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Digital Image Processing

Image Improvement

Département Génie Electrique
5GE - TdSi

Summary

Image Improvement

- Image improvement denotes three types of image manipulation processes:
  - Image enhancement entails operations that improve the appearance to a human viewer, or operations to convert an image to a format better suited to machine processing
  - Image restoration has commonly been defined as the modification of an observed image in order to compensate for defects in the imaging system that produced the observed image
  - Geometrical image modification includes image magnification, minification, rotation and nonlinear spatial warping

Image Improvement

- Image enhancement
  - Contrast and histogram
  - Noise cleaning
  - Edge enhancement
  - *Color/multispectral image enhancement*
- Image restoration
- Geometrical image modification

→ *Book: Digital Image Processing, Pratt, Ed Wiley, 4rd edition, 2007*

Image Enhancement

- Improve the visual appearance of an image or to convert the image to a form better suited for analysis by a human or a machine
- A lot of techniques exist
- There is no general unifying theory of image enhancement at present because there is no general standard of image quality that can serve as a design criterion for an image enhancement processor

Contrast improvement

- The most common defects of photographic or electronic images is poor contrast resulting from a reduced, and perhaps nonlinear, image amplitude range

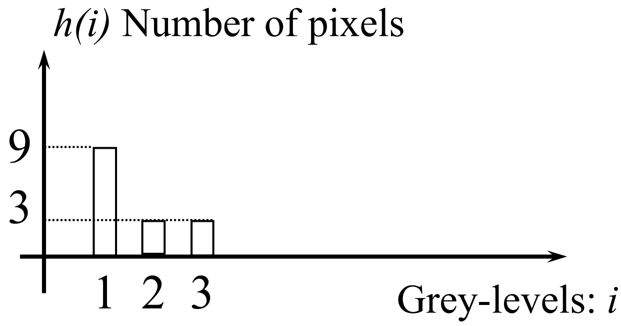
→ Image contrast can often be improved by amplitude rescaling of each pixel
  - → Histogram
  - → Transformation functions

Histogram

- Number of pixels that have a given intensity value

1	2	1	3	1
2	1	1	3	1
1	2	3	1	1

image

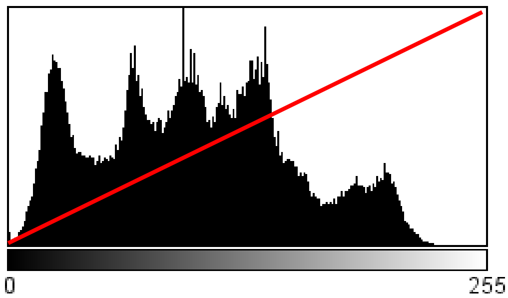


- Similar to the probability density function

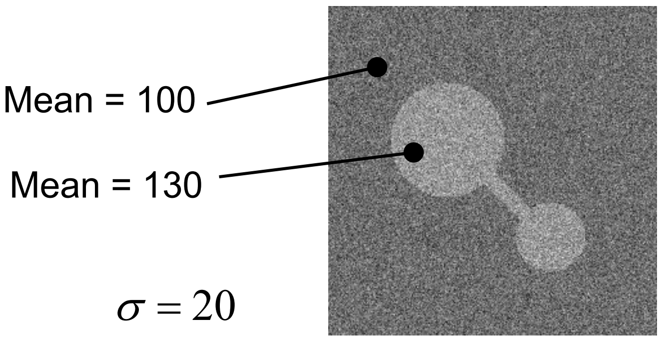
$$p(i) = h(i) / nb\_of\_pixels$$

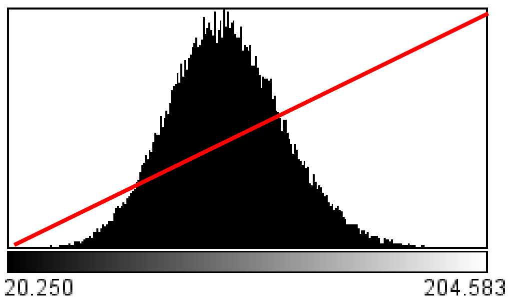
Histogram

- Examples



Count: 262144 Min: 0
Mean: 99.434 Max: 243
StdDev: 52.585 Mode: 93 (2760)

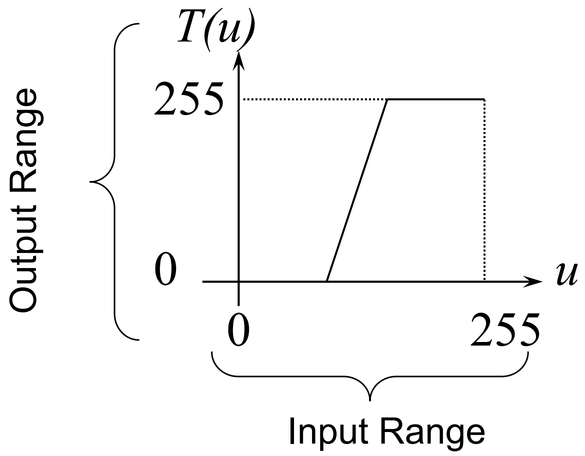


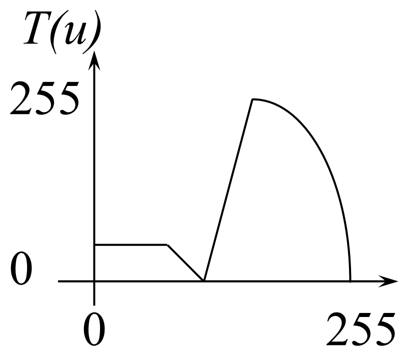


Count: 65536 Min: 20.250
Mean: 104.219 Max: 204.583
StdDev: 22.501 Mode: 103.416 (908)
Bins: 256 Bin Width: 0.720

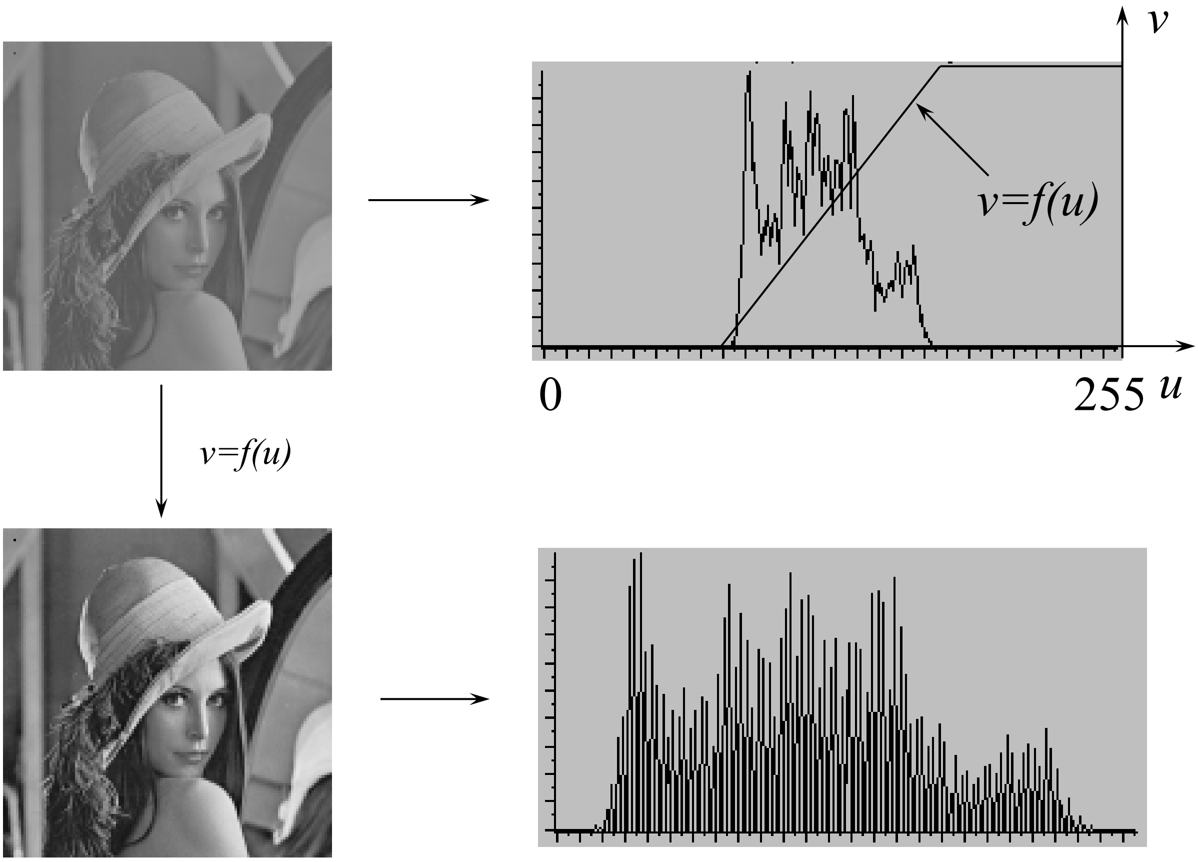
Histogram manipulation

- Use a transformation function
  - Try an existing (classical) one
  - Build your own!



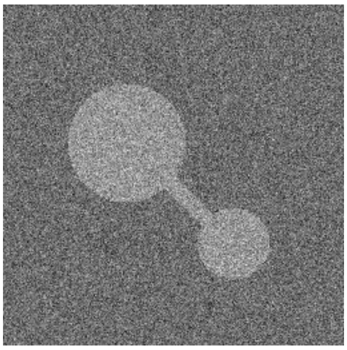


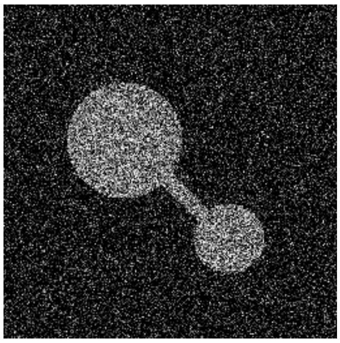
Histogram manipulation

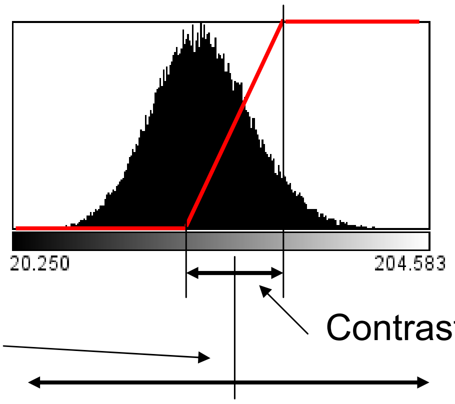


Histogram manipulation

- Brightness and contrast







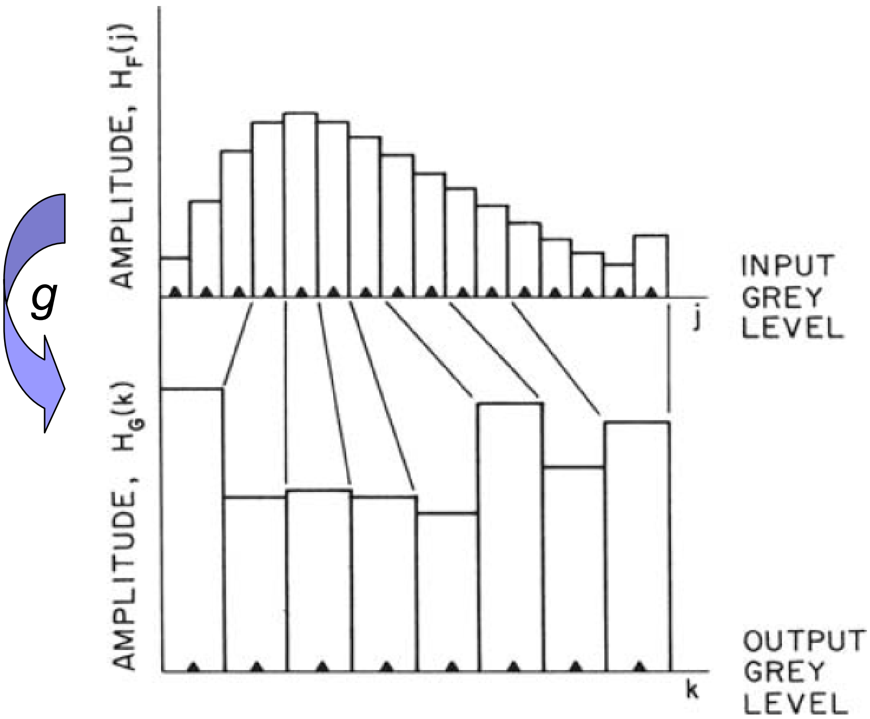
Contrast (window)

Brightness (level)
middle value of the contrast window

→Demo
→Best choice ?

