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Magnetic resonance image reconstruction using analytic image representation

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Abstract

Partial *k*-space acquisition is a subject of great interest in magnetic resonance imaging (MRI). This type of acquisition usually consists of, in addition to the positive half of *k*-space, acquiring a number of negative frequencies beyond the *k*-space center, thus requiring an increase of acquisition time. Furthermore, the quality of the reconstructed image depends on the number of negative frequencies acquired. To date it has not been demonstrated that the non-acquisition of negative frequencies of *k*-space can produce useful reconstructed images. We propose a novel approach for reconstructing MR images from partial *k*-space. It is based on the notion of analytic image and presents the particularity of using only exactly half of *k*-space without involving any negative frequencies or phase estimation. Furthermore, it can be implemented at no cost. The benefits are illustrated by experiments on both simulated and real human brain image.

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1. Introduction

Magnetic resonance imaging (MRI) plays an increasingly important role in medicine and biology. The raw output of a magnetic resonance (MR) imager, called the *k*-space data, belongs to the Fourier domain. A reconstruction algorithm is needed to obtain the spatial domain image that is ultimately visualized. Let N be a positive even integer. Given an $N \times N$ image X in the spatial domain,

$$X(i,j),\ i \in \{0,\dots,N-1\}$$

we denote by $\underline{X}$ the *k*-space representation of X:

$$\underline{X}(m,n),\ m,n \in \{-N/2,\dots,N/2-1\}.$$

Of course, $\underline{X} = \mathrm{DFT}(X)$, where DFT is the discrete Fourier transform operator. In conventional MRI, data acquisition covers only part of the *k*-space in order to reduce acquisition time, which is particularly important for imaging organs in motion and for patients' comfort as well. This is known as partial Fourier imaging. Typically, the *k*-space is truncated in the phase-encoded direction, says the vertical direction, which leads to data of the form

$$\underline{X}_{m_1,m_2}(m,n) = \begin{cases} \underline{X}(m,n) & \text{if } m \in \{m_1,\dots,m_2\}, \\ \text{undefined} & \text{otherwise} \end{cases} \tag{1}$$

where $m_1, m_2 \in \{-N/2,\dots,N/2-1\}$ and $0 < m_2 - m_1 < N-1$ (m_1 and m_2 are called the lower and upper truncation frequencies, respectively). In practice, m_2 is usually set to $N/2-1$, m_1 is always a negative integer [1–7]. It has not yet been demonstrated that the negative frequencies ranging from m_1 to -1 can be safely ignored (see Ref. [7] for a more detailed description of conventional MRI reconstruction techniques).

In this paper, we propose a novel method for reconstructing MR images. The approach relies on the notion of analytic image and allows to set the lower truncation frequency to zero. More specifically, only half the *k*-space is needed to be sampled, which further reduces acquisition

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time while maintaining good reconstruction quality, in comparison with the conventional Hermitian Conjugation (HC) technique.

2. Method

2.1. Notion of analytic image

The notion of analytic signal is well-known in 1-D signal processing. An analytic signal z is a complex signal satisfying one of the following two equivalent properties:

(1)

$$\mathrm{Im}(z) = H(\mathrm{Re}(z)),$$

(2)

$$F(z)(u) = \begin{cases} 2F(\mathrm{Re}(z))(u) & \text{if } u \geqslant, \\ 0 & \text{otherwise} \end{cases}$$

where H denotes the Hilbert transform operator, and F the Fourier transform (FT) operator. Let $f(t)$ be a real signal. The analytic signal $z(t)$ corresponding to $f(t)$ is simply

$$z(t) = f(t) + H(f)(t) \tag{2}$$

where

$$H(f)(t) = \frac{1}{\pi}\,\mathrm{p.v.}\int_{-\infty}^{+\infty} \frac{f(\tau)}{t-\tau}\,\mathrm{d}\tau \tag{3}$$

(p.v. stands for the Cauchy principal value).

