

Digital Image Processing

Exercises

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5GE - TdSi*

Fundamentals

- 2.04: You are hired to design the front end of an imaging system for studying the boundary shapes of cells, bacteria, viruses and proteins. The front end consists, in this case, of the illumination source(s) and corresponding imaging camera(s). The diameters of circles required to enclose individual specimens in each of these categories are 50, 1, 0.1, and 0.01 micrometer, respectively.
 - (a) Can you solve the imaging aspects of this problem with a single sensor and camera? If yes, specify the illumination wavelength band and the type of camera needed. ("Type" means the band of the electromagnetic spectrum to which the camera is most sensitive (ie. Infrared))
 - (b) If no, what type of illumination sources and corresponding imaging sensors would you recommend? Specify the light sources and cameras as requested in part (a). (Use the minimum number of illumination sources and cameras needed to solve the problem)



Fundamentals

- 2.06: An automobile manufacturer is automating the placement of certain components on the bumpers of a limited-edition line of sports cars. The components are color coordinated, so the robots need to know the color of each car (only: green, blue, red and white) in order to select the appropriate bumper component. You are hired to propose a solution based on imaging.
 - How would you solve the problem of automatically determining the color of each car (keeping in mind that cost is the most important consideration in your choice of components)?



Fundamentals

- 2.09: A common measure of transmission for digital data is the *baud rate*, defined as the number of bits transmitted per second. Generally, transmission is accomplished in packets consisting of a start bit, a byte (8 bits) of information, and a stop bit. Using this fact answer the following:
 - (a) How many minutes would it take to transmit a 1024x1024 image with a 256 intensity levels using a 56K baud modem?
 - (b) What would the time be at 3000K baud? (medium speed of a phone DSL)

Fundamentals

- 2.20: Let $g(x,y)$ denote a corrupted image formed by the addition of noise to a noiseless image $f(x,y)$, that is:

$$g(x, y) = f(x, y) + \eta(x, y)$$

Where the assumption is that at every pair of coordinates (x,y) the noise is uncorrelated and has zero average value. With:

$$\bar{g}(x, y) = \frac{1}{K} \sum_{i=1}^K g_i(x, y)$$

□ (a) Prove the validity of $E[\bar{g}(x, y)] = f(x, y)$

□ (b) Prove the validity of $\sigma^2_{\bar{g}(x,y)} = \frac{1}{K} \sigma^2_{\eta(x,y)}$

Uncorrelated random variables z_i, z_j have their covariance $E[(z_i - m_i)(z_j - m_j)] = 0$

Hints: expected value of a sum is the sum of the expected values

Fundamentals

- 2.22: Image subtraction is used often in industrial applications for detecting missing components in product assembly. The approach is to store a “golden” image that corresponds to a correct assembly. This image is then subtracted from incoming images of the same product. Ideally, the differences would be zero if the new products are assembled correctly. Difference images for products with missing components would be nonzero in the area where they differ from the golden image.

□ **What conditions have to be met in practice for this method to work?**

Intensity transformation

- 3.1: Give a single intensity transformation function for spreading the intensities of an image so the lowest intensity is 0 and the highest is L-1.
- 3.5: What effect would setting to zero the lower-order bit planes have on the histogram of an image in general?
- 3.6: Explain why the discrete histogram equalization technique does not, in general, yield a flat histogram.
- 3.19:
 - (a) Develop a procedure for computing the median of an nxn neighborhood.
 - (b) Propose a technique for updating the median as the center of the neighborhood is moved from pixel to pixel.
- 3.28: Show that subtracting the Laplacian from an image is proportional to unsharp masking (use Laplacian with a negative central value).

Filtering

- Prove that the convolution of a digital image by a filter having the sum of its elements equal to zero, is a zero mean image.
- Prove the validity of $f(x, y) \Leftrightarrow F(u, v)$
$$f(x - x_0, y - y_0) \Leftrightarrow F(u, v) \cdot e^{-2j\pi(ux_0/M + vy_0/N)}$$
- Prove the validity of $f(r, \theta + \theta_0) \Leftrightarrow F(\omega, \varphi + \theta_0)$
where $x = r \cos \theta; y = r \sin \theta; u = \omega \cos \varphi; v = \omega \sin \varphi$

Filtering

- 4.27: Consider a 3x3 spatial mask that averages the four closest neighbors of a point (x,y), but excludes the point itself from the average.
 - (a) Find the equivalent filter, $H(u,v)$, in the frequency domain
 - (b) Show that your result is a lowpass filter.
- 4.31: A continuous Gaussian lowpass filter in the continuous frequency domain has the transfer function
$$H(\mu, \nu) = A e^{-\left(\mu^2 + \nu^2\right) / 2\sigma^2}$$
 - Show that the corresponding filter in the spatial domain is
$$h(t, z) = 2A\pi\sigma^2 e^{-2\pi^2\sigma^2(t^2 + z^2)}$$