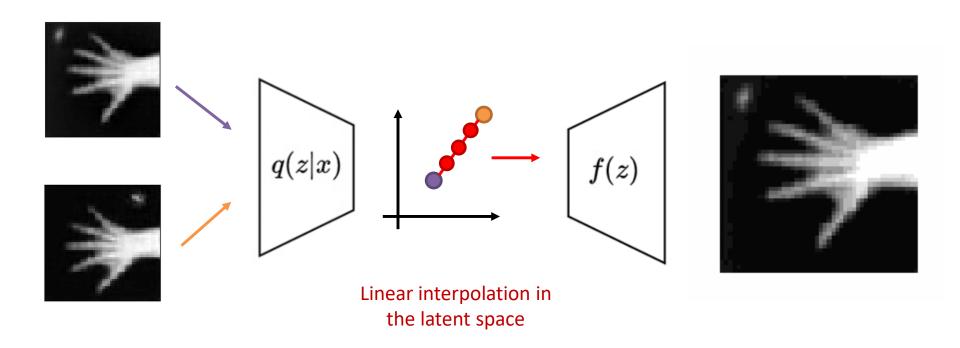


Sampling from the latent space  $\mathcal{N}(0, I)$  does not guarantee to obtain a reconstructed image from the target distribution

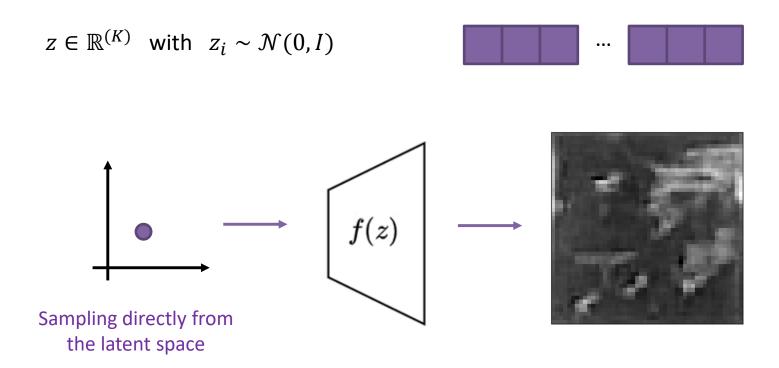
- Illustration from Mednist dataset
  - (train,valid,test) = (1491,373,223)
  - Input image size: 48x48 / latent space K=432 (compression factor around 5)



Linear interpolation between two real images



Sampling directly from the latent space



A soft constraint on the latent space to remain close to  $\mathcal{N}(0,I)$  is not sufficient to build generative models that effectively learn a target distribution

# The denoising diffusion probabilistic models

### **DDPM**

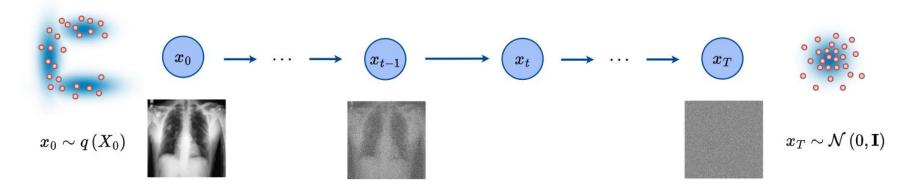
All the mathematics are described in the following blog

https://creatis-myriad.github.io/tutorials/2023-11-30-tutorial-ddpm.html

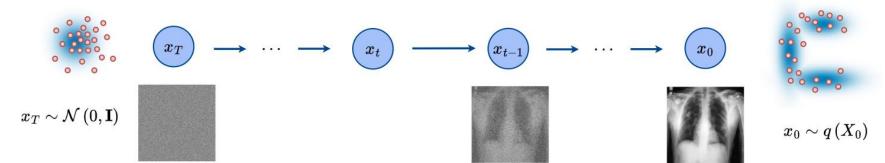
#### Basic idea of denoising diffusion model

How can a hard constraint be enforced to ensure a direct transformation from the latent space (modeled as a Gaussian) to the target distribution?

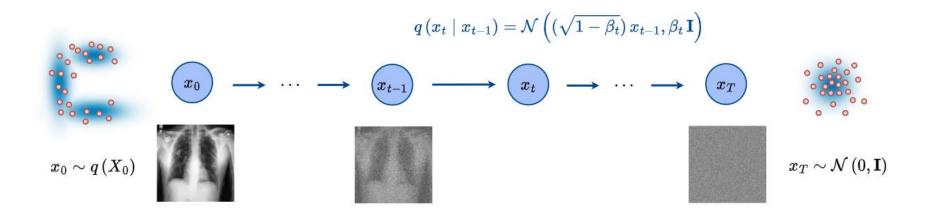
#### Noising process



#### Denoising process



#### Noising process (forward diffusion process)



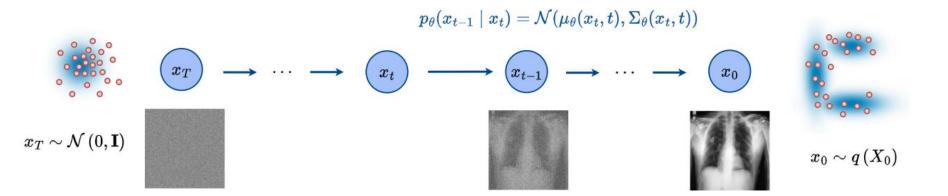
Defined as a sequence of normal distributions

$$q\left(x_{t}\mid x_{t-1}
ight)=\mathcal{N}\left(\left(\sqrt{1-eta_{t}}
ight)x_{t-1},eta_{t}\mathbf{I}
ight)$$

lacktriangle Forward process variances  $eta_1, \cdots$  ,  $eta_T$  with values from 0 to 1

$$egin{array}{ll} ext{if} & eta_t = 0, \quad ext{then} & q(x_t \mid x_{t-1}) = x_{t-1} \ ext{if} & eta_t = 1, \quad ext{then} & q(x_t \mid x_{t-1}) = \mathcal{N}(0, \mathbf{I}) \end{array}$$