Histogram manipulation

- Grey level histogram equalization



'*g*' function ?

$$\int_{g_{min}}^{g} p_g(g).dg = \int_{f_{min}}^{f} p_f(f).df$$

↓

$$\int_{g_{min}}^{g} p_g(g).dg = P_f(f)$$

Histogram manipulation

- Histogram equalization $$\int_{g_{min}}^{g} p_g(g).dg = P_f(f)$$
  - Examples
    - The output density is forced to be the uniform density

$$p_g(g) = \frac{1}{g_{max} - g_{min}} \quad g_{min} \le g \le g_{max}$$

$$g = (g_{max} - g_{min})P_f(f) + g_{min}$$

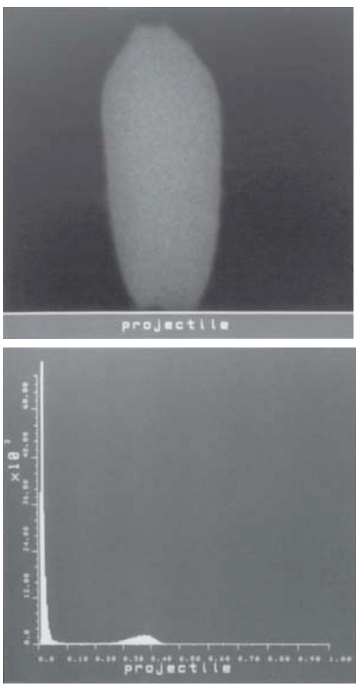
    - Other functions for the output density (exponential, logarithmic)

$$p_g(g) = \alpha \exp\{-\alpha(g - g_{min})\} \quad g \le g_{min} \quad \longrightarrow \quad g = g_{min} - \frac{1}{\alpha}\ln\{1 - P_f(f)\}$$

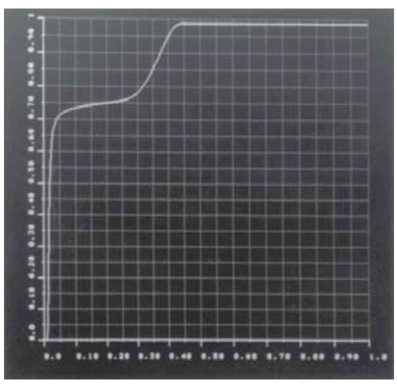
$$p_g(g) = \frac{1}{g[\ln\{g_{max}\} - \ln\{g_{min}\}]} \quad \longrightarrow \quad g = g_{min}\left(\frac{g_{max}}{g_{min}}\right)^{P_f(f)}$$

Histogram manipulation

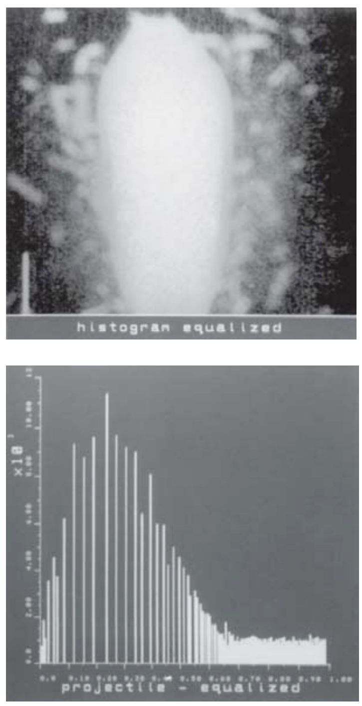
- Histogram equalization, example



X-ray projectile image and histogram



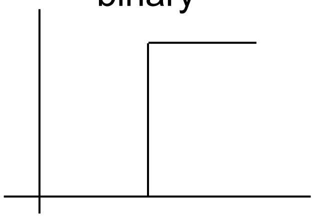
Transfer function ($P_f(f)$)

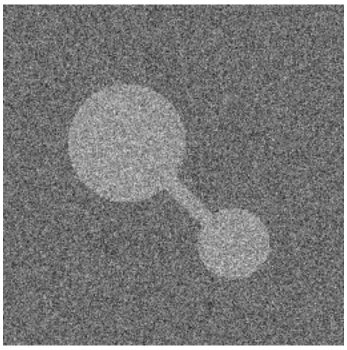


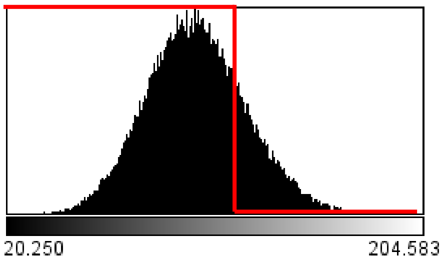
Histogram manipulation

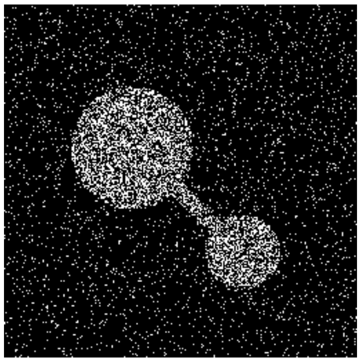
- Threshold

binary





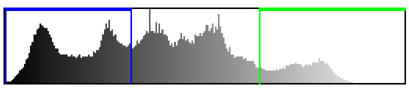




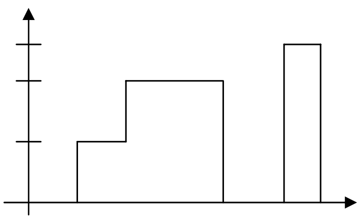
Histogram manipulation

- Threshold

LUT



n regions



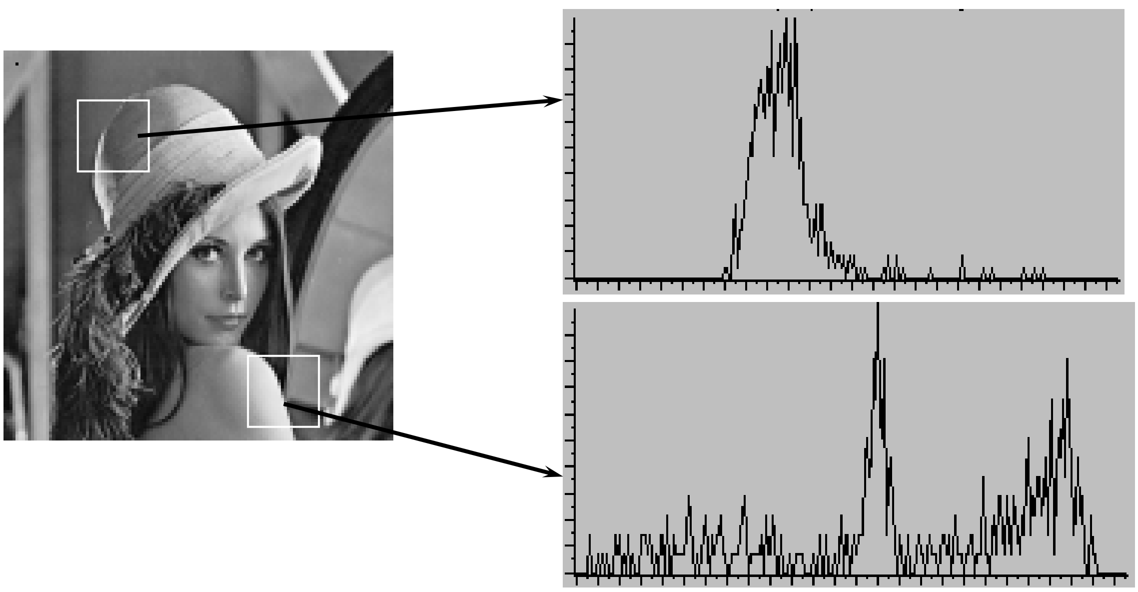
Histogram manipulation

- Limitations
  - Histogram equalization is not well adapted for good quality images
  - Histogram threshold is not a noise removing technique!
  - Histogram equalization should be adaptive!
    - Some methods exist (local equalization)

Local Histogram analysis

- features measured on the smallest neighborhood (1 pixel)
  grey-level (NG), color, quantitative value (Bq/cc, ...) ...

- features measured on a neighborhood → local histogram



Common computed values from the density probability function p(x) (based on histogram, local or not)

- Moments

$$m_i = E[x^i] = \sum_{n=0}^{N-1} x^i p(x)$$

N: number of grey levels

- Centered Moments

$$\hat{m}_i = E[(x - E[x])^i] = \sum_{n=0}^{N-1} (x - m_1)^i p(x)$$

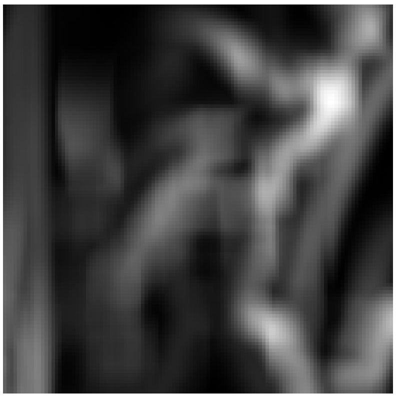
- Entropy

$$H = -\sum_{x=0}^{N-1} p(x) \log_2(p(x))$$

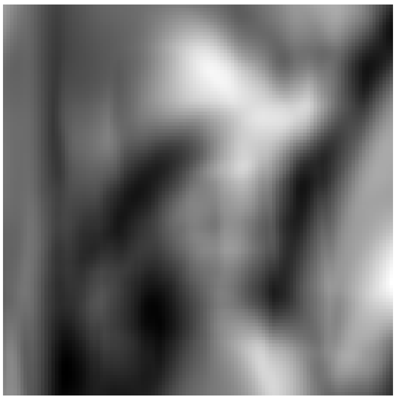
- Absolute moments, median value, max/min value, mode, percentiles, invariant moments (Legendre, …) ...