The notion of analytic signal can be extended to the 2-D case [8]. Let $f(x,y)$ be a 2-D signal. We say that $z(x,y)$ is an analytic image associated with $f(x,y)$ if it can be written under the form

$$z(x,y) = f(x,y) + jH_2(f)(x,y) \tag{4}$$

where the operator H_2, which we call a 2-D Hilbert transform operator by analogy with the 1-D case, reduces to the 2-D convolution of f by some kernel function h. Equivalently, in the spectral domain,

$$F_2(z)(u,v) = 2\rho(u,v)F_2(f)(u,v) \tag{5}$$

where F_2 is the 2-D FT operator and

$$\rho(u,v) = \tfrac{1}{2}(1 + jF_2(h)(u,v)). \tag{6}$$

2.2. MR image reconstruction

The partial k-space data resulting from a typical acquisition scheme is composed of all non-negative frequencies together with a few negative frequencies in the phase-encoded direction, as depicted in Fig. 1a. Our goal is to maintain good image reconstruction quality in the extreme case where only non-negative frequencies are considered, as illustrated in Fig. 1b. Define the function ρ in Eq. (6) by

$$\rho(u,v) = \tfrac{1}{2}(1 + \mathrm{sgn}(u)). \tag{7}$$

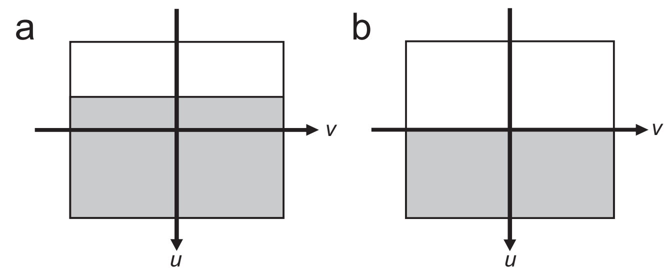

Fig. 1. (a) Typical partial k-space sampling coverage and (b) spectral support of the analytic image representation considered.

Then, the associated kernel function h in Eq. (6) is given by

$$F_2(h)(u,v) = -j\,\mathrm{sgn}(u) \tag{8}$$

or, equivalently,

$$h(x,y) = \frac{\delta(y)}{\pi}\,\mathrm{p.v.}\frac{1}{x} \tag{9}$$

where δ is the delta function. The corresponding analytic image is

$$z(x,y) = f(x,y) + \frac{j}{\pi}\,\mathrm{p.v.}\int_{-\infty}^{+\infty} \frac{f(\tau,y)}{x-\tau}\,\mathrm{d}\tau \tag{10}$$

and the image f in the spatial domain can therefore be reconstructed by taking the real part of z. More precisely,

$$f = \mathrm{Re}(F_2^{-1}(\varphi))$$
$$\text{with} \quad \varphi(u,v) = (1 + \mathrm{sgn}(u))F_2(f)(u,v). \tag{11}$$

Applying this result to our problems gives the following estimate for the MR image in the spatial domain:

$$\hat{X} = \mathrm{Re}(\mathrm{DFT}^{-1}(\varphi))$$
$$\text{with} \quad \varphi(m,n) = (1 + \mathrm{sgn}(m))\underline{X}_{m_1,m_2}(m,n) \tag{12}$$

where $\underline{X}_{m_1,m_2}$ is defined by Eq. (1).

3. Results

In our experiments, we applied the proposed reconstruction method (12) to five k-space data sets. Two of them are simulated data, to which was introduced a slowly varying phase to generate a complex image whose k-space does not exhibit Hermitian symmetry. The other three data sets are real data on a physical phantom and human head. The raw k-space data corresponding to the real MR data were acquired using a GE 1.5 T MRI System. The reconstruction quality was assessed both using qualitative (visual comparison) and quantitative root mean square (RMS) error criteria. The proposed method was also compared with the HC reconstruction technique that consists in filling in missing data using HC, blending the acquired data with the symmetry data at the transition, computing the inverse DFT of the thus obtained k-space, and taking the magnitude of the result.