#### **Denoising process**



lacktriangle The denoising process  $p_{ heta}$  is learned by the model

$$p_{ heta}(x_{t-1} \mid x_t) = \mathcal{N}(\mu_{ heta}(x_t, t), \Sigma_{ heta}(x_t, t))$$

- ► Knowing  $x_{t-1}$ , we need to predict  $μ_θ$  and  $Σ_θ$ 
  - $\Sigma_{\theta} = \sigma_t^2 I$  with  $\sigma_t = \beta_t$  for simplification purposes
  - Predicting  $\mu_{\theta}$  involves estimating the added noise  $\varepsilon_t$  from  $x_{t-1}$  to  $x_t$

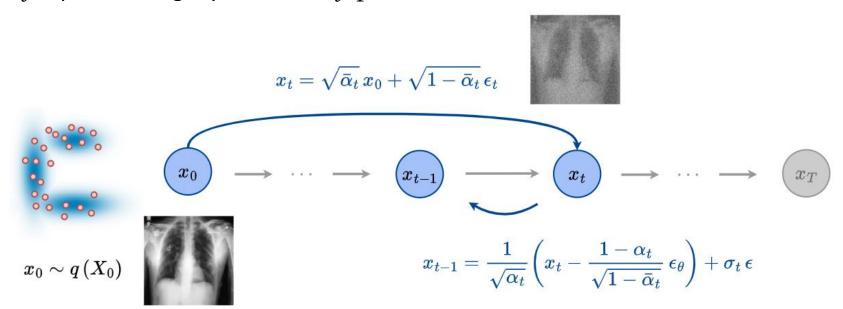
#### Training procedure

- ► Choose a random step  $t \in \{0, \dots, T\}$
- $\triangleright$  Add t steps of noise to our input image  $x_0$ , and obtain a noisy image  $x_t$

$$\left(x_t = \sqrt{ar{lpha}_t}\,x_0 + \sqrt{1-ar{lpha}_t}\,\epsilon_t\,
ight)$$

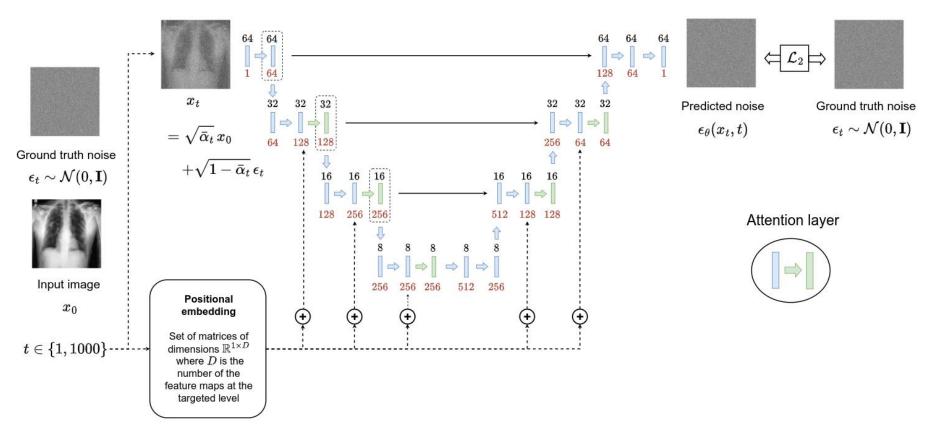
$$\left\{egin{array}{l} lpha_t = 1 - eta_t \ ar{lpha}_t = \prod_{k=1}^t lpha_k \end{array}
ight. \left\{egin{array}{l} \epsilon_t = \mathcal{N}\left(0, \mathbf{I}
ight) \ ext{added noise from } x_{t-1} ext{ to } x_t \end{array}
ight.$$

A U-Net model is trained to predict the noise pattern  $\varepsilon_{\theta}$  that needs to be subtracted to  $x_t$  to predict a slightly denoised  $x_{t-1}$ 



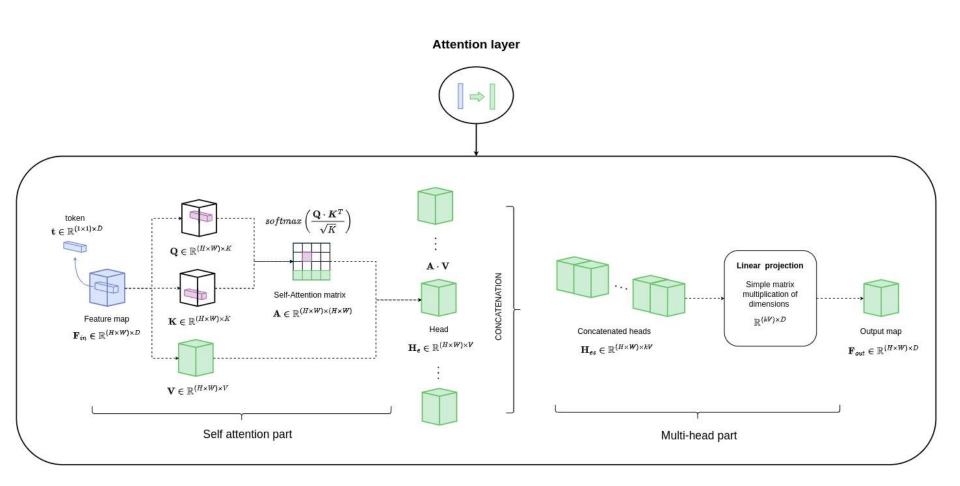
Standard U-Net with attention layers and position encoding to integrate temporal information