128x128 pixels

Computed on a 16x16 neigborhood



Variance



Mean

Noise cleaning

- Mean filter (linear filtering)
- Median filter (nonlinear filtering)
- Frequency domain filter (LP, HP, band-rejection (*notch*), …)
- …

Noise cleaning

- Many types of noise…

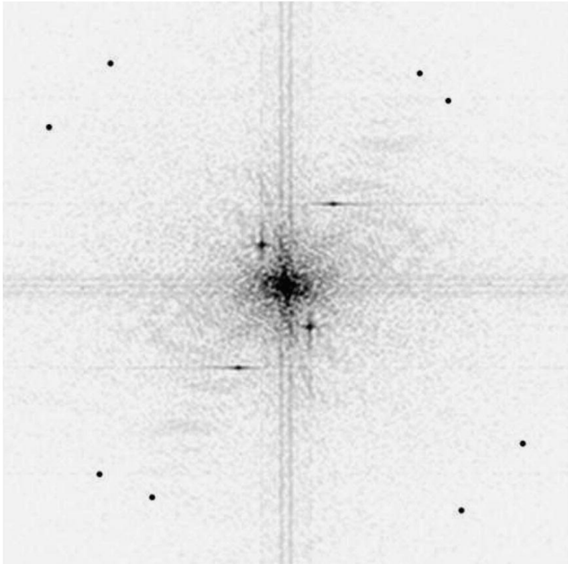
Gaussian Noise
(sd 25)

Salt and pepper noise

Noise cleaning

- Periodic and quasi periodic noise

Quasi periodic noise



Fourier amplitude spectrum

Linear filtering

- Mean filter

$$g(i,j) = \sum_{(k,l)\in W}\sum h(k,l)\, f(i-k, j-l)$$

W: 25 neighbors

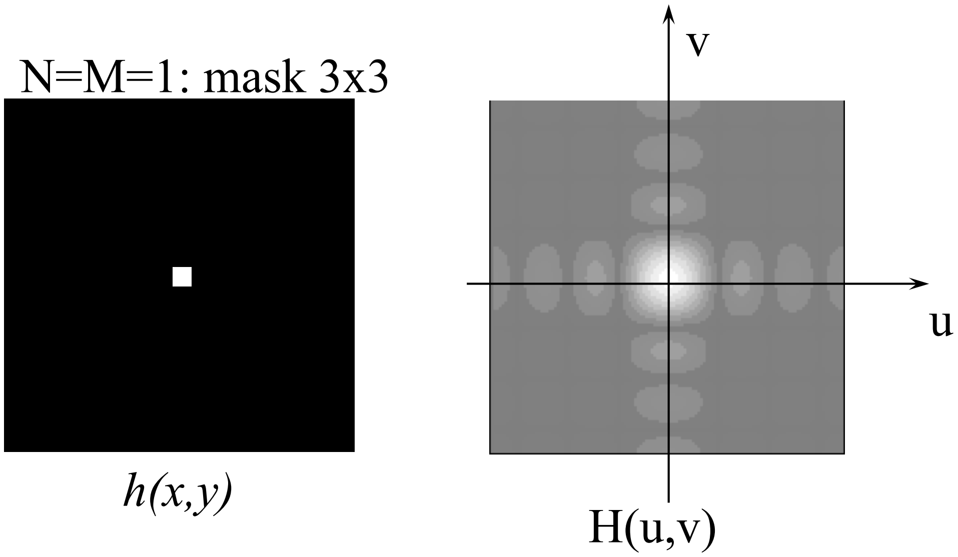
H=(1/25).**I**

Result on Gaussian noise

Result on Salt & Pepper noise

Linear filtering

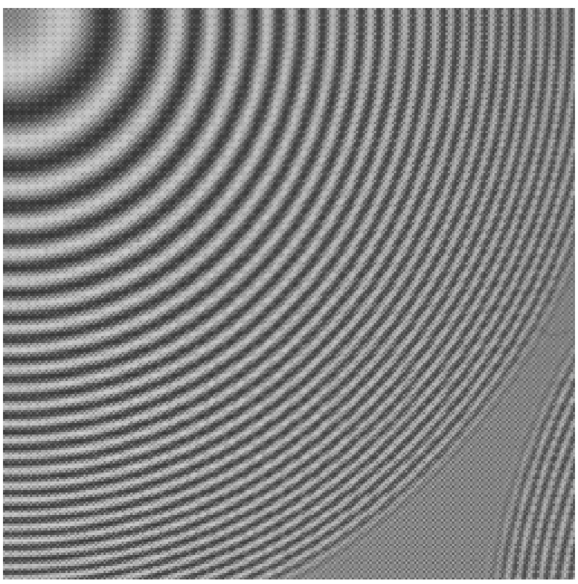
- Mean filter and DFT

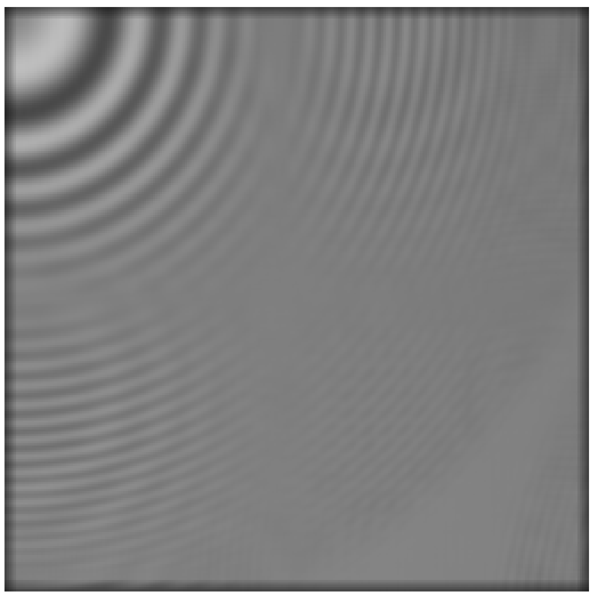


➔ *Anisotropic low pass filter with poor selectivity*

Linear filtering

- Mean filter anisotropy, (mask 11x11)





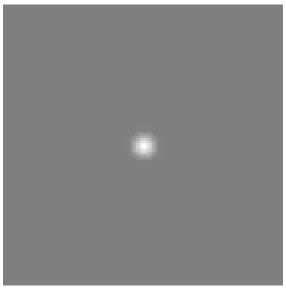
Linear filtering

- Gaussian Filter

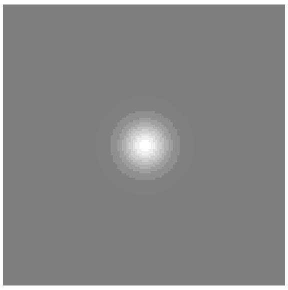
$$h(x,y) = \frac{1}{2\pi\sigma^2}\exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)$$

Filter with inifinite impulse response! → Approximate the ideal filter by truncating and windowing the infinite impulse response to make a FIR

h(x,y)



H(u,v)



Isotropic low pass filter with poor selectivity
H(u,v) is a gaussian

➔Many other types of filter
- ➔high pass, low pass, band-stop, derivative…
- ➔Ideal, Butterworth,…

Non linear filtering

- Median filter
  - Replace the central value by the median value of the neighborhood

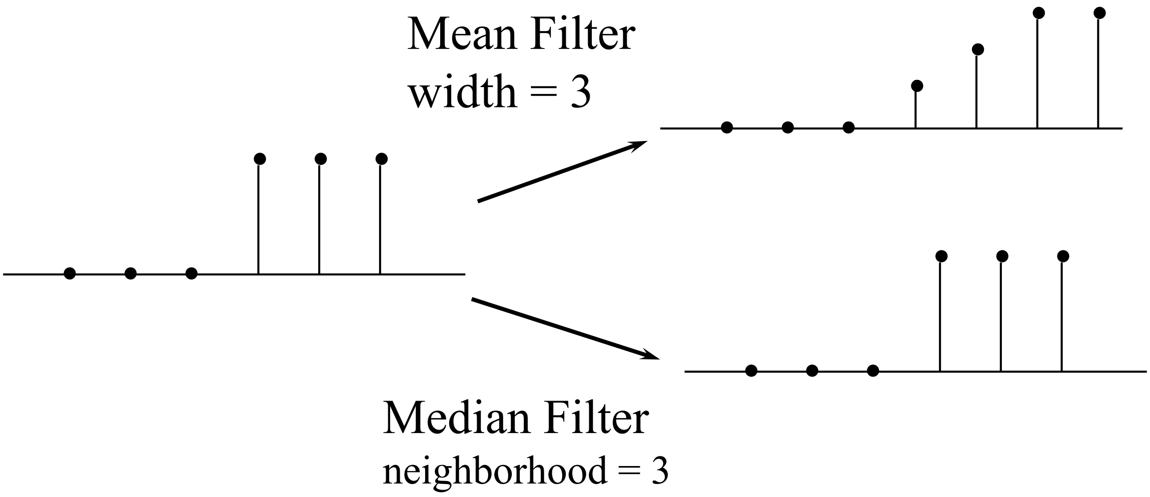
1	3	2	4
3	1	4	2
2	3	2	1
1	2	3	3

→ 1 1 2 2 [2] 3 3 3 4 →

.	.	.	.
.	2	.	.
.	.	.	.
.	.	.	.

Non linear filtering

- Median filter
  - Advantage of median filtering over linear filtering: edges are preserved



Non linear filtering

- Median filter

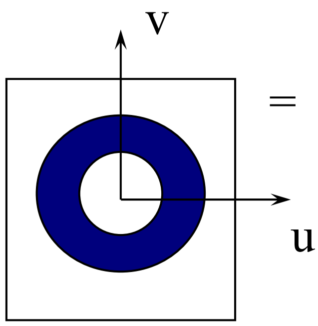
Result on Gaussian noise W=25

W=9 Result on salt & pepper noise

Frequency domain filtering

- Approach
  - Region selection in the frequency space (u,v)

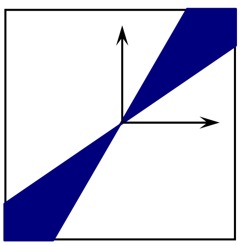
f(i,j) $\xrightarrow{\text{DFT}}$ F(u,v) x H(u,v) = F'(u,v) $\xrightarrow{\text{DFT}^{-1}}$ f'(i,j)



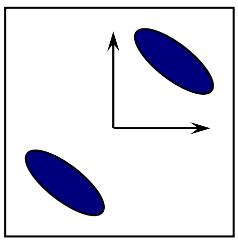
- to keep f'(i,j) real, regions must be symmetric about the origin
- In frequency space (u,v), region boundaries can be
  - •Steep (but … oscillations can appear: Gibbs)
  - •Smooth (but less selective)