Fig. 2 shows the half k-space reconstructions of physical phantom images. The complete k-space data of the

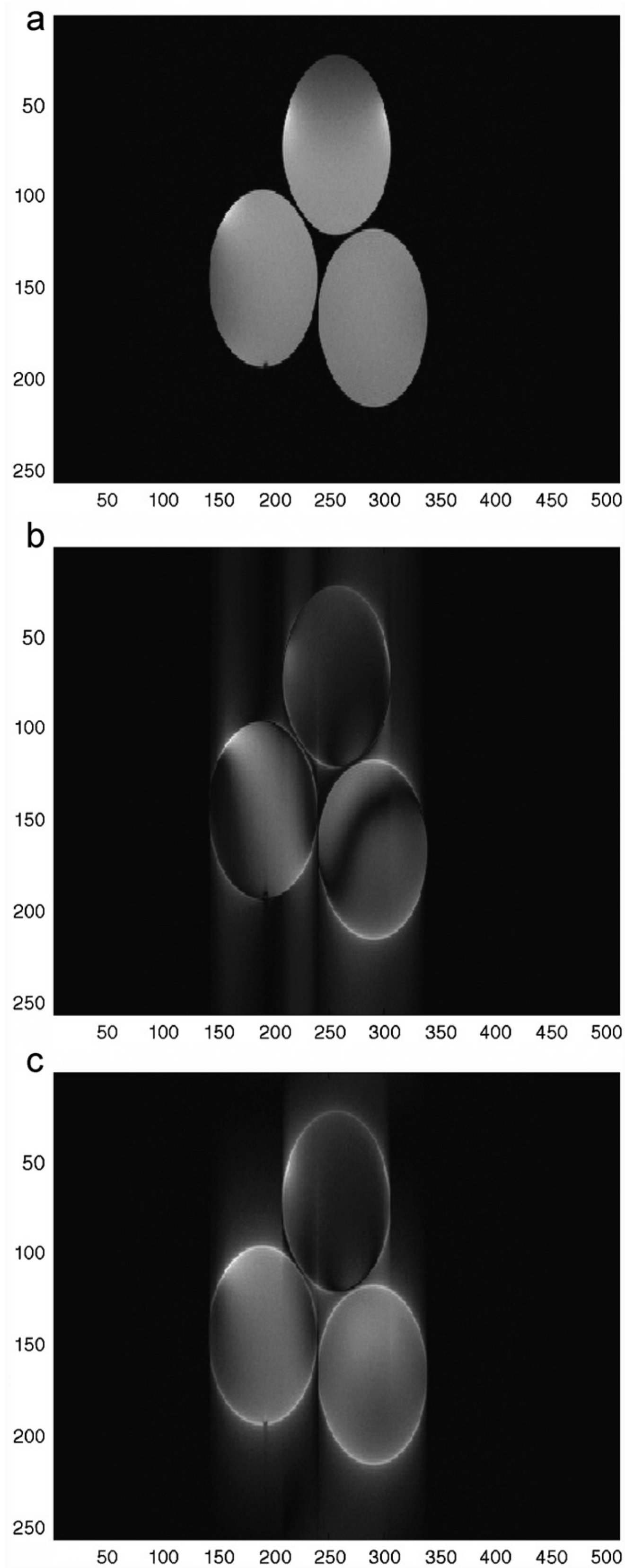

Fig. 2. Real phantom image reconstructed from exactly half k-space: (a) original image reconstructed from the complete k-space and (b) and (c) images reconstructed using, respectively, the HC and the proposed techniques.