 $\rightarrow$  Integration of t is necessary because the added noise varies over time

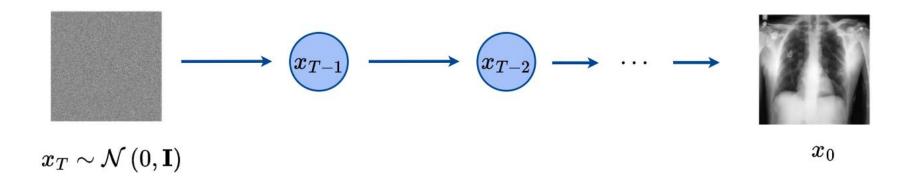


#### Architecture

#### → Attention layer



#### Inference: generation of synthetic data



- ► Generate a random image  $x_T \sim \mathcal{N}(0, I) \in \mathbb{R}^{N \times M}$
- $\blacktriangleright$  At each step from T to 0, use the U-Net model to compute

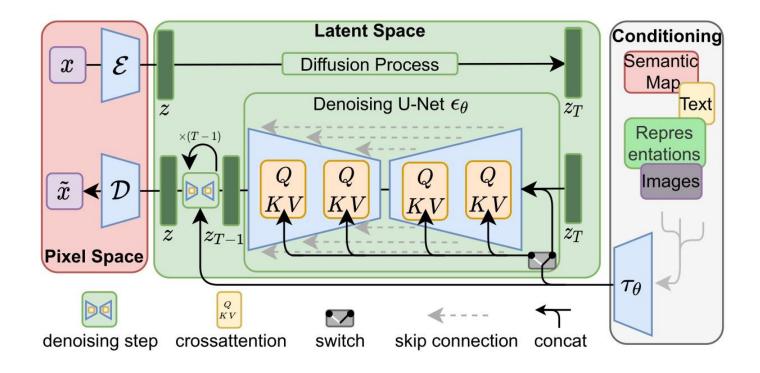
$$x_{t-1} = rac{1}{\sqrt{lpha_t}}igg(x_t - rac{1-lpha_t}{\sqrt{1-arlpha_t}}\,oldsymbol{\epsilon_ heta}(x_t,t)igg) + \sigma_t\,\epsilon$$
 U-Net

with 
$$\epsilon \sim \mathcal{N}\left(0,\mathbf{I}
ight)$$
 and  $\left\{egin{array}{l} lpha_t = 1 - eta_t \ ar{lpha}_t = \prod_{k=1}^t lpha_k \end{array}
ight.$ 

## Practical application

## Latent diffusion models

- VAE is learned independently of DDPM and its architecture is fixed
  - Efficiently reduce the dimensionality of the input space
  - Efficiently initiate the Gaussian diffusion process
- LDM architecture



#### Properties

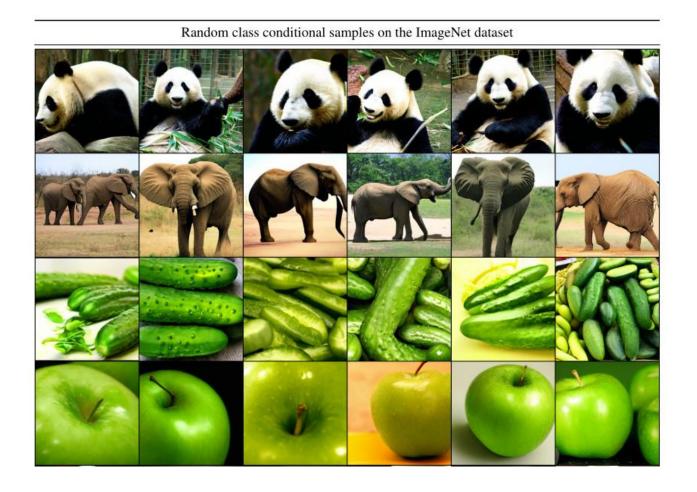
Parameters	LDM – $256 \times 256$
z dimensions	<b>64</b> × <b>64</b> × 3
Diffusion steps	1000
Noise scheduler ( $eta_t$ )	linear
Number of parameters	274 Million
Channels	224
Channel multiplier	1, 2, 3, 4
Levels for attention	2, 3, 4
Number of head	1
Batch size	48
Iterations	<b>410</b> k
Learning rate	9.6 $e^{-5}$

Random generation of synthetic images without conditioning learned from the CelebA-HQ database