Frequency domain filtering

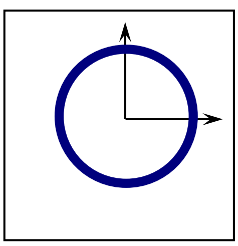
- Approach
  - Many forms for regions



Choice of one direction



Choice of the direction and frequency bands

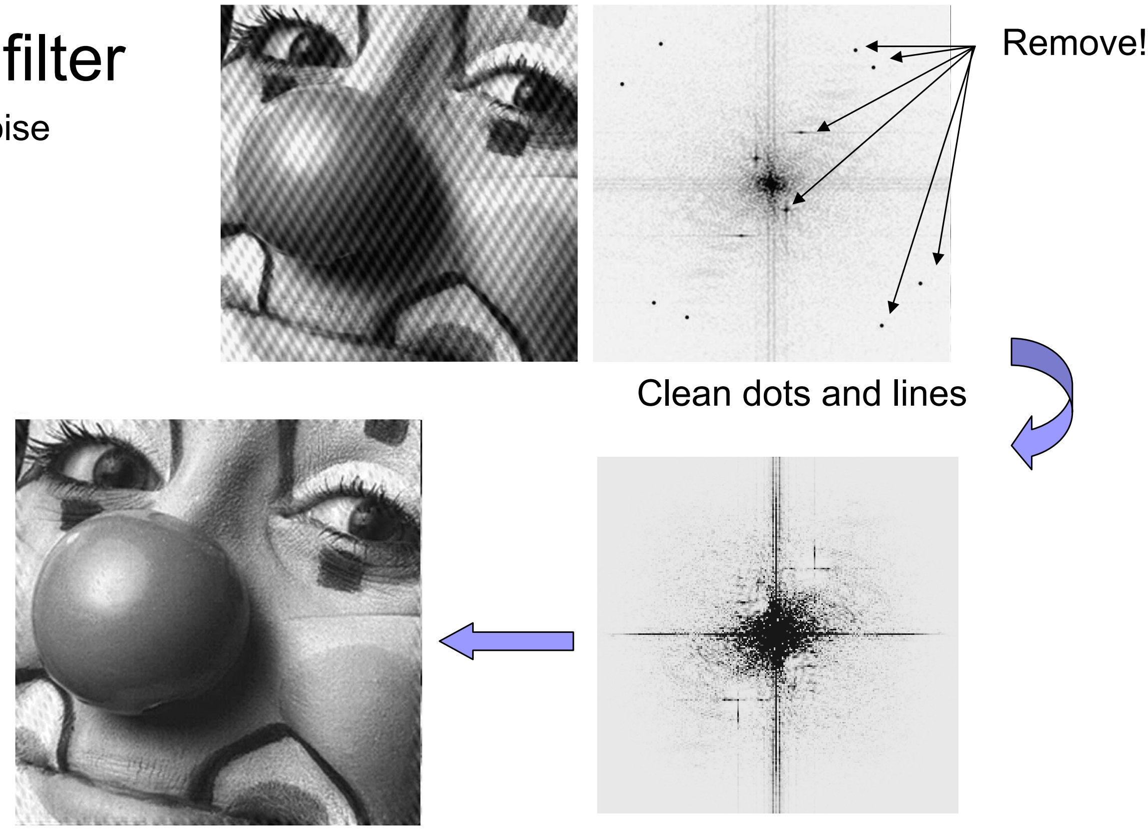


Choice of frequency bands

→Low-pass filters
→High-pass filters
→Band-pass filters
→Band-reject filters…

Frequency domain filtering

- Notch filter
  - Periodic noise



Edge enhancement

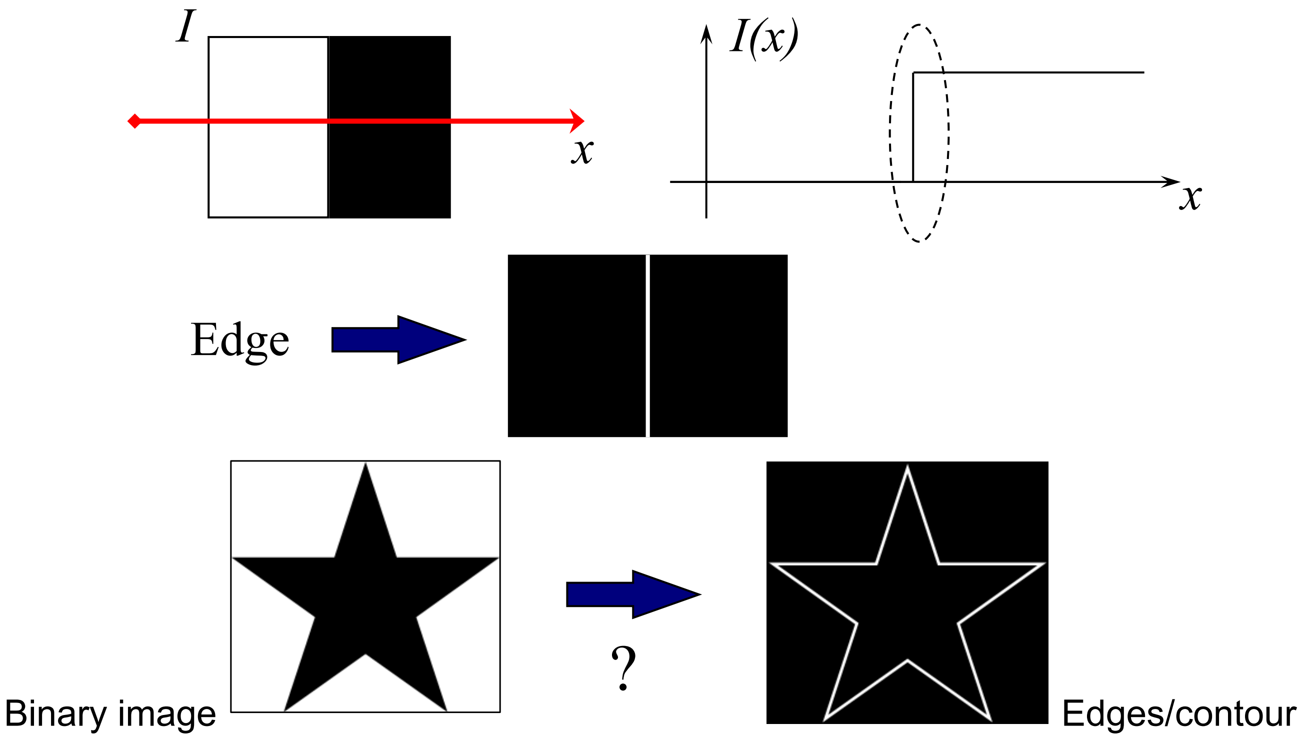
➔ ***Edges***: Changes or discontinuities of amplitude in an image

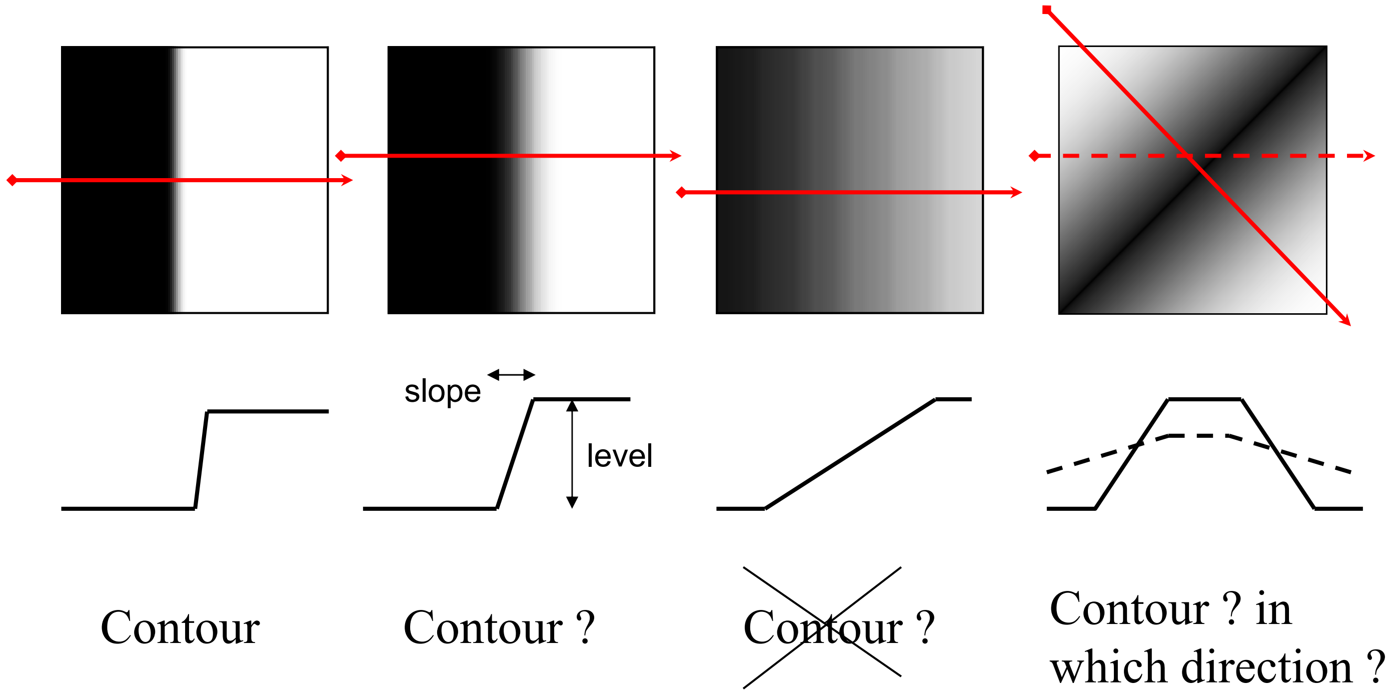
➔ Edges provide an indication of the physical extent of objects within the image ➔ ***Contours***

- Edge detection
  - Differential detection
  - Model fitting
- Edge enhancement filters

Edge enhancement

- Edge detection

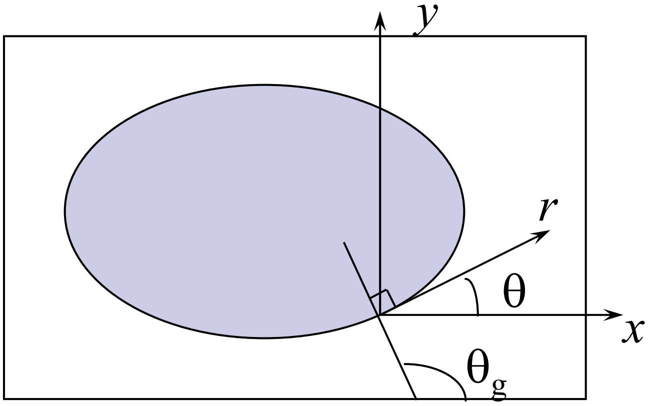




➔What are a change or a discontinuity ?
➔What about the direction (in image) ?