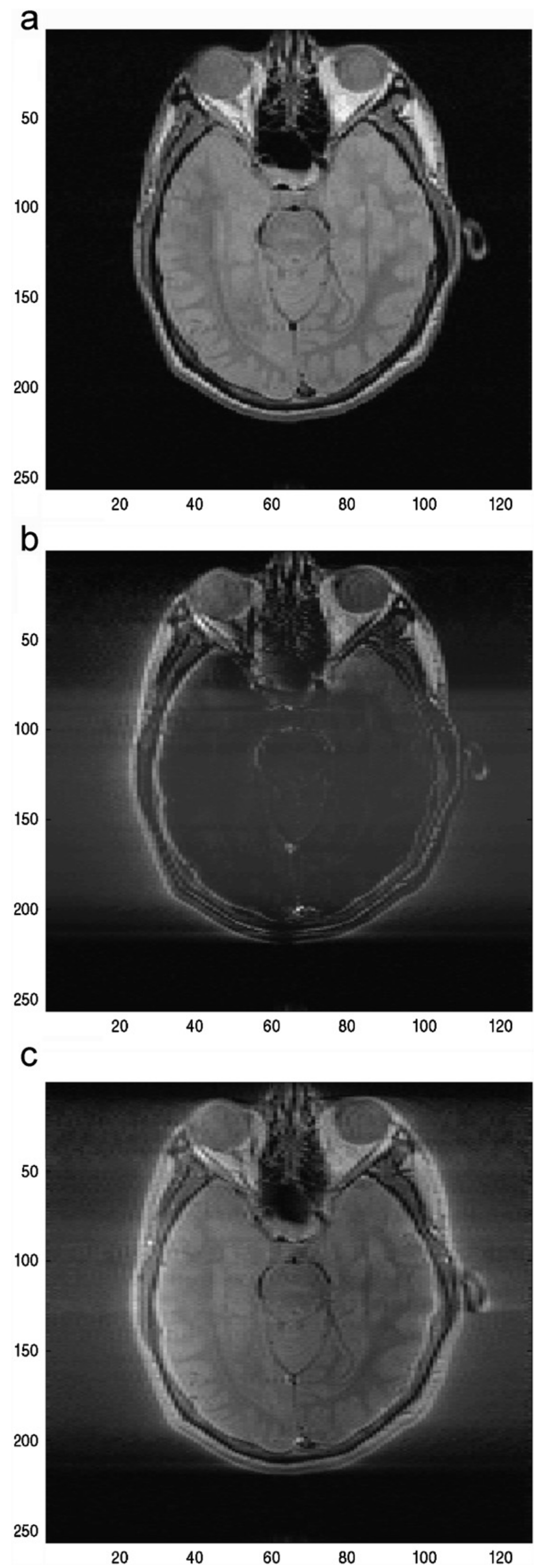

Fig. 3. Real human brain image reconstructed from exactly half k-space: (a) original image reconstructed from the complete k-space and (b) and (c) images reconstructed using, respectively, the HC and the proposed techniques.

phantom is of size 256×512. Fig. 2a gives the image reconstructed from the complete k-space data. It is considered as the best standard image one can obtain. The images reconstructed by considering only the non-negative frequency lines in the phase-encoded direction (as depicted in Fig. 1b) are shown in Fig. 2b (using the HC technique) and c (using the proposed method). In both cases, it was not possible to reconstruct exactly the original image using only one half k-space. Nevertheless, the proposed method has yielded significantly better results, by exhibiting much few shadow effects ($\text{RMS} = 0.00026$ for the HC, and $\text{RMS} = 0.00023$ for the proposed method).

Fig. 3 gives the reconstruction results in the case of the real human brain data. Fig. 3a shows the image reconstructed from the complete k-space data (128×256). Figs. 3b and c correspond to the reconstructions from the exactly half the k-space, using respectively the conventional HC (Fig. 3b) and proposed (Fig. 3c) methods. Again, as in the case of the phantom image, the quality of the image reconstructed using the proposed method is much closer to that of the original image, whereas, with the HC technique, the reconstructed image is not useful at all, because of loss of image details and reconstruction artefacts ($\text{RMS} = 0.0018$ for the HC, and $\text{RMS} = 0.0014$ for the proposed method).

4. Conclusion

We have proposed a magnetic resonance image reconstruction method based on the notion of analytic image. When using only one and exactly one half k-space, without involving any negative frequencies or phase estimation, the proposed method yields substantially better reconstruction results than the conventional HC technique. Besides, the idea can be implemented at no cost. All these would open interesting perspectives to applications involving the use of half k-space.

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