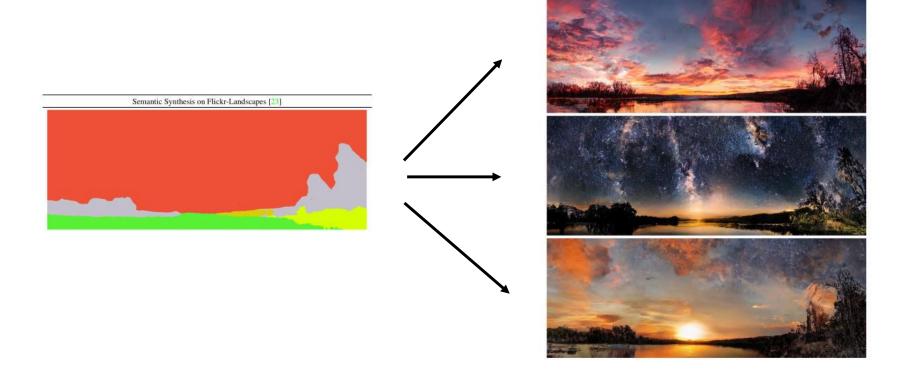
Random samples on the CelebA-HQ dataset



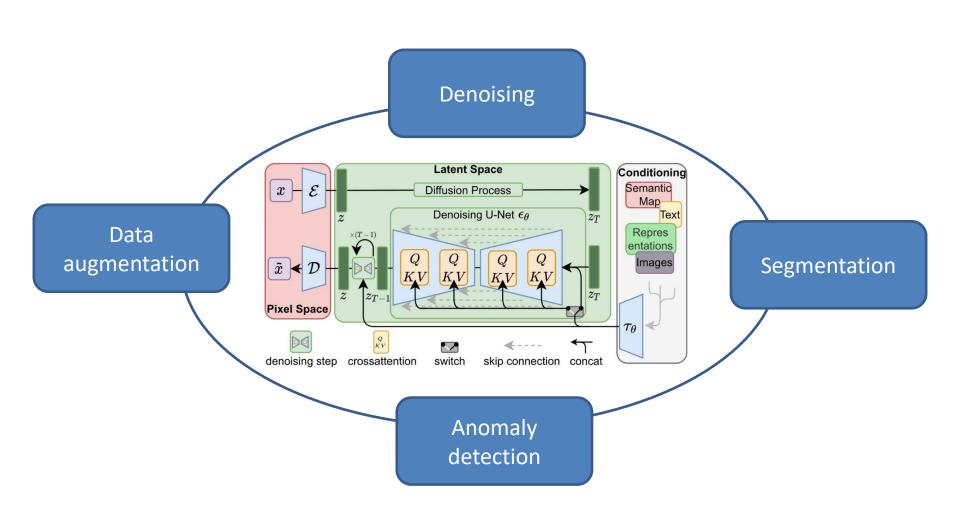
Random generation of synthetic images with conditioning on the class learned from the ImageNet database

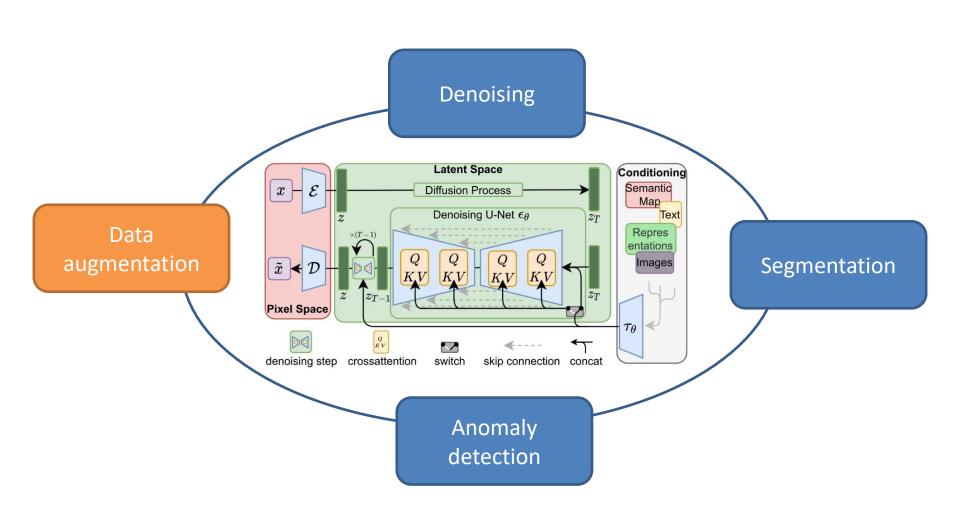


Random generation of synthetic images with conditioning on masks learned from the Flickr-landscapes database

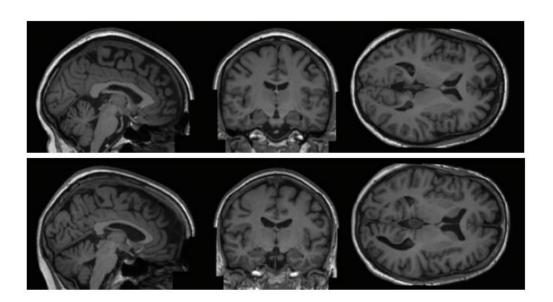


## Medical applications

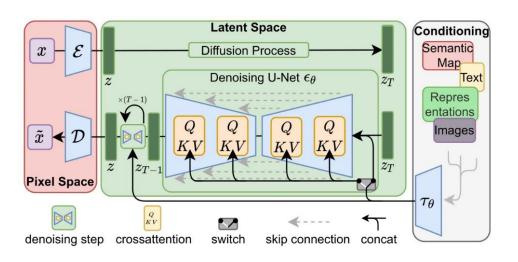




- Synthetic dataset generation for brain MR volumes [Walter et al., MICCAI workshop 2022]
- UK Biobank dataset
  - ➤ 3D MR volumes (T1w)
  - ► Training: 31,740 patients
  - with covariables: age (44 to 82 years), gender (53% women), brain structure volumes
  - Quality of synthetic data measured using FID: Fréchet Inception Distribution