Definition of continuous contour and gradient

f(x,y)



Continuous one-dimensional gradient **g**(*x,y*) of *f(x,y)* along a line normal to the edge slope which is at an angle θ with respect to the horizontal axis:

$$\frac{\partial f(x,y)}{\partial r} \max \mathit{for}\ \theta_g = \theta + \frac{\pi}{2} \Rightarrow \frac{\partial}{\partial \theta}\left(\frac{\partial f(x,y)}{\partial r}\right) = 0$$

$$\mathbf{g}(x,y) = \nabla f(x,y) = \begin{Bmatrix} \partial f(x,y)/\partial x \\ \partial f(x,y)/\partial y \end{Bmatrix}$$

$$\frac{\partial f}{\partial r} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial r} = \frac{\partial f}{\partial x}\cos(\theta) + \frac{\partial f}{\partial y}\sin(\theta)$$

$$\frac{\partial}{\partial \theta}\left(\frac{\partial f(x,y)}{\partial r}\right) = 0 \Rightarrow -\frac{\partial f}{\partial x}\sin(\theta) + \frac{\partial f}{\partial y}\cos(\theta) = 0$$

Direction

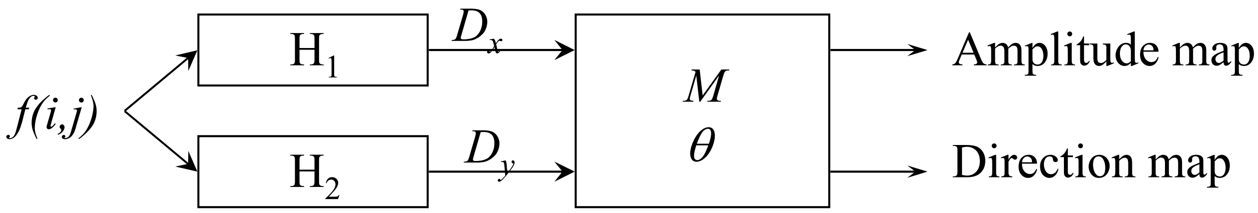
$$\theta_g = \arctan\left(\frac{\partial f/\partial y}{\partial f/\partial x}\right)$$

Amplitude

$$\left(\frac{\partial f}{\partial r}\right)_{\max} = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

Gradient in discrete domain

- For each pixel (i,j): gradient computation in two orthogonal directions → D_x , D_y
- Gradient amplitude $M = \sqrt{D_x^{\,2} + D_y^{\,2}}$
- Gradient direction $\theta = \text{Arctan}\left(\dfrac{D_y}{D_x}\right)$



→ For computational efficiency, the gradient amplitude is sometimes approximated by the magnitudes combination

$$M = |D_x| + |D_y|$$

→ If the gradient amplitude M is large enough (i.e., above some threshold value), an edge is deemed present

→ The direction (angle) map is used to follow the contour

→ Many H1 H2 operators exist:

$$\frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Pixel difference

$$D_x(i,j) = f(i,j) - f(i-1,j)$$
$$D_y(i,j) = f(i,j) - f(i,j-1)$$

Separated pixel difference

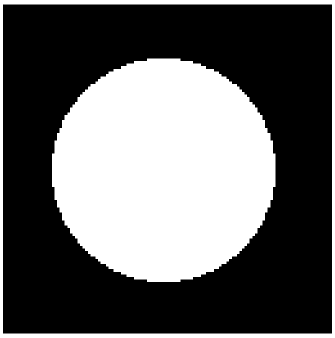
$$D_x(i,j) = f(i+1,j) - f(i-1,j)$$
$$D_y(i,j) = f(i,j+1) - f(i,j-1)$$

→ Convolution windows!

H_1, H_2

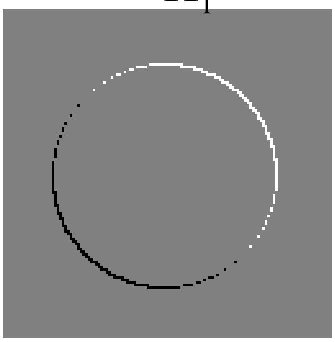
Operator	Row gradient H_1	Column gradient H_2
Pixel difference	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
Separated pixel difference	$\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$
Roberts	$\begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
Prewitt	$\frac{1}{3}\begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}$	$\frac{1}{3}\begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$
Sobel	$\frac{1}{4}\begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$	$\frac{1}{4}\begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$
Frei–Chen	$\frac{1}{2+\sqrt{2}}\begin{bmatrix} 1 & 0 & -1 \\ \sqrt{2} & 0 & -\sqrt{2} \\ 1 & 0 & -1 \end{bmatrix}$	$\frac{1}{2+\sqrt{2}}\begin{bmatrix} -1 & -\sqrt{2} & -1 \\ 0 & 0 & 0 \\ 1 & \sqrt{2} & 1 \end{bmatrix}$

Roberts

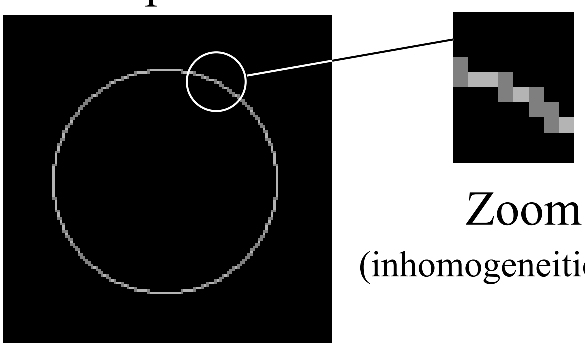


128x128

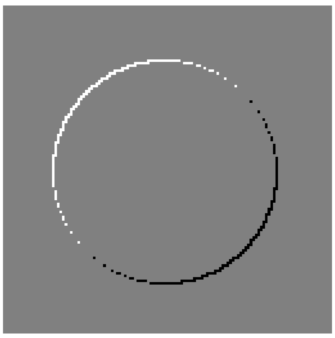
H_1



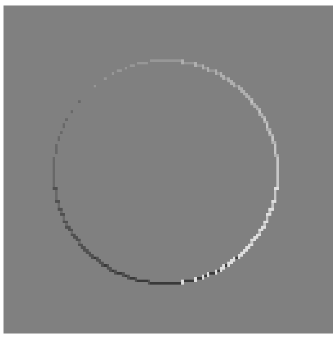
Amplitude



Zoom
(inhomogeneities!)



H_2



Direction

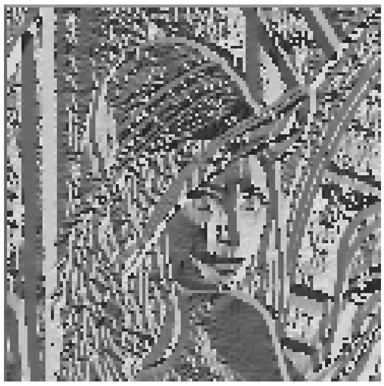
white=π/4
…
grey = π +π/4
…
black = 2π+π/4

Roberts

H$_1$

Amplitude

H$_2$



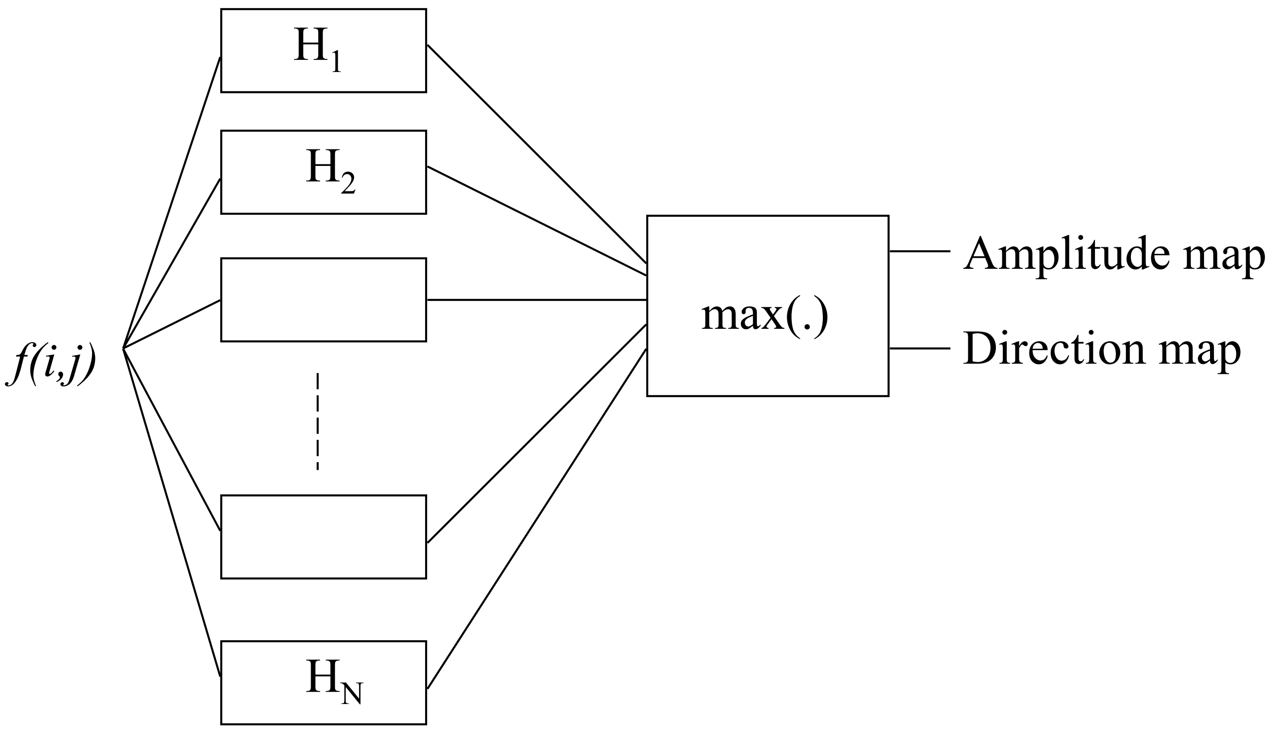
Direction

→ Increasing the contrast, many edges appear due to noise

➔ Edge detectors are high-pass filters

Compass operator

- Computation of the gradient in N directions
- Selection of the maximum value



- Examples

Gradient direction	Prewitt compass gradient	Kirsch	Robinson 3-level	Robinson 5-level
East H_1	$\begin{bmatrix}1 & 1 & -1\\ 1 & -2 & -1\\ 1 & 1 & -1\end{bmatrix}$	$\begin{bmatrix}5 & -3 & -3\\ 5 & 0 & -3\\ 5 & -3 & -3\end{bmatrix}$	$\begin{bmatrix}1 & 0 & -1\\ 1 & 0 & -1\\ 1 & 0 & -1\end{bmatrix}$	$\begin{bmatrix}1 & 0 & -1\\ 2 & 0 & -2\\ 1 & 0 & -1\end{bmatrix}$
Northeast H_2	$\begin{bmatrix}1 & -1 & -1\\ 1 & -2 & -1\\ 1 & 1 & 1\end{bmatrix}$	$\begin{bmatrix}-3 & -3 & -3\\ 5 & 0 & -3\\ 5 & 5 & -3\end{bmatrix}$	$\begin{bmatrix}0 & -1 & -1\\ 1 & 0 & -1\\ 1 & 1 & 0\end{bmatrix}$	$\begin{bmatrix}0 & -1 & -2\\ 1 & 0 & -1\\ 2 & 1 & 0\end{bmatrix}$
North H_3	$\begin{bmatrix}-1 & -1 & -1\\ 1 & -2 & 1\\ 1 & 1 & 1\end{bmatrix}$	$\begin{bmatrix}-3 & -3 & -3\\ -3 & 0 & -3\\ 5 & 5 & 5\end{bmatrix}$	$\begin{bmatrix}-1 & -1 & -1\\ 0 & 0 & 0\\ 1 & 1 & 1\end{bmatrix}$	$\begin{bmatrix}-1 & -2 & -1\\ 0 & 0 & 0\\ 1 & 2 & 1\end{bmatrix}$
Northwest H_4	$\begin{bmatrix}-1 & -1 & 1\\ -1 & -2 & 1\\ 1 & 1 & 1\end{bmatrix}$	$\begin{bmatrix}-3 & -3 & -3\\ -3 & 0 & 5\\ -3 & 5 & 5\end{bmatrix}$	$\begin{bmatrix}-1 & -1 & 0\\ -1 & 0 & 1\\ 0 & 1 & 1\end{bmatrix}$	$\begin{bmatrix}-2 & -1 & 0\\ -1 & 0 & 1\\ 0 & 1 & 2\end{bmatrix}$
West H_5	$\begin{bmatrix}-1 & 1 & 1\\ -1 & -2 & 1\\ -1 & 1 & 1\end{bmatrix}$	$\begin{bmatrix}-3 & -3 & 5\\ -3 & 0 & 5\\ -3 & -3 & 5\end{bmatrix}$	$\begin{bmatrix}-1 & 0 & 1\\ -1 & 0 & 1\\ -1 & 0 & 1\end{bmatrix}$	$\begin{bmatrix}-1 & 0 & 1\\ -2 & 0 & 2\\ -1 & 0 & 1\end{bmatrix}$
Southwest H_6	$\begin{bmatrix}1 & 1 & 1\\ -1 & -2 & 1\\ -1 & -1 & 1\end{bmatrix}$	$\begin{bmatrix}-3 & 5 & 5\\ -3 & 0 & 5\\ -3 & -3 & -3\end{bmatrix}$	$\begin{bmatrix}0 & 1 & 1\\ -1 & 0 & 1\\ -1 & -1 & 0\end{bmatrix}$	$\begin{bmatrix}0 & 1 & 2\\ -1 & 0 & 1\\ -2 & -1 & 0\end{bmatrix}$
South H_7	$\begin{bmatrix}1 & 1 & 1\\ 1 & -2 & 1\\ -1 & -1 & -1\end{bmatrix}$	$\begin{bmatrix}5 & 5 & 5\\ -3 & 0 & -3\\ -3 & -3 & -3\end{bmatrix}$	$\begin{bmatrix}1 & 1 & 1\\ 0 & 0 & 0\\ -1 & -1 & -1\end{bmatrix}$	$\begin{bmatrix}1 & 2 & 1\\ 0 & 0 & 0\\ -1 & -2 & -1\end{bmatrix}$
Southeast H_8	$\begin{bmatrix}1 & 1 & 1\\ 1 & -2 & -1\\ 1 & -1 & -1\end{bmatrix}$	$\begin{bmatrix}5 & 5 & -3\\ 5 & 0 & -3\\ -3 & -3 & -3\end{bmatrix}$	$\begin{bmatrix}1 & 1 & 0\\ 1 & 0 & -1\\ 0 & -1 & -1\end{bmatrix}$	$\begin{bmatrix}2 & 1 & 0\\ 1 & 0 & -1\\ 0 & -1 & -2\end{bmatrix}$
Scale factor	$\frac{1}{5}$	$\frac{1}{15}$	$\frac{1}{3}$	$\frac{1}{4}$

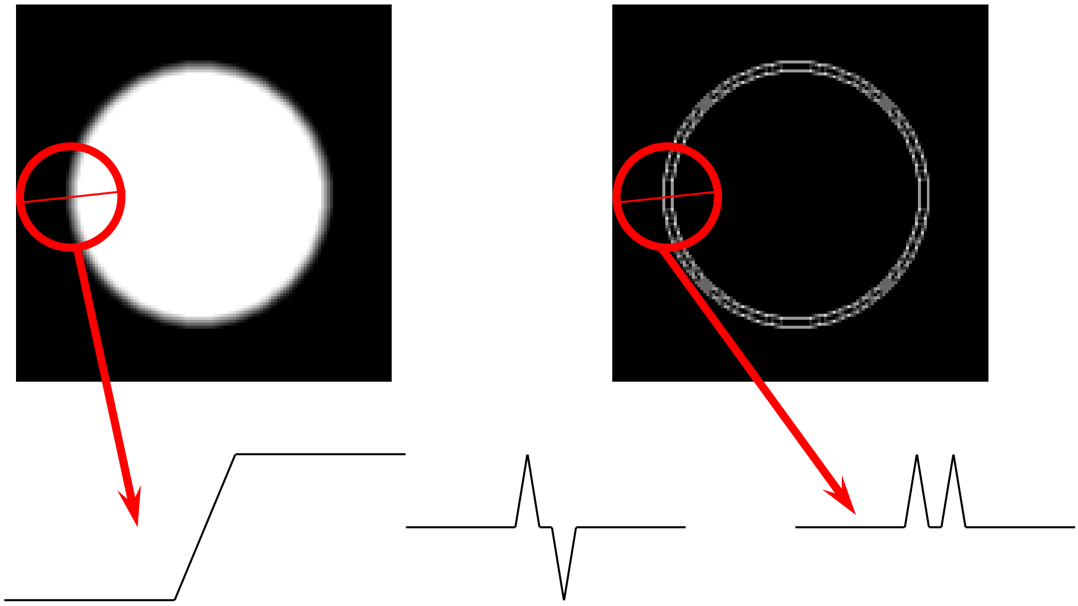
□ Laplacian

$$\Delta f(x,y) = \nabla^2 f(x,y) = \frac{\partial^2 f(x,y)}{\partial x^2} + \frac{\partial^2 f(x,y)}{\partial y^2}$$

$$\left.\begin{matrix}\begin{bmatrix}0&0&0\\0&1&-1\\0&0&0\end{bmatrix}\otimes\begin{bmatrix}0&0&0\\0&1&-1\\0&0&0\end{bmatrix}\\ \begin{bmatrix}0&-1&0\\0&1&0\\0&0&0\end{bmatrix}\otimes\begin{bmatrix}0&-1&0\\0&1&0\\0&0&0\end{bmatrix}\end{matrix}\right\}\begin{bmatrix}0&1&0\\1&-4&1\\0&1&0\end{bmatrix}$$

$$\begin{bmatrix}0&-1&0\\-1&4&-1\\0&-1&0\end{bmatrix}$$

then absolute value

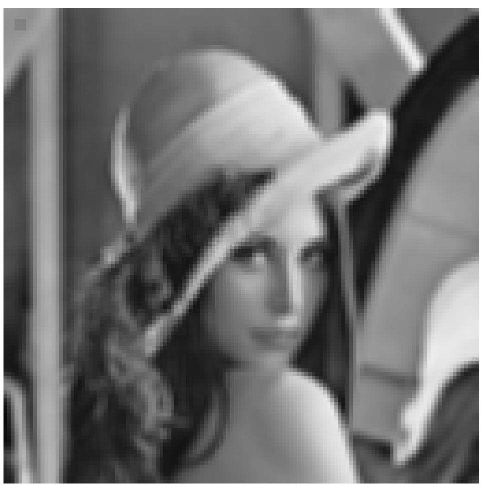


■ Emphasis filter

0 -1 0
-1 4 -1 →
0 -1 0

0 -1 0
-1 5 -1 = Input Image + Laplacian
0 -1 0

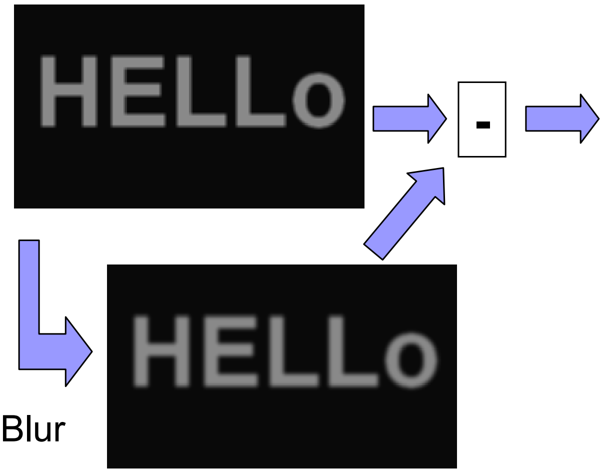
➔ Enhancement of high frequencies



- Unsharp masking, Highboost Filtering
  - Used by the printing and publishing industry

1- Blur the original image
2- Substract the blurred image from the original (the result is called the mask)
3- Add the mask (multiplied by k) to the original

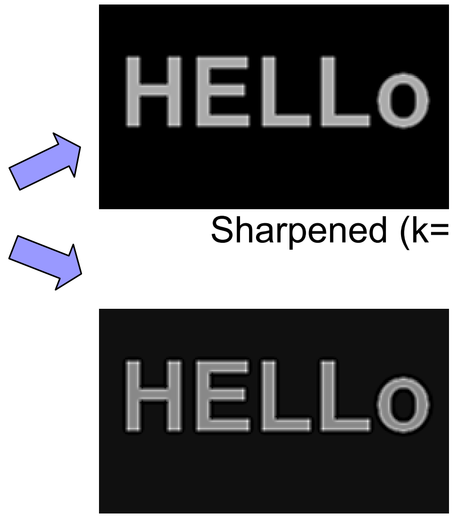
original



Blur

mask

Sharpened (k=1)

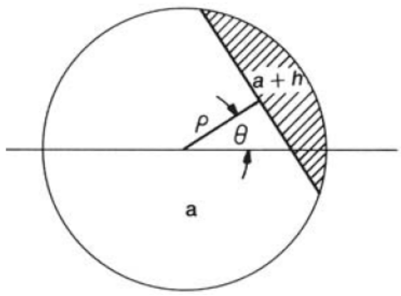


Highboost (k>1)

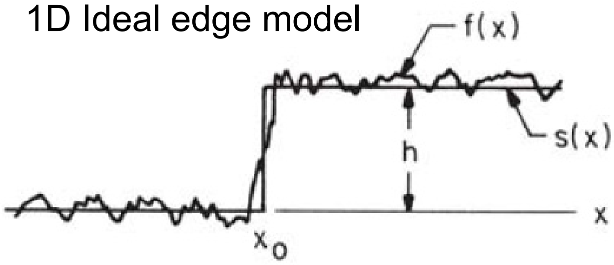
Edge fitting

- Image data *f* can be fitted to an ideal edge model *s*

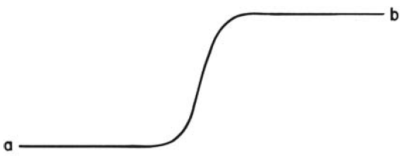
2D Ideal edge model



1D Ideal edge model



Hyperbolic Edge model 1D



→ An edge is assumed present if the Mean Square Error is below a threshold value

$$MSE = \int_{x_0-L}^{x_0+L} [f(x)-s(x)]^2 .dx$$

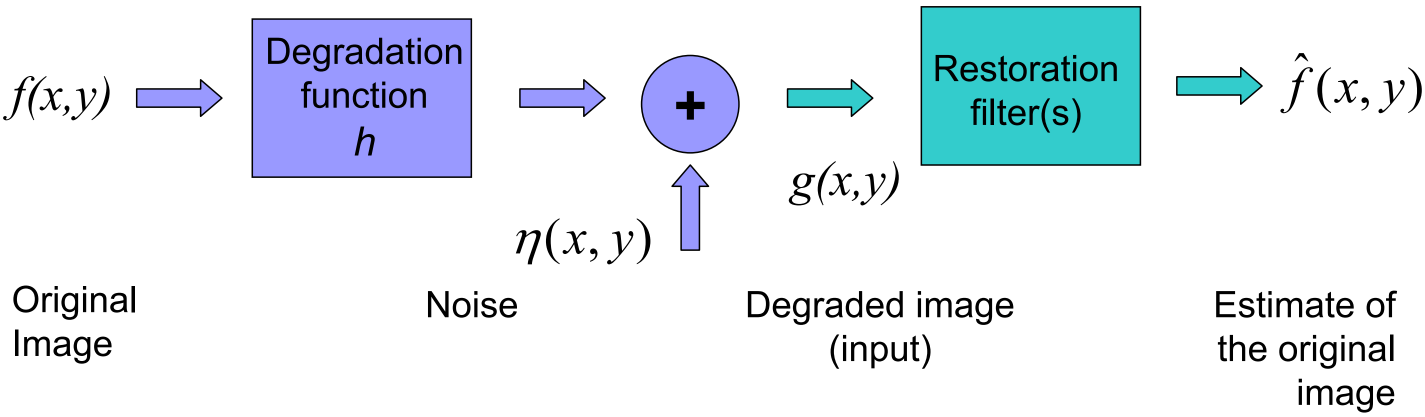
Model+minimization… image restoration

Image Restoration

- Image restoration attempts to recover an image that has been degraded using a priori knowledge of degradation phenomenon
  - →Modeling the degradation
  - →Applying the inverse process (in order to recover the original image)

➔ Involves formulating a criterion of goodness that will yield an optimal estimate of the desired result

- A model of the image degradation/restoration process



$$g(x,y) = h(x,y) \otimes f(x,y) + \eta(x,y)$$
$$G(u,v) = H(u,v).F(u,v) + N(u,v)$$

Degradation
(Sensor, digitizer, display, movement)

Restoration

$\eta(x, y)$

- Noise Models
  - Gaussian (normal) noise $p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(z-\bar{z})^2/2\sigma^2}$
  - Uniform noise $p(z) = \begin{cases} \frac{1}{b-a} & \text{if } a \le z \le b \\ 0 & \text{otherwise} \end{cases}$
  - Impulse (salt and pepper) noise $p(z) = \begin{cases} P_a & \text{for } z = a \\ P_b & \text{for } z = b \\ 0 & \text{otherwise} \end{cases}$
  - Erlang (gamma) noise
  - Rayleigh noise $p(z) = \begin{cases} \frac{2}{b}.(z-a)e^{-(z-a)^2/b} & \text{for } z \ge a \\ 0 & \text{for } z < a \end{cases}$

Restoration: presence of noise only

$$g(x, y) = f(x, y) + \eta(x, y)$$

- Mean filters
  - Arithmetic, geometric, harmonic

$$\frac{1}{m.n} \sum_{i,j \in S_{xy}} g(i, j) \qquad \left[\prod_{i,j \in S_{xy}} g(i, j) \right]^{\frac{1}{m.n}} \qquad \frac{m.n}{\sum_{i,j \in S_{xy}} \frac{1}{g(i, j)}}$$

- Order statistic filters
  - Median, min & max, midpoint

$$\hat{f}(x, y) = \underset{s,t \in S_{xy}}{median}\left(g(s, t)\right)$$

$$\hat{f}(x, y) = \max_{s,t \in S_{xy}}\left(g(s, t)\right)$$

$$\hat{f}(x, y) = \min_{s,t \in S_{xy}}\left(g(s, t)\right)$$

$$\frac{1}{2}\max + \frac{1}{2}\min$$

- Adaptive filters
  - Local noise reduction, adaptive median, …

Adaptive Median Filter

Notations

z_{min} = minimum intensity value in S_{xy}
z_{max} = maximum intensity value in S_{xy}
z_{med} = median of intensity values in S_{xy}
z_{xy} = intensity value at coordinates (x,y)
S_{max} = maximum allowed size of S_{xy}

Algorithm for a pixel (x,y)

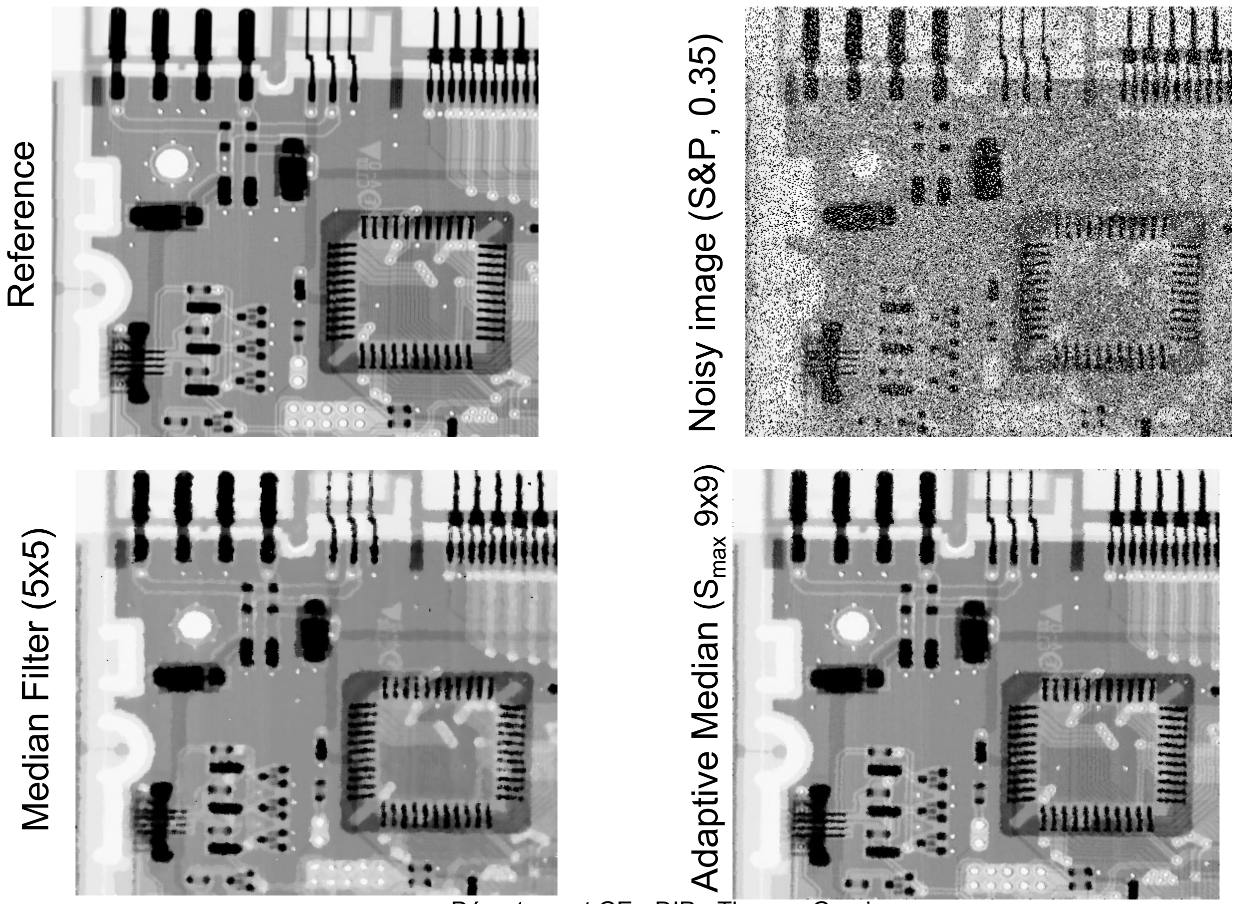
Stage 1 :

Compute z_{min}, z_{max}, z_{med}
$A1 = z_{med} - z_{min}$
$A2 = z_{med} - z_{max}$
if A1>0 and A2<0 → stage 2
increase the window size
if window size < S_{max} → stage 1
else output z_{med}

Stage 2:

$B1 = z_{xy} - z_{min}$
$B2 = z_{xy} - z_{max}$
if B1>0 and B2<0
output z_{xy}
else output z_{med}

Result of Adaptive Median filter



Restoration: Periodic noise reduction

$$g(x,y) = f(x,y) + \eta(x,y)$$

- By frequency domain filtering
  - Bandreject filter
  - Notch filter → optimum notch

Build H_{NP} (Notch Pass) by placing a notch pass filter at the location of each spike.
Interference noise pattern is:

$$N(u,v) = \mathrm{H_{NP}}(\mathrm{u,v}).\mathrm{G}(\mathrm{u,v})$$

then $\eta(\mathrm{x,y}) = \mathrm{FT}^{-1}\left[N(u,v)\right]$

thus $\hat{f}(x,y) = g(x,y) - w(x,y).\eta(\mathrm{x,y})$

Estimate of f(x,y)

Weighted function (minimizes the effect of components not present in the estimate of η)

➔ **How to select w(x,y) ?**

- w(x,y) ?

w(x,y) is selected so that the local variance of the estimate of *f* is minimized (optimum choice of w)

$$\frac{\partial \sigma^2(x,y)}{\partial w(x,y)} = 0$$

$$\Rightarrow \quad w(x,y) = \frac{\overline{g(x,y).\eta(x,y)} - \overline{g}(x,y).\overline{\eta}(x,y)}{\overline{\eta^2}(x,y) - \overline{\eta}^2(x,y)}$$

→Prove the validity of this equation

Hints:

→estimate the variance in a small neighborhood

→assume that w remains essentially constant over the neighborhood

Restoration: linear, position-invariant degradation

$$g(x,y) = H[f(x,y)] + \eta(x,y)$$

- H is linear,

$$H[af_1(x,y) + bf_2(x,y)] = aH[f_1(x,y)] + bH[f_2(x,y)]$$

- If H is position invariant then (for any a and b):

$$H[\delta(x-a, y-b] = h(x-a, y-b)$$

➔ **g(x,y):**

$$g(x,y) = h(x,y) \otimes f(x,y) + \eta(x,y)$$

$$G(u,v) = H(u,v).F(u,v) + N(u,v)$$

Estimating the degradation function

Blind deconvolution

- 3 principal ways
  - Observation $H_s(u,v) = \frac{G_s(u,v)}{\hat{F}_s(u,v)}$
    - Small rectangular section containing samples structures (part of an object, background)
  - Experimentation $H(u,v) = \frac{G(u,v)}{A}$
    - Obtain the impulse response of the degradation function by imaging an impulse (small dot of light)
  - Modeling
    - Mathematical model that take into account environmental conditions that cause degradation
    - Derive a mathematical model starting from basic principles

And after ?