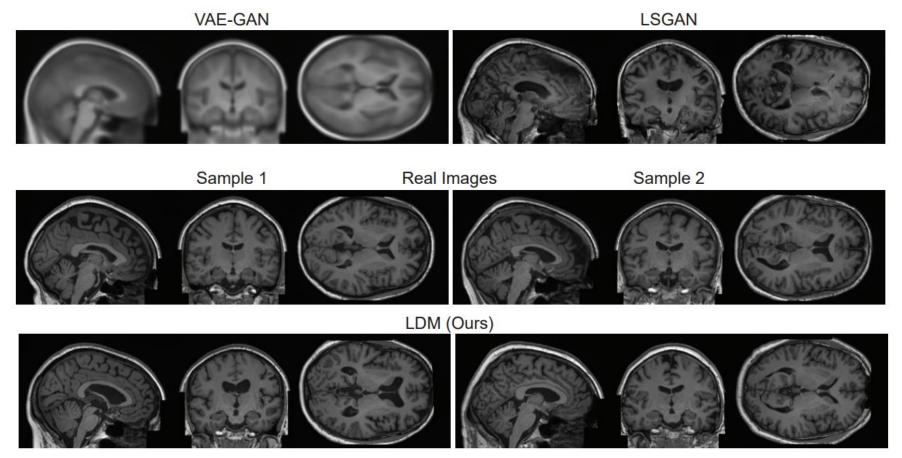


- VAE
  - ➤ 3D convolutions
  - Latent space dimension: 20 x 28 x 20
- DDPM
  - 3D convolutions
  - ► T=1000 time steps
  - Conditioning: vector encoding of each covariable

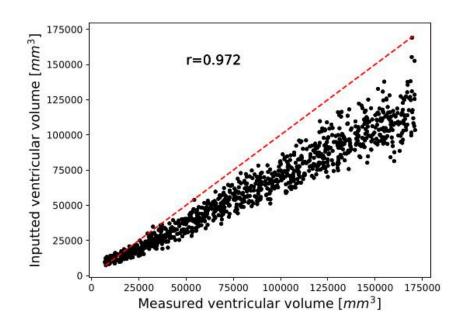


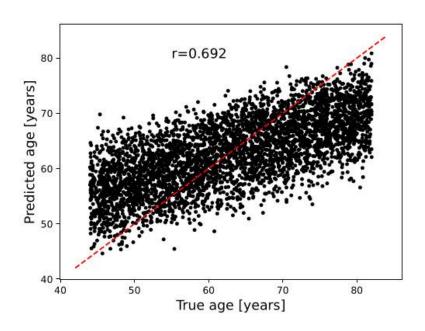
- Results
  - ► FID: generated from 1,000 samples drawn from each of the two distributions to be compared

	$\mathbf{FID}\downarrow$
LSGAN	0.0231
VAE-GAN	0.1576
LDM	0.0076
Real images	0.0005

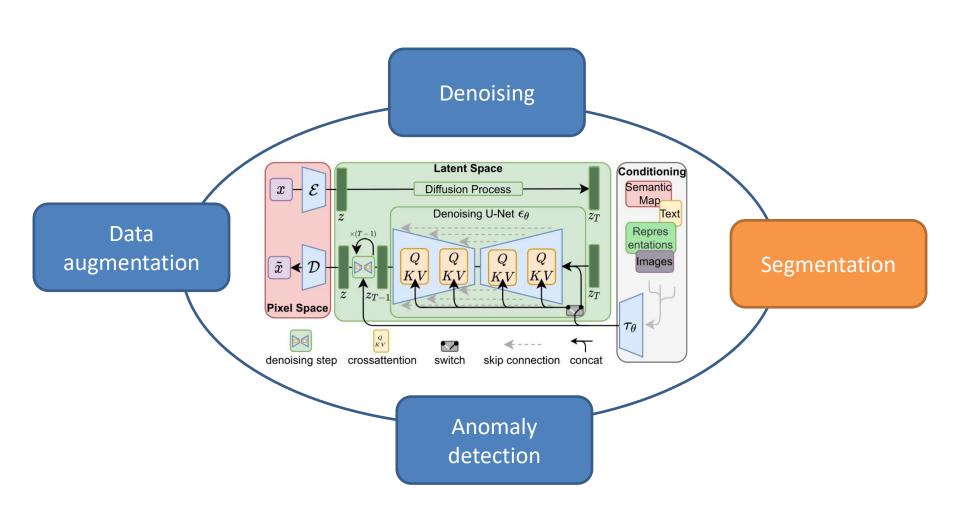


- Results
  - SynthSeg model was used to automatically measure brain volumes from synthetic data
  - ➤ A 3D CNN trained from the UK biobank was used to automatically predict the age from the synthetic data

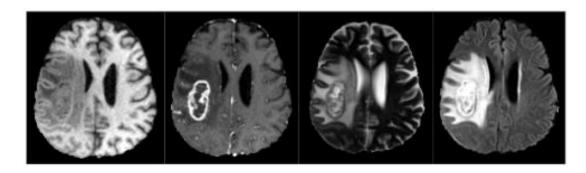




- Synthetic dataset of 100,000 human brain was generated and made publicly available with the conditioning information
- Promote data sharing with privacy guarantees



- Segmentation of tumors from MR images [Wolleb et al., MIDL 2022]
- BRATS2020 dataset
  - ▶ 4 different MR sequences per patient (T1, T2, T1ce, FLAIR)
  - Training: 332 patients with 3D volumes sequences => 16,998 2D images
  - Testing: 37 patients with 3D volumes sequences => 1,082 2D images

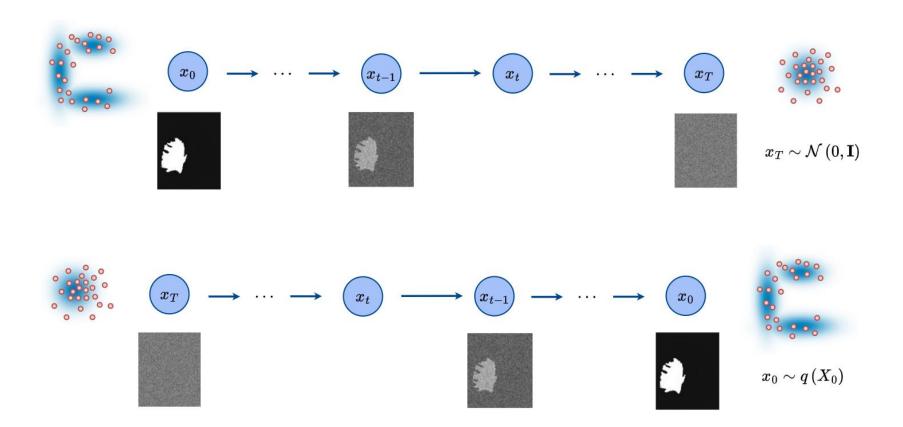


4 MR inputs per patient (T1, T2, T1ec, FLAIR)

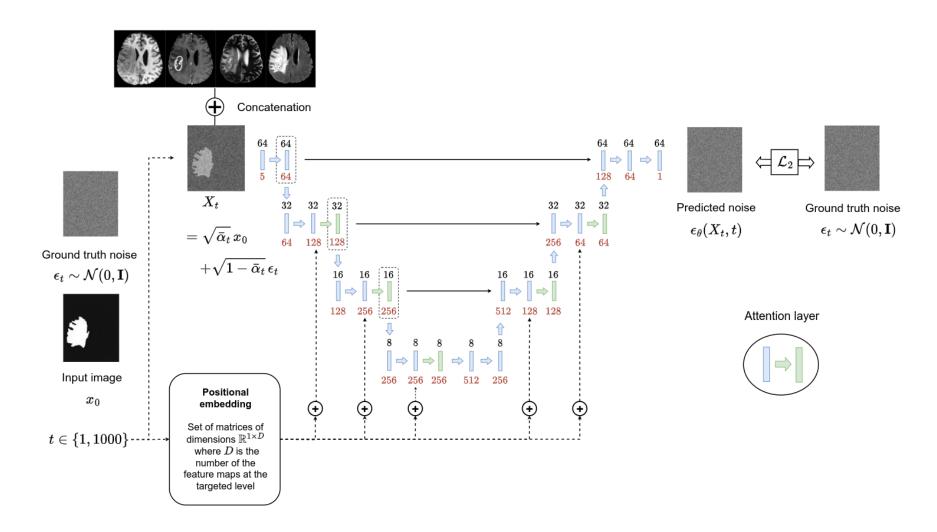


Mask output

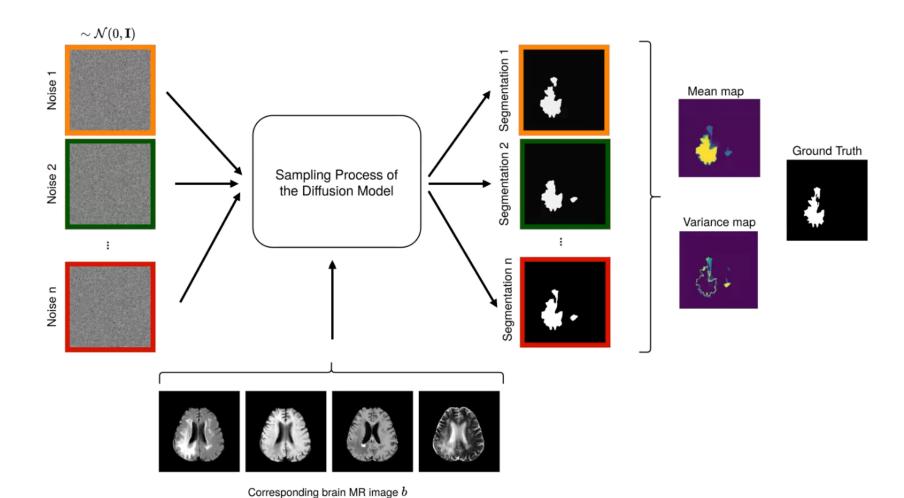
Learn the underlying distribution of tumor segmentation masks



Conditioning with the 4 MR images using concatenation scheme

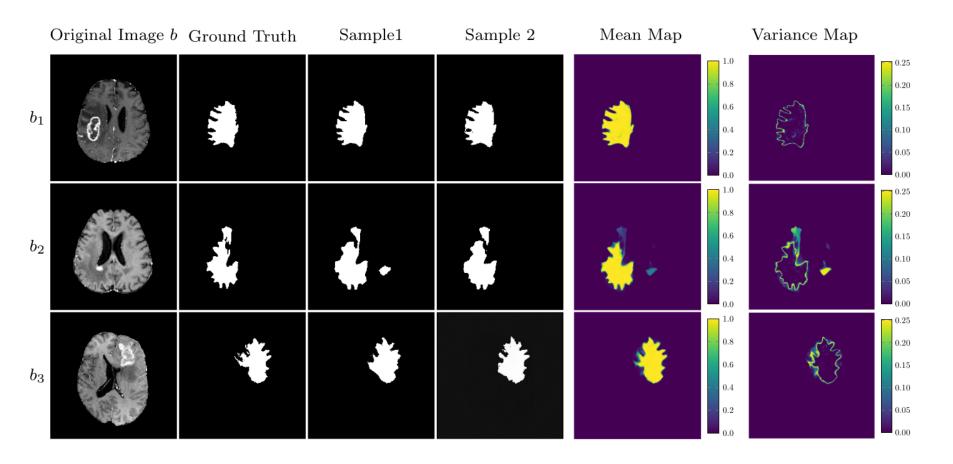


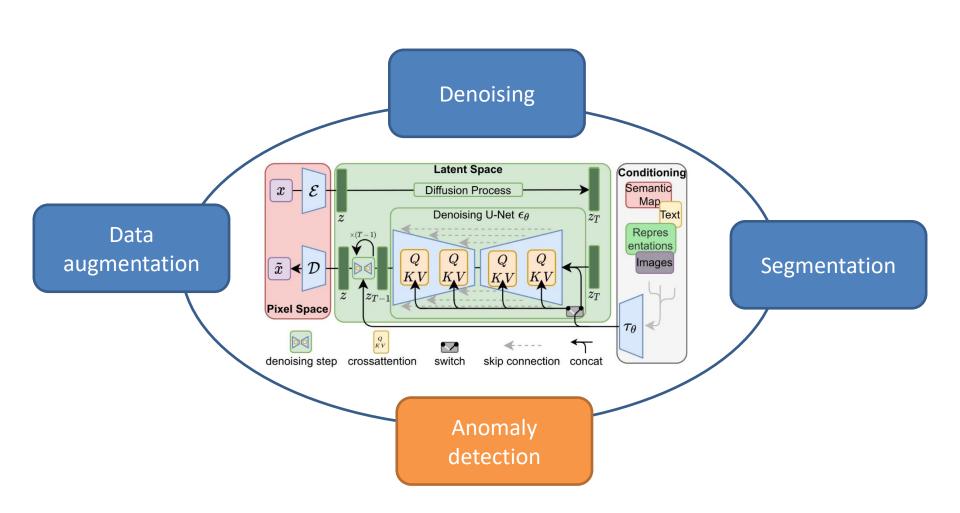
At inference time: modelling of the segmentation uncertainty



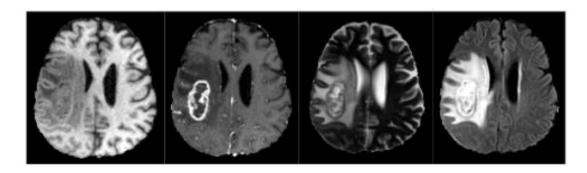
## Diffusion models for image segmentation

#### Results





- Anomaly detection from MR images [Wolleb et al., MICCAI 2024]
- BRATS2020 dataset
  - ▶ 4 different MR sequences per patient (T1, T2, T1ce, FLAIR)
  - ► Training: 332 patients with 3D volumes sequences => 16,998 2D images
  - > 5,598 healthy 2D slices (without tumor) / 10,607 disease 2D slices

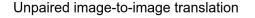


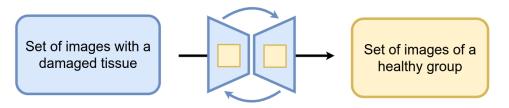
4 MR inputs per patient (T1, T2, T1ec, FLAIR)

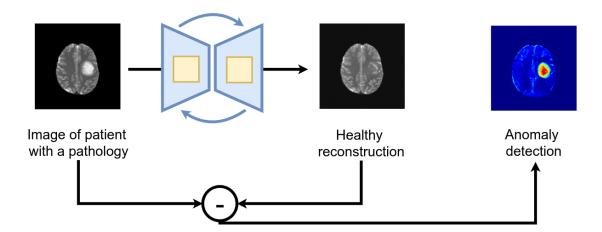


Mask output

#### General idea







How to preserve spatial anatomical information using a diffusion process?

- Denoising Diffusion Implicit Models (DDIM)
  - Reformulation of the diffusion process
  - Remove the random component  $\sigma_t \epsilon$

$$x_{t-1} = \sqrt{\bar{\alpha}_{t-1}} \left( \frac{x_t - \sqrt{1 - \bar{\alpha}_t} \cdot \epsilon_{\theta}(x_t, t)}{\sqrt{\bar{\alpha}_t}} \right) + \sqrt{1 - \bar{\alpha}_{t-1}} \cdot \epsilon_{\theta}(x_t, t)$$

$$x_{t+1} = x_t + \sqrt{\bar{\alpha}_{t+1}} \left[ \left( \sqrt{\frac{1}{\bar{\alpha}}} - \sqrt{\frac{1}{\bar{\alpha}}} \right) x_t + \left( \sqrt{\frac{1}{\bar{\alpha}}} - 1 - \sqrt{\frac{1}{\bar{\alpha}}} - 1 \right) \epsilon_{\theta}(x_t, t) \right]$$

$$x_{t+1} = x_t + \sqrt{\bar{\alpha}_{t+1}} \left[ \left( \sqrt{\frac{1}{\bar{\alpha}_t}} - \sqrt{\frac{1}{\bar{\alpha}_{t+1}}} \right) x_t + \left( \sqrt{\frac{1}{\bar{\alpha}_{t+1}} - 1} - \sqrt{\frac{1}{\bar{\alpha}_t} - 1} \right) \epsilon_{\theta}(x_t, t) \right]$$

Make the diffusion process deterministic



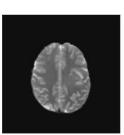
Iterative noise encoding

for 
$$t=0,\cdots T$$

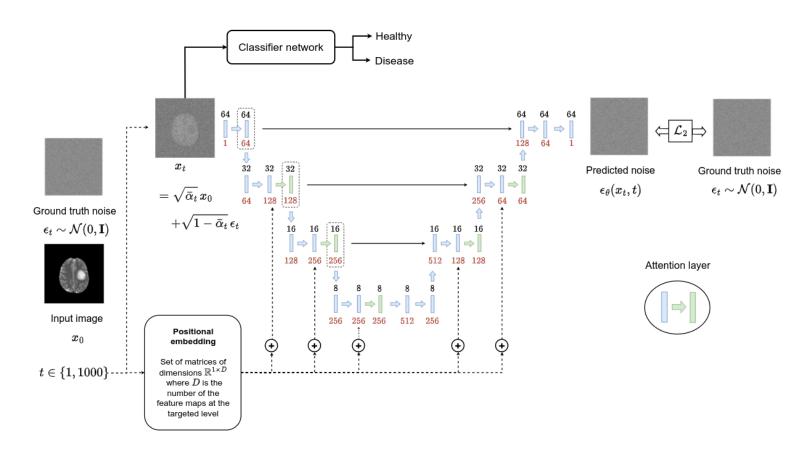


Iterative noise decoding

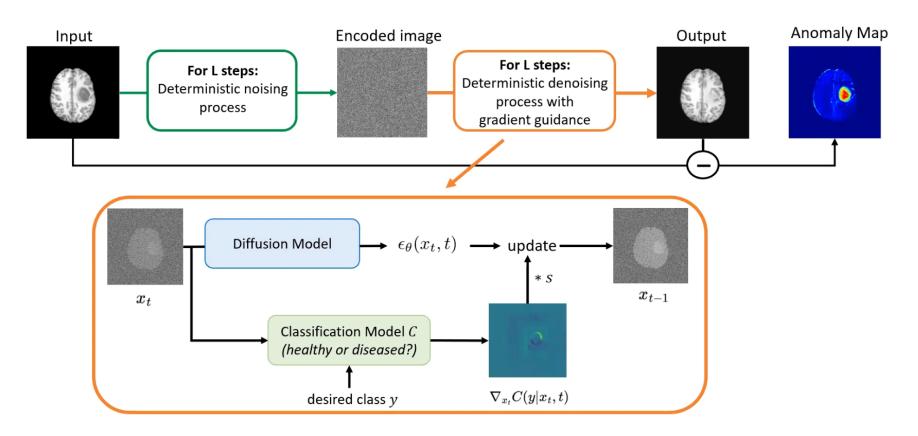
for 
$$t=T,\cdots 0$$



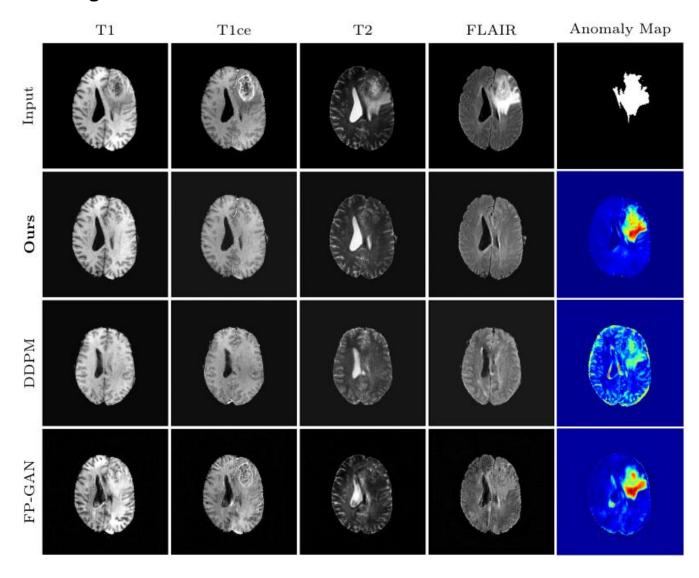
- Main algorithm part 1
  - Train a classical DDPM on the dataset containing healthy and disease images
  - Train a classifier network  ${\it C}$  to predict the class label (healthy vs disease) from any noisy images  $x_t$



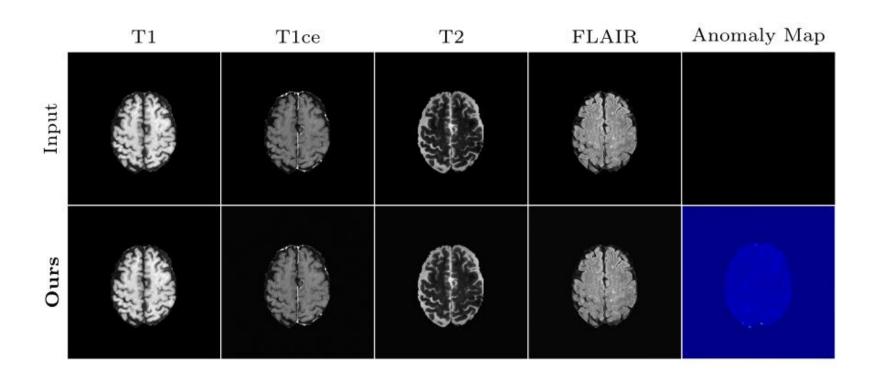
- Main algorithm part 2
  - Use DDIM process
  - Compute the gradient of the classifier to guide the removing of anomaly regions



Result on an image with a tumor



Result on an image without any tumor



# That's all folks

## What is the purpose of diffusion models?

Recent extensions for video synthesis

https://lumiere-video.github.io/#section\_image\_to\_video

### Image-to-Video

\* Hover over the video to see the input image and prompt.



### Latent diffusion model (LDM)

- Random generation of synthetic images with conditioning on text learned from LAION-400M database
  - → Using the BERT tokenizer
  - → This model has over 1.45 billion parameters!



'A painting of the last supper by Picasso.'