- Inverse filtering
  - Without noise $\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)}$
  - With noise $\hat{F}(u,v) = F(u,v) + \frac{N(u,v)}{H(u,v)}$
    - We have to known N!
    - What happen for small values of H(u,v) ?
- Minimum Mean Square Error (Wiener) Filtering
- Constrained Least Squares Filtering
- …

Geometrical image modification

- Spatial transformations
  - Example
    - Shrink image to half its size $(x', y') = T\{(x, y)\} = (x/2, y/2)$
  - **Affine transform**:

$$[x', y', 1] = [x, y, 1] \cdot \begin{bmatrix} t_{11} & t_{12} & 0 \\ t_{21} & t_{22} & 0 \\ t_{31} & t_{32} & 1 \end{bmatrix}$$

  - Higher order

$$[x', y', 1] = [x, y, x^2, y^2, xy, \ldots, 1].\mathbf{T}$$

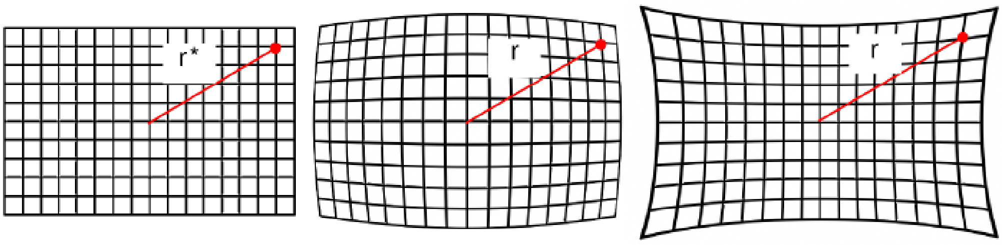
➔ Estimate (or compute) the inverse matrix

➔ If needed, use interpolation

- Higher order transforms

Applications :

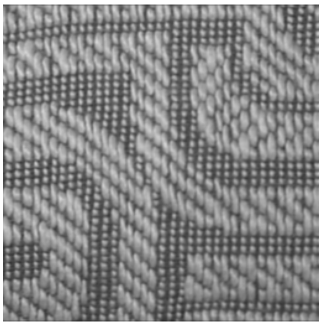
Lens distortion correction, perspective

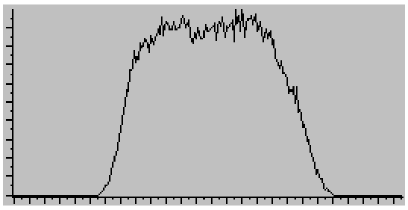


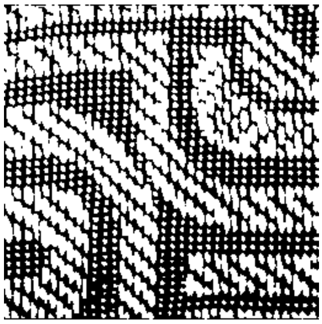
- End …
  - Before texture

Analyse de texture

Région ~~≠~~ zone de NG ou de couleur homogène







Seuillage !

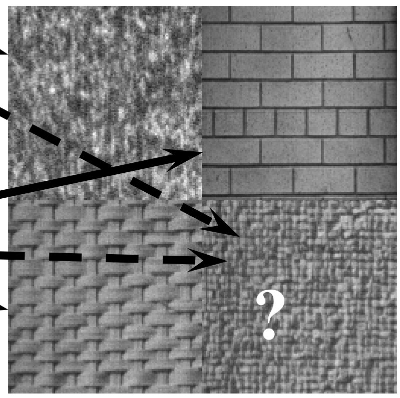
Détection de contour !

<u>Texture</u> = information visuelle qualitative:
Grossière, fine, tachetée, marbrée, régulière, périodique...
<u>Région homogène</u>: Assemblage plus ou moins régulier de primitives plus ou moins similaires.

<u>Analyse de texture</u> = formalisation de ces critères

<u>Texture microscopique</u>: Aspect chaotique mais régulier, primitive de base réduite.

<u>Texture macroscopique</u>: primitive de base évidente, assemblage régulier.



Méthodes d'analyse de texture:

Structurelles: recherche de primitives de base bien définies et de leur organisation (règles de placement)
Méthodes peu utilisées

Stochastiques: primitives mal définies et organisation +/- aléatoire.

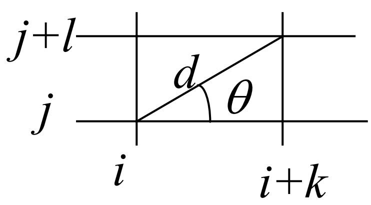
Principe: évaluation d'un paramètre dans une petite région (fenêtre de taille dépendant de la texture (!)): Analyse fréquentielle, statistiques, comptage d'événements, corrélation,....

> Pas de modèle général de texture ⟶ Nombreuses méthodes ad-hoc.

Exemple de méthode: Matrices de co-occurence

Statistique du second ordre:

$Pr.(f(i,j)=a \text{ et } f(i+k,j+l)=b)=p\,(k,l;a,b) = p\,(d,\theta;\,a,b)$



$d = 1\ ,\ \theta = 0°\quad (k=1,l=0)$

0	0	1	1
0	0	1	1
0	2	2	2
2	2	3	3

a \ b	0	1	2	3
0	2	2	1	0
1	0	2	0	0
2	0	0	3	1
3	0	0	0	1

a \	0	1	2	3
0	4	2	1	0
1	2	4	0	0
2	1	0	6	1
3	0	0	1	2

(en symétrique $\theta = 0°\ d = 1$ *et* $d= -1$)

Quelques Paramètres extraits des matrices de co-occurence

Moyenne locale: $\sum_{i=1}^{NG}\sum_{j=1}^{i}(i+j)p(i,j)$ (i,j : ligne et colonne de la matrice de co-occurrence p)

Energie ou second moment: $\sum_{i=1}^{NG}\sum_{j=1}^{i} p(i,j)^2$

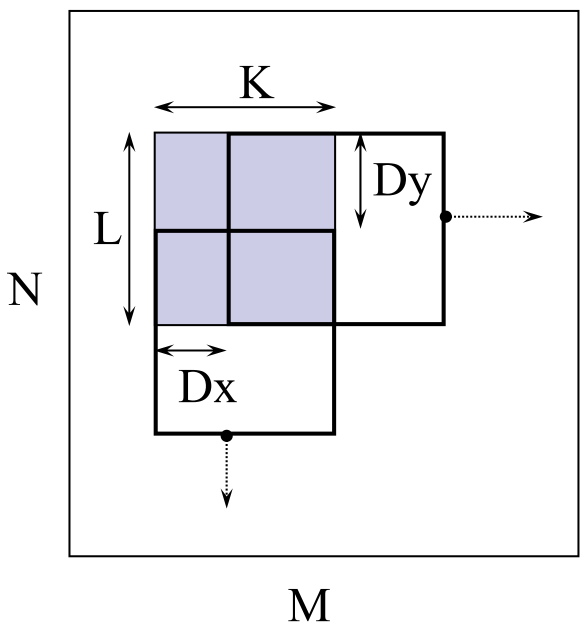
Inertie ou moment d'ordre deux des différences : $\sum_{i=1}^{NG}\sum_{j=1}^{i}(i-j)^2\ p(i,j)$

Autocorrélation: $\sum_{i=1}^{NG}\sum_{j=1}^{i} i.j\ p(i,j)$

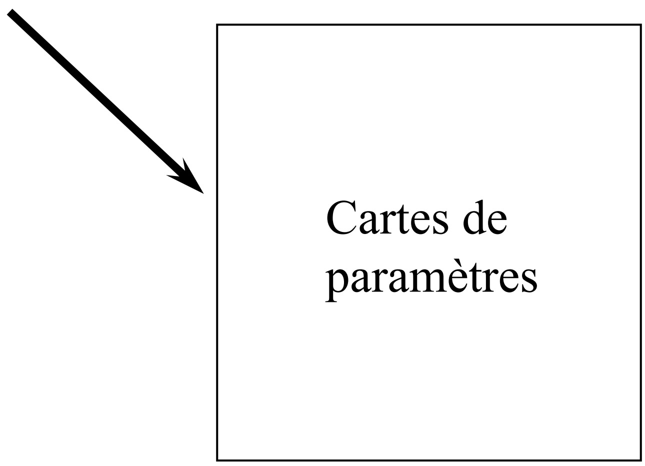
Contraste: $\sum_{i=1}^{NG}\sum_{j=1}^{i}(i+j)^2\ p(i,j)$

- Il y en a d'autres ....
- L'interprétation visuelle est difficile.

Application de l'analyse de texture

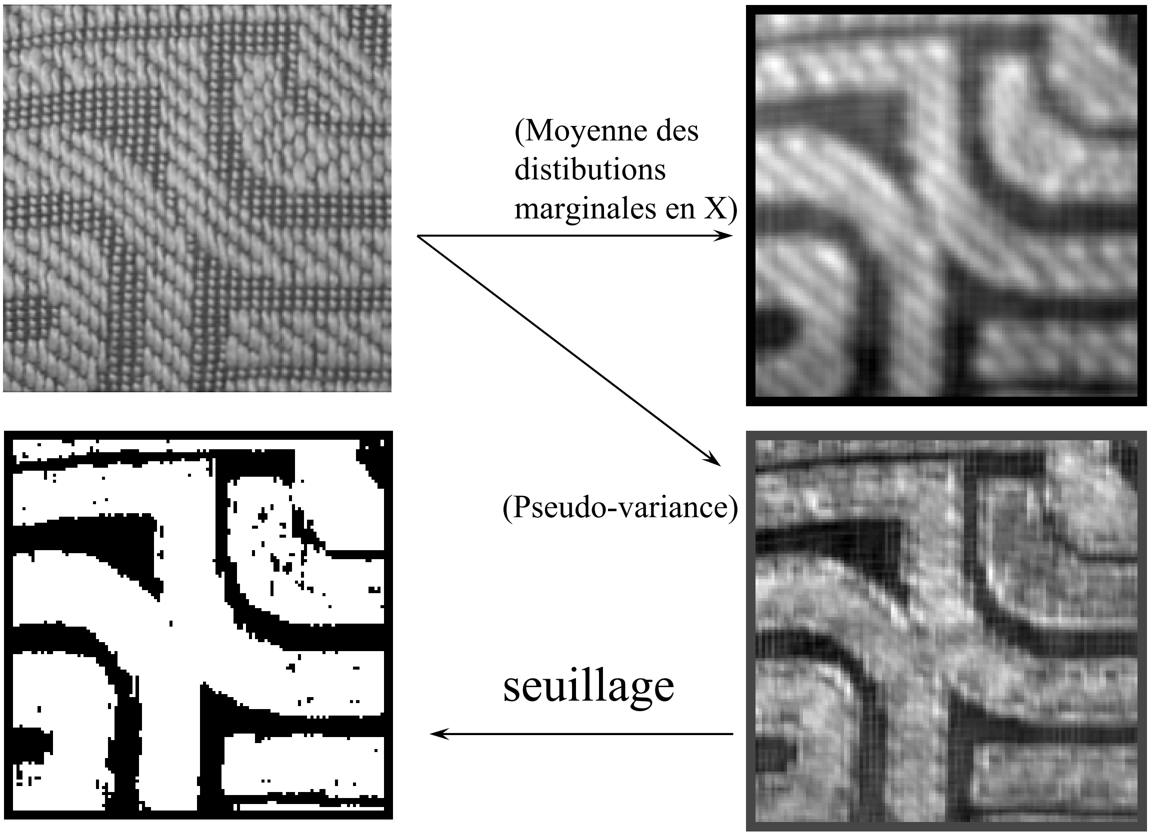


Mesure de paramètres dans une fenêtre de taille K,L
Avec un pas de déplacement Dx, Dy



Application des matrices de co-occurence

Fenêtre 16x16, pas 2x2, *k=1, l=0*

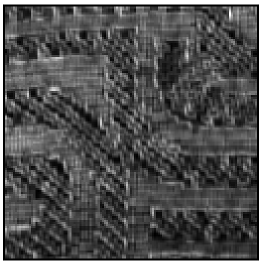


Influence des paramètres

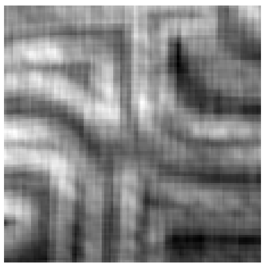
- Exemple : Choix de la taille de la fenêtre

(Matrice de co-occurrence : Pseudo-variance)


4x4


8x8


16x16


32x32

Le choix et les réglages des paramètres sont difficiles. Il faut souvent faire de nombreux essais.

Les paramètres obtenus doivent être pertinents pour l'opération suivante de **segmentation